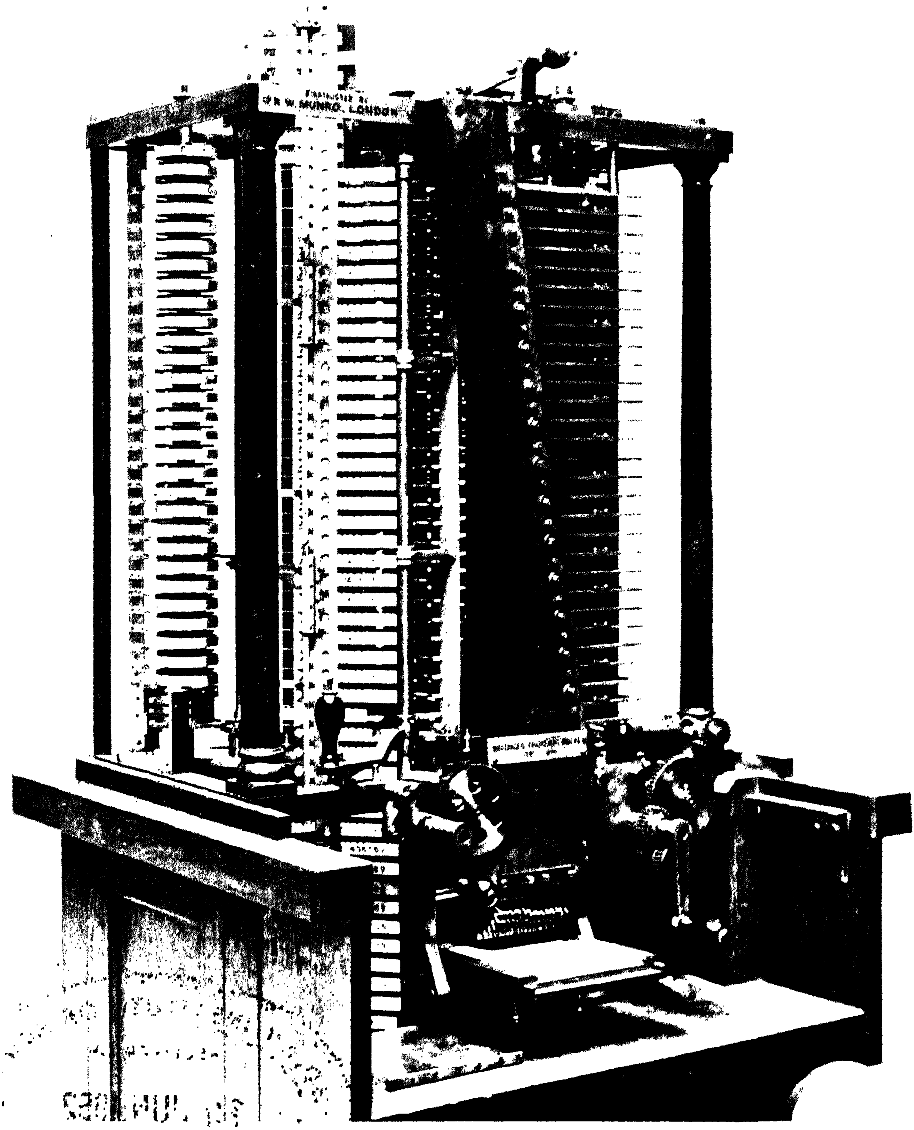

THE BABBAGE ANALYTICAL ENGINE

One of the most interesting episodes in the history of the construction of calculating machines is found in the invention of an analytical engine by Charles Babbage (1792-1871). This ambitious project to perfect a machine to evaluate any algebraic formula for any given values of the variables was never completed. Major-General H. P. Babbage, the youngest son of the inventor, constructed (1880 to 1910) a part of the engine known as the "Mill" and exhibited specimens of its work. It is this machine which is shown in the above picture. For further reference see: *Monthly Notices of the Royal Astronomical Society*, vol. 70 (1910), pp. 517-520.

TABLES OF THE HIGHER MATHEMATICAL FUNCTIONS

COMPUTED AND COMPILED

UNDER THE DIRECTION OF

HAROLD T. DAVIS

INDIANA UNIVERSITY AND THE COWLES COMMISSION
FOR RESEARCH IN ECONOMICS

WITH THE CO-OPERATION OF:

MURIEL E. ADAMS : JOSEPH T. ALTENHOF : CATHERINE BENNETT
JAMES A. COOLEY : FRANK K. EDMONDSON
EDWARD E. EDWARDS : EDWIN L. GODFREY : ARIEL M. GREEN
DR. H. E. H. GREENLEAF : ELMER L. HAGERTY
MABEL E. INCO : ESTHER KANTZ : LUCY C. KANTZ
DR. WILLIAM J. KIRKHAM : DR. VORIS V. LATSHAW
ANNA M. LESCISIN : RALPH E. MCCLAIN : JAMES S. MENTZER
LLOYD G. MITTON : EDWARD B. MORRIS
PAUL W. OVERMAN : DR. IRENE PRICE : ELZA W. SCOTTEN
DANIEL SCHIMMELPFENNIG : MARION B. SHELLY
DOROTHY J. STEPHAN : HARRY F. TAYLOR : RICHARD E. THOMPSON
RAYMOND J. TERKHORN : MILO WELLS
JOHN R. WETNIGHT : KATHRYN E. WITHERS

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Euripides (Hippolytus, 615).

PREFACE

Of the making of tables there is no end! In this volume the reader will find the continuation of the project initiated with the publication in 1933 of volume 1 of *Tables of the Higher Mathematical Functions*. In that work there appeared an historical introduction, a summary of mathematical methods used in table computation, an account of interpolation with auxiliary tables, a bibliography, and twelve tables relating to the Gamma and Psi Functions. The present work includes 37 additional tables covering the Polygamma functions [derivatives of $\log \Gamma(x)$], the Bernoulli polynomials and numbers, the Euler polynomials and numbers, and certain functions useful in unweighted polynomial approximation. These latter are closely related to the Legendre polynomials, being essentially the analogue of the Legendre polynomials for finite summation. In this work a supplementary bibliography is included, which contains titles either overlooked in the compilation of the original bibliography or which have appeared since the publication of Volume 1.

The director regrets the overlapping of the present tables of the polygamma functions with the admirable tables of these functions published in Volume 1 of the computations issued by the British Association for the Advancement of Science. The tables included in the present volume were far advanced when the work of the British Association appeared. Tables of the reciprocals of the first five powers of the first thousand integers had been computed to fifteen or more significant figures and these made the compilation of tables of the polygamma functions relatively easy.

In the publication of a work on mathematical tables the greatest struggle must be waged against error. One is perpetually amazed at the many ingresses available to this incubus of the computer. This is particularly true in a project which embraces the computation of so many diverse functions, computations which are being made simultaneously by different computers. First the basic formulas must be carefully prepared and auxiliary tables computed and checked. After the tabular values have been found, these must be checked by some device independent of the one employed in the original calculation. Duplicate computation is resorted to only

when other methods appear too laborious or impractical as in the case of tabular values in the neighborhood of the poles of an analytic function. Differencing, the best check, although a tedious process with available statistical machinery, is employed in most instances. Finally, after the tables have been completed and checked, they must be transferred to copy sheets for the printer and then proofread from the original note books. This latter is, perhaps, the most exacting task of all and may be the most fruitful source of errors unless it is warily undertaken.

The field of exact computation has been exceedingly fortunate in enlisting the devoted interest of Dr. L. J. Comrie, Superintendent of the British Almanac Office, who states that he has "the vicious (?) habit of examining published tables closely, in order to show whether or not the author is to be trusted." While the detection of errors in one's work is not pleasant, there is no denying the supreme importance of this quest and computational science is highly indebted to Dr. Comrie for his self-imposed and thankless task. Dr. Comrie went to the trouble of computing 2000 values in a scrutiny of volume 1 of this work. He selected $\Gamma(x)$, as given in Table 2, and $\Psi(x)$, as given in Table 8. The examination revealed 10-th decimal errors in the values of $\Gamma(x)$ ranging in value from —2 to +2 units, which was according to expectation since the table was computed by taking antilogarithms (using Vega) of the 12-figure values given by Legendre. One discrepancy of —3 units was attributed to a computational error and another of +4 units was due to an error in Legendre's table. The examination of the second table revealed three errors of 1 unit in the 10-th place and three places where there was a difference of 0.51 units of the last decimal. Unfortunately a more serious error was detected in the interval from 1.431 to 1.439 inclusive, where the sixth decimal is too small by 1. This error is to be attributed to the method of subtabulation (page 87, vol. 1), where the computer failed to detect the discrepancy between the original and the built up value of the function at the end of the interval. A proof-reading error is also reported in the 6-th decimal of the logarithm at 1.791.

From this it will be obvious to the reader that the elimination of computational error from a large set of tables is a task of considerable magnitude. Therefore, in the construction of tables, it is incumbent upon the author to take every precaution possible, and,

in addition, readers who discover errors should consider it a duty to report them. The director of this work wishes to express his further indebtedness to Dr. Comrie for his help in securing the picture of J. W. L. Glaisher which appears in this volume.

The director wishes also to express his great debt to the faithful laboratory assistants whose names appear on the title page of this work. Specific credit for the work which they have done is given elsewhere in the book. Without their enthusiastic help, freely given in the long hours of arduous computation and check, this work could never have reached its present state. In addition to these members of the laboratory staff, the director also wishes to express his appreciation to his sister, Miss Marjorie Davis, and to Miss Margaret McNeill, Miss Martha Belschner, Miss Emma Manning, Forrest Danson and Herbert Newhall, who assisted in the proof reading.

A debt of gratitude is also due to President W. L. Bryan and to Dean Fernandus Payne of Indiana University who have supported the project since its inception. The latter has stimulated the flagging spirits of the director at times when they needed encouragement. Valuable suggestions have also been received from time to time from Professor K. P. Williams of the Mathematics Department of Indiana University.

As was stated in the preface to volume 1, the printer plays an important rôle in the final appearance of the completed work. The Dentan Printing Company has been unusually patient in meeting all suggestions made by the director and has exerted every effort to keep the book free from error. Special mention should also be made of D. L. Taggart, who for the most part put the manuscript into type, and to F. L. Lewis who superintended the corrections of the proof and the final make-up of the forms.

Indiana University, 1935 H. T. Davis.

TABLES OF THE HIGHER MATHEMATICAL FUNCTIONS

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THE
POLYGAMMA FUNCTIONS

THE POLYGAMMA FUNCTIONS.

1. Definition. We shall mean by the *polygamma functions*, the psi function and its derivatives. The *digamma* function is $\Psi(x)$* itself. The *trigamma* function is $\Psi'(x)$, the *tetragamma* function $\Psi''(x)$, the *pentagamma* function $\Psi^{(3)}(x)$, the *hexagamma* function $\Psi^{(4)}(x)$, etc. The names *digamma* and *trigamma* originated in a paper by Eleanor Pairman. (See Bibliography, vol 1), who, however, defines $\Psi(x)$ as the derivative of $\log x! = \log \Gamma(x+1)$, i. e., $\Psi(x) = d \log x!/dx$. This same definition has been adopted in the tables of the British Association [See Bibliography, vol. 1: British Association, (1)].

2. Properties. We shall note first the fundamental difference equations:

$$\Psi^{(n)}(x+1) = \Psi^{(n)}(x) + (-1)^n n!/x^{n+1} , \qquad (2.1)$$

$$\Psi^{(n)}(1-x) + (-1)^{n+1} \Psi^{(n)}(x) = (-1)^n \pi \frac{d^n}{dx^n} \cot \pi x . \qquad (2.2)$$

For the four functions which we shall specifically treat in this volume, the last equation becomes:

$$\Psi'(1-x) + \Psi'(x) = \pi^2 \csc^2 \pi x ,$$
$$\Psi''(1-x) - \Psi''(x) = 2\pi^3 \csc^2 \pi x \cot \pi x ,$$
$$\Psi^{(3)}(1-x) + \Psi^{(3)}(x) = 2\pi^4 \csc^2 \pi x (3 \csc^2 \pi x - 2) ,$$
$$\Psi^{(4)}(1-x) - \Psi^{(4)}(x) = 8\pi^5 \csc^2 \pi x \cot \pi x (3 \csc^2 \pi x - 1) .$$

If we employ the abbreviation,

$$S_r = 1/1^r + 1/2^r + 1/3^r + 1/4^r + \qquad S_1 = C ,†$$

*The reader will find a description of $\Psi(x)$ and tables in vol. I of this work.

†For numerical values see p. 280, vol. 1 or Table 33 of this volume.

then we have,

$$\Psi^{(m)}(1) = (-1)^{m+1}\, m!\, S_{m+1} \;, \quad \Psi^{(2m-1)}(1) = (2\pi)^{2m} B_m/4m \;;$$

$$\Psi^{(m)}(n+1) = (-1)^{m}\, m!\,[-S_{m+1} + 1 + 1/2^{m+1} + 1/3^{m+1} + \cdots + 1/n^{m+1}] \;.$$

$$\Psi^{(n)}(1/2) = (-1)^{n+1}\, n!\,(2^{n+1}-1)S_{n+1}, \quad n > 0 \;,$$

$$\Psi^{(2n-1)}(1/2) = (2\pi)^{2n}\,(2^{2n}-1)B_n/4n \;,$$

$$\Psi'(1/2) = \pi^2/2 \;, \quad \Psi^{(3)}(1/2) = \pi^4 \;, \quad \Psi^{(5)}(1/2) = 8\pi^6 \;.$$

It is easily seen that none of the polygamma functions except the psi function has a zero on the positive real axis. The polygamma functions of even order, $\Psi^{(2m)}(x)$, $m > 0$, have an infinite number of zeros on the negative real axis in the neighborhood of $-n + 1/2$, n an integer. If we designate these zeros by x_n we have the following asymptotic expansions. For the tetragamma function,

$$x_n \backsim -n + 1/2 + 1/2\pi^4 n^2 \;,$$

and for the hexagamma function,

$$x_n \backsim -n + 1/2 + 3/8\pi^6 n^4 \;.$$

These results are obtained by an easy generalization of an argument employed by C. Hermite in obtaining the asymptotic expansion of the roots of the psi function.*

For multiple values of the argument the following formula holds:

$$\Psi^{(m)}(nx) = (1/n^{m+1})\,[\Psi^{(m)}(x) + \Psi^{(m)}(x+1/n) + \Psi^{(m)}(x+2/n) + \cdots + \Psi^{(m)}(x+1-1/n)] \;, \qquad m > 0 \;.$$

A relationship of similar kind is furnished by the equation,†

$$-\tfrac{1}{2}\,\beta^{(m)}(nx) = (1/n^{m+1})\,[\Psi^{(m)}(x) - \Psi^{(m)}(x+1/n) + \Psi^{(m)}(x+2/n) - \Psi^{(m)}(x+3/n) + \cdots + (-1)^{n-1}\,\Psi^{(m)}(x+1-1/n)] \;,$$

*Sur l'intégrale Eulérienne de seconde espèce. *Journal für Mathematik*, vol. 90 (1881), pp. 332-338. See, in particular, the *Postscriptum*.

†The function $\beta(x)$ is designated by $g(x)$ by Nörlund.

where n is an even number and $\beta^{(m)}(x)$ is the function,

$$\beta^{(m)}(x) = (-1)^m\, 2(m!) \sum_{r=0}^{\infty} (-1)^r/(x+r)^{m+1} .$$

If x is replaced by $x/2$ and n by 2, the above identity reduces to the following:

$$2^m\, \beta^{(m)}(x) = \Psi^{(m)}\left(\frac{x+1}{2}\right) - \Psi^{(m)}\left(\frac{x}{2}\right) .$$

We also note the fundamental expansion,

$$\Psi^{(n)}(x) = (-1)^{n+1}\, n! \sum_{r=0}^{\infty} \frac{1}{(x+r)^{n+1}} , \quad n = 1, 2, \cdots .$$

3. Integral Representations. The polygamma functions are variously represented by means of definite integrals, $n > 0$,

$$\Psi^{(n)}(x) = \int_0^1 \{\log^n t\, t^{x-1}/(t-1)\}dt, \qquad R(x) > 0;$$

$$\Psi^{(n)}(x) = \int_0^1 t^{x-1} \log^{n-1} t \log[t/(1-t)]\,[x \log t + n]dt, \; R(x) > 0;$$

$$\Psi^{(n)}(x) = (-1)^{n+1} \int_0^\infty \log^n(1+t)\, \frac{(1+t)^{-x}}{t}\, dt , \quad R(x) > 0;$$

$$\Psi^{(n)}(x) - \Psi^{(n)}(y) = \int_0^1 \log^n t\,[(t^{y-1} - t^{x-1})/(1-t)]\, dt , \quad R(x), \mathrm{R}(y) > 0;$$

$$\Psi^{(n)}(x) - \Psi^{(n)}(y) = (-1)^n \int_0^\infty \log^n(1+t)\,[(1+t)^{-y} - (1+t)^{-x}]\,dt/t , \quad R(x), R(y) > 0 .$$

4. Development in Power Series. The polygamma functions have the following development in power series about the origin:

$$\Psi^{(n)}(x+1) = (-1)^{n+1}[n!\; S_{n+1} - (n+1)!\; S_{n+2}\, x/1! + (n+2)!\; S_{n+3}\, x^2/2! - \cdots\cdots] ,$$

for $|x| < 1$.

We also note the following closely related expansion:

$$\Psi^{(n)}(x+1) =$$

$$(-1)^n\, n!/2x^{n+1} - \frac{1}{2}\frac{d^n}{dx^n}[\pi \cot \pi x - 1/(1-x^2)] + \Phi^{(n)}(x) \quad ,$$

where we abbreviate,

$$\Phi^{(2n-1)}(x) = (2n)!(1-S_{2n+1})x + (2n+2)!(1-S_{2n+3})x^3/3! + \cdots$$

$$\Phi^{(2n)}(x) = (2n)!(1-S_{2n+1}) + (2n+2)!(1-S_{2n+3})x^2/2! + \cdots$$

5. *Derivatives of the Gamma Function.* It will be noted that the derivatives of the gamma function, $\Gamma^{(n)}(x)$, can be computed from the values of the polygamma functions and the gamma function itself. Formulas for the first five derivatives are given below as follows:

$$\Gamma^{(1)}(x) = \Psi(x)\,\Gamma(x) \quad ,$$

$$\Gamma^{(2)}(x) = [\Psi^{(1)}(x) + \Psi^2(x)]\,\Gamma(x) \quad ,$$

$$\Gamma^{(3)}(x) = [\Psi^{(2)}(x) + 3\,\Psi(x)\,\Psi^{(1)}(x) + \Psi^3(x)]\,\Gamma(x) \quad ,$$

$$\Gamma^{(4)}(x) = [\Psi^{(3)}(x) + 4\,\Psi(x)\,\Psi^{(2)}(x) + 6\,\Psi^{(1)}(x)\,\Psi^2(x) + 3\{\Psi^{(1)}(x)\}^2 + \Psi^4(x)]\,\Gamma(x) \quad ,$$

$$\Gamma^{(5)}(x) = [\Psi^{(4)}(x) + 5\,\Psi(x)\,\Psi^{(3)}(x) + 10\,\Psi^2(x)\,\Psi^{(2)}(x) + 15\,\Psi(x)\{\Psi^{(1)}(x)\}^2 + 10\,\Psi^{(1)}(x)\,\Psi^3(x) + 10\,\Psi^{(1)}(x)\,\Psi^{(2)}(x) + \Psi^5(x)]\,\Gamma(x) \quad .$$

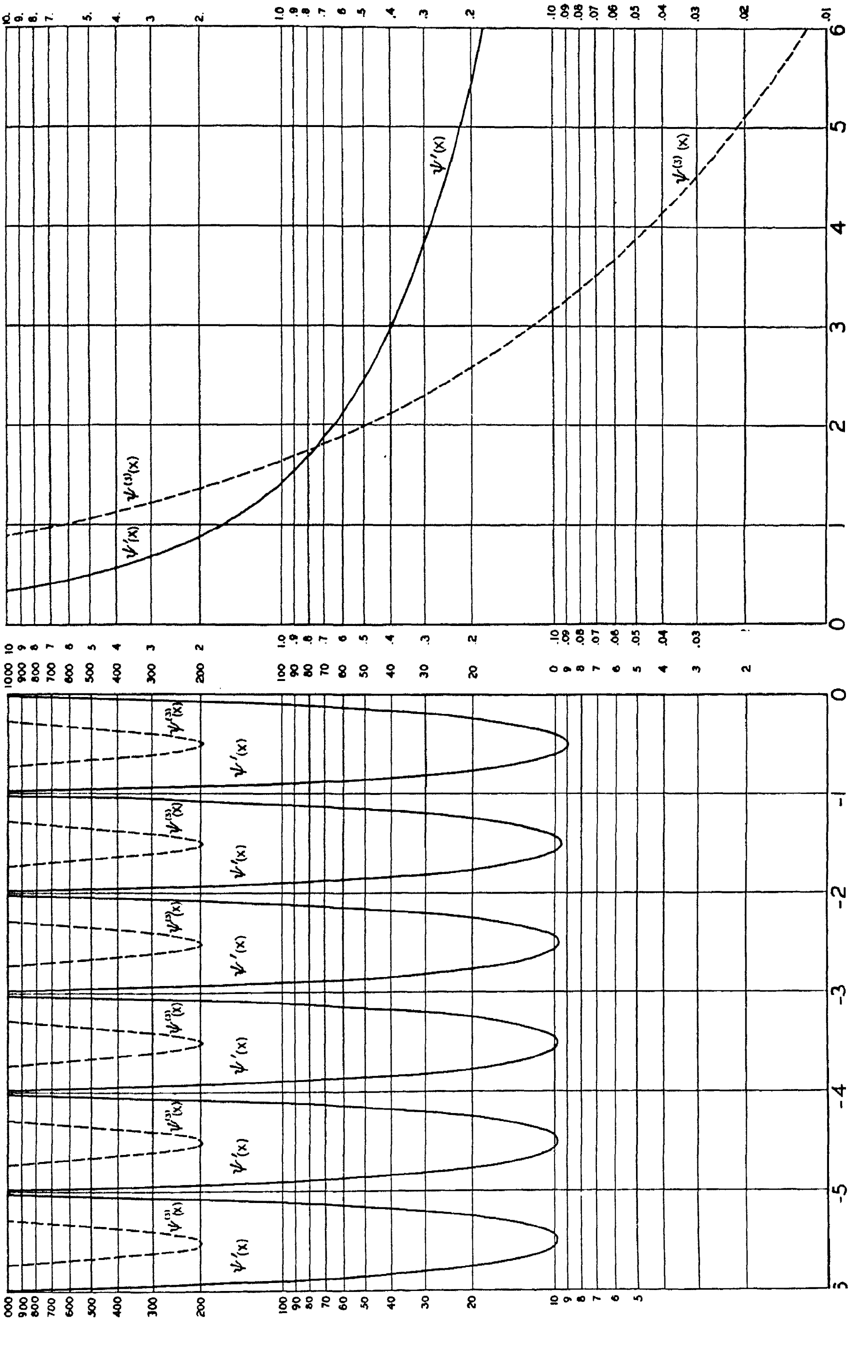
6. *Asymptotic Expansions.* The polygamma functions have the following asymptotic expansion:

$$\frac{d^n}{dx^n}\Psi(x) \sim (-1)^{n-1}\Big[(n-1)!/x^n + n!/2x^{n+1} + \sum_{m=1}^{\infty}(-1)^{m-1}B_m(2m+n-1)!/(2m)!\cdot x^{2m+n}\Big] \quad , \quad n > 0 \; .$$

The remainder after m terms of the summation can be shown to equal

$$R_m = [(2m+n)!/(2m)!](-1)^n \int_0^\infty [B_{2m}^*(t)/(x+t)^{2m+n+1}]\,dt \quad ,$$

where $B_m^*(x)$ designates the function of unit period which coin-



THE TRIGAMMA AND PENTAGAMMA FUNCTIONS

cides with $B_n(x)$, the Bernoulli polynomial of mth degree, in the interval $0 \leqq x < 1$.*

The general formula leads to the following explicit expansions:

$$
\begin{aligned}
d\,\Psi(x)/dx &\backsim 1/x + 1/2\cdot x^2 + B_1/x^3 - B_2/x^5 + B_3/x^7 \\
&\qquad - B_4/x^9 + \cdots, \\
&\backsim 1/x + 1/2x^2 + 1/6\cdot x^3 - 1/30\cdot x^5 + 1/42\cdot x^7 \\
&\qquad - 1/30\cdot x^9 + \cdots ; \\
d^2\,\Psi(x)/dx^2 &\backsim - 1/x^2 - 1/x^3 - 3\,B_1/x^4 + 5\,B_2/x^6 - 7\,B_3/x^8 \\
&\qquad + \cdots, \\
&\backsim - 1/x^2 - 1/x^3 - 1/2\cdot x^4 + 1/6\,x^6 - 1/6\,x^8 \\
&\qquad + 3/10\,x^{10} - 5/6\,x^{12} + \cdots ; \\
d^3\,\Psi(x)/dx^3 &\backsim 2/x^3 + 3!/2\cdot x^4 + 4!\,\mathrm{B}_1/2!\cdot x^5 - 6!\,\mathrm{B}_2/4!\cdot x^7 \\
&\qquad + 8!\,B_3/6!\cdot x^9 - \cdots, \\
&\backsim 2/x^3 + 3/x^4 + 2/x^5 - 1/x^7 + 4/3\cdot x^9 - 3/x^{11} \\
&\qquad + 10/x^{13} - \cdots ; \\
d^4\,\Psi(x)/dx^4 &\backsim - 3!/x^4 - 4!/2\cdot x^5 - 5!\,B_1/2!\cdot x^6 + 7!\,B_2/4!\cdot x^8 \\
&\qquad - 9!\,B_3/6!\,x^{10} + 11!\,B_4/8!\,x^{12} - \cdots, \\
&\backsim - 6/x^4 - 12/x^5 - 10/x^6 + 7/x^8 - 12/x^{10} \\
&\qquad + 33/x^{12} - 130/x^{14} + \cdots .
\end{aligned}
$$

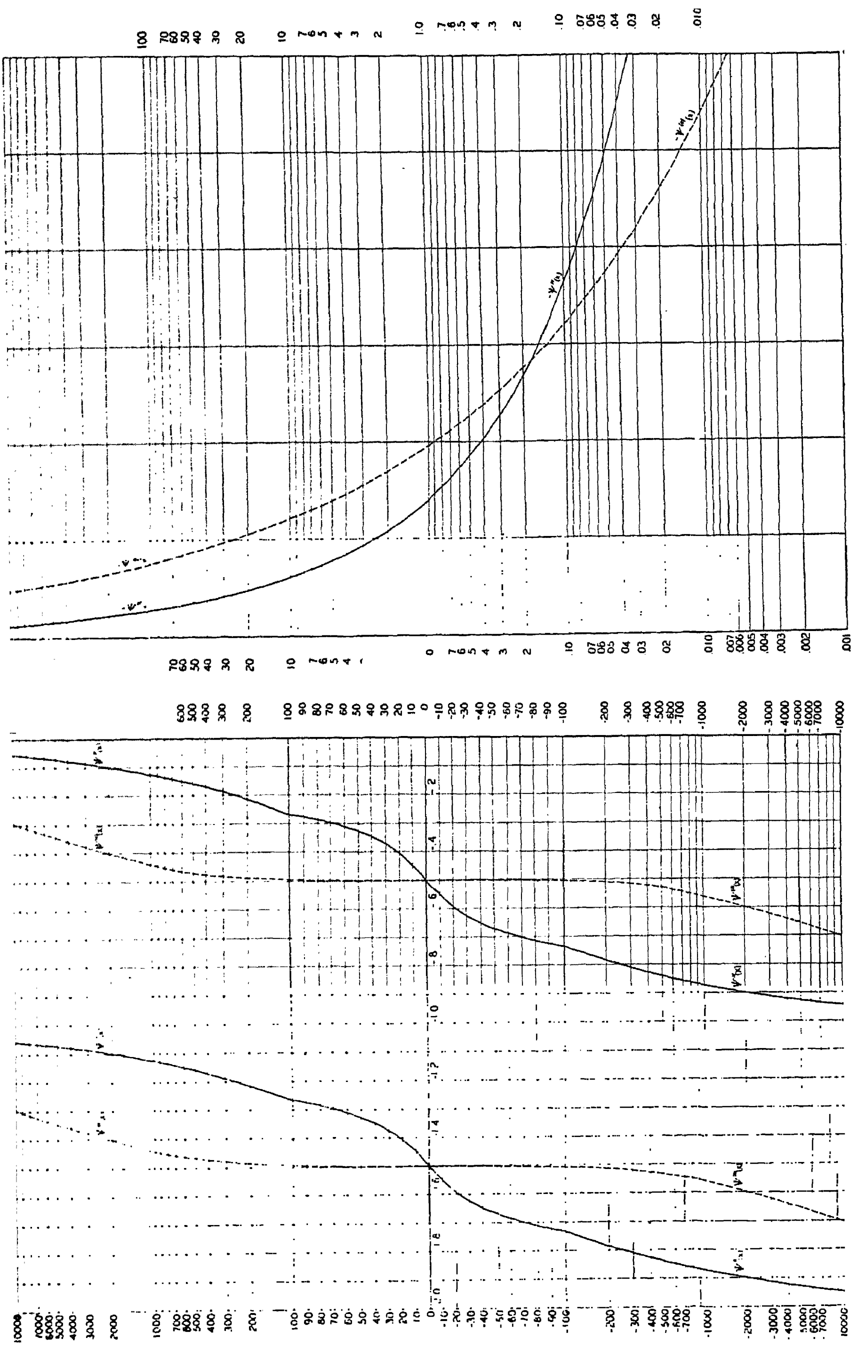
Another asymptotic formula of considerable usefulness has been developed by A. Lodge and employed by him in computing tables for the British Association for the Advancement of Science. (See Bibliography, vol. 1). This formula follows:

$$
\begin{aligned}
\Psi^{(n)}(x+1) &\backsim (-1)^{n+1}\tfrac{1}{2}(n-1)!\,[1/x^n + 1/(x+1)^n] \\
&- \sum_{r=1}^{\infty} (-1)^{r+n} B_r[(2r+n-2)!/(2r-1)!]\,[1/x^{2r+n-1} \\
&\qquad - 1/(x+1)^{2r+n-1}], \quad n > 0 .
\end{aligned}
$$

7. *Application to the Summation of Series.* The polygamma functions find a special use when applied to summations of the following form:

---

*For a more adequate description see section 4 of *Bernoulli Polynomials and Bernoulli Numbers.*



THE TETRAGAMMA AND GAMMA FUNCTIONS

$$\mathrm{S} = \sum^{\infty} S(n) \quad ,$$

in which we define:

$$S(n) = P(n) / [\prod_{m=1}^{p} (n+v_m) \prod_{m=1}^{q} (n+w_m)^2 \prod_{m=1}^{r} (n+x_m)^3$$
$$\prod_{m=1}^{s} (n+y_m)^4 \prod_{m=1}^{t} (n+z_m)^5] \quad ,$$

where $P(n)$ is a polynomial in n of degree $p + 2q + 3r + 4s + 5t$ -2.

Employing the theory of partial fractions, we are always able to expand $S(n)$ in the following form:

$$S(n) = \sum_{m=1}^{p} A_m / (n+v_m) + \sum_{m=1}^{q} [B_{1m} / (n+w_m) + B_{2m} / (n+w_m)^2]$$
$$+ \sum_{m=1}^{r} C_{1m} / (n+x_m) + C_{2m} / (n+x_m)^2 + C_{3m} / (n+x_m)^3]$$
$$+ \sum_{m=1}^{s} [D_{1m} / (n+y_m) + D_{2m} / (n+y_m)^2 + D_{3m} / (n+y_m)^3$$
$$+ D_{4m} / (n+y_m)^4]$$
$$+ \sum_{m=1}^{t} [E_{1m} / (n+z_m) + E_{2m} / (n+z_m)^2 + E_{3m} / (n+z_m)^3$$
$$+ E_{4m} / (n+z_m)^4 + E_{5m} / (n+z_m)^5] \quad .$$

The sum, S, is then expressed in terms of these coefficients as follows:

$$S = - \sum_{m=1}^{p} A_m \, \Psi(1+v_m)$$
$$- \sum_{m=1}^{q} B_{1m} \, \Psi(1+w_m) + \sum_{m=1}^{q} B_{2m} \, \Psi'(1+w_m)$$
$$- \sum_{m=1}^{r} C_{2m} \, \Psi(1+x_m) + \sum_{m=1}^{r} C_{2m} \, \Psi'(1+x_m)$$

$$- \sum_{m=1}^{r} \frac{C_{3m}}{2!} \Psi''(1+x_m) - \sum_{m=1}^{s} D_{1m} \Psi(1+y_m)$$

$$+ \sum^{s} D_{2m} \Psi'(1+y_m) - \sum^{s} \frac{D_{3m}}{2!} \Psi''(1+y_m)$$

$$+ \sum_{m=1}^{s} \frac{D_{4m}}{3!} \Psi'''(1+y_m) \qquad \sum^{t} E_{1m} \Psi(1+z_m)$$

$$+ \sum_{m=1} E_{2m} \Psi'(1+z_m) - \sum_{m=1}^{t} \frac{E_{3m}}{2!} \Psi''(1+z_m)$$

$$+ \sum_{m=1} \frac{E_{4m}}{3!} \Psi^{(3)}(1+z_m) - \sum_{m=1}^{t} \frac{E_{5m}}{4!} \Psi^{(4)}(1+z_m) \ .$$

As an example of this application, let us evaluate the sum,

$$S = \sum_{n=1}^{\infty} (5-2n)/[(n+1)^3 (n+3)] \ .$$

Expanding by the theory of partial fractions,

$$(5-2n)/[(n+1)^3 (n+3)] = -11/8(n+3) + 11/8(n+1)$$
$$- 11/4(n+1)^2 + 7/2(n+1)^3 \ ,$$

we obtain immediately the sum,

$$S = (11/8)\ \Psi(4) - (11/8)\ \Psi(2) - (11/4)\ \Psi'(2)$$
$$- (7/4)\ \Psi''(2) = .07946\ 38106.$$

8. Generalization of Gauss's Theorem for the Computation of $\Psi(x)$. In section 11 of the description of the Psi function in volume 1 of this work, there was set forth an elegant theorem which K. F. Gauss developed in order to compute values for $\Psi(x)$. This theorem may be extended to the general polygamma functions as follows:*

If p and q are integers, $p < q$, and if n is greater than zero, then the following formulas hold:

*For a proof of this theorem see H. T. Davis: An Extension to Polygamma Functions of a Theorem of Gauss. *Bulletin of American Math. Soc.*, vol. 41 (1935), pp. 243-247.

$$\Psi^{(n)}(p/q) + \Psi^{(n)}(1-p/q)$$
$$= (-1)^{n+1}\, 2 \cdot n!\, q^n \sum_{r=1}^{q-1} \cos(2\pi rp/q)\, L^{(n+1)}(2\pi r/q) + 2\, q^n\, \Psi^{(n)}(1) \quad ,$$

where we employ the abbreviation,

$$L^{(m)}(x) = \cos x + \cos 2x/2^m + \cos 3x/3^m + \cos 4x/4^m + \cdots \quad ;$$

$$\Psi^{(n)}(p/q) - \Psi^{(n)}(1-p/q)$$
$$= (-1)^{n+1}\, 2 \cdot n!\, q^n \sum_{r=1}^{q-1} \sin(2\pi rp/q)\, M^{(n+1)}(2\pi r/q) \quad ,$$

where we abbreviate,

$$M^{(m)}(x) = \sin x + \sin 2x/2^m + \sin 3x/3^m + \cdots \; .$$

In connection with this theorem we note the following formulas:

$$\int_0^x L^{(n)}(t)\, dt = M^{(n+1)}(x) \quad ;$$

$$\int_0^x M^{(n)}(t)\, dt = -L^{(n+1)}(x) + S_{n+1} \quad ,$$

where we use the customary notation,

$$S_r = 1/1^r + 1/2^r + 1/3^r + 1/4^r + \cdots, \quad S_1 = C \; .$$

From these it follows that we have,

$$L^{(1)}(x) = -\log(2 \sin \tfrac{1}{2} x) \quad , \quad M^{(1)}(x) = \frac{1}{2}(\pi - x) \quad , \quad 0 < x < 2\pi;$$

$$L^{(2)}(x) = \tfrac{1}{4}(x-\pi)^2 - \frac{1}{12}\pi^2, \quad M^{(3)}(x) = \frac{1}{12}\{(x-\pi)^3 - \pi^2 x + \pi^3\} \quad , \qquad 0 \leq x \leq 2\pi \quad ;$$

$$L^{(4)}(x) = \frac{1}{48}\{2\pi^2(x-\pi)^2 - (x-\pi)^4 - \frac{7}{15}\pi^4\} \quad , \qquad 0 \leq x \leq 2\pi \quad ;$$

$$M^{(5)}(x) = \frac{1}{720}\{10\,\pi^2(x-\pi)^3 - 3(x-\pi)^5 - 7\,\pi^4 x + 7\pi^5\} \quad , \qquad 0 \leq x \leq 2\pi$$

We note that in the general case $L^{(m)}(x)$ for even exponents and $M^{(m)}(x)$ for odd exponents can be expressed in terms of Bernoulli polynomials as follows:

$$L^{(2n)}(x) = (-1)^{n-1} 2^{2n-1} \pi^{2n} B_{2n}(x/2\pi)/(2n)!\ ,$$
$$M^{(2n+1)}(x) = (-1)^{n+1} 2^{2n} \pi^{2n+1} B_{2n+1}(x/2\pi)/(2n+1)!\ .$$

In this connection we also record the following identities:

$$\sin x + \sin 3x/3^{2n+1} + \sin 5x/5^{2n+1} + \cdots$$
$$= (-1)^{n} \tfrac{1}{2} \pi^{2n+1} [B_{2n+1}(x/\pi) - 2^{2n+1} B_{2n+1}(x/2\pi)]/(2n+1)!\ ,$$
$$\cos x + \cos 3x/3^{2n} + \cos 5x/5^{2n} + \cdots$$
$$= (-1)^{n} \tfrac{1}{2} \pi^{2n} [B_{2n}(x/\pi) - 2^{2n} B_{2n}(x/2\pi)]/(2n)!\ .$$

Applying the extended theorem of Gauss to the cases, $q = 2$, 3, and 4, we are able to deduce the following results:

$$\Psi^{(n)}(1/2) = (-1)^{n+1} n!\ (2^{n+1} - 1)\ S_{n+1}\ ;$$
$$\Psi^{(n)}(1/4) = \tfrac{1}{2}\ (-1)^{n+1} n!\ 4^{n+1} [T_{n+1} + (1 - 1/2^{n+1})\ S_{n+1}]\ ,$$
$$\Psi^{(n)}(3/4) = \tfrac{1}{2}\ (-1)^{n+1} n!\ 4^{n+1} [-T_{n+1} + (1 - 1/2^{n+1})\ S_{n+1}]\ ;$$
$$\Psi^{(n)}(1/3) = \tfrac{1}{2}\ (-1)^{n+1} n!\ [3^{n+1} \sigma_{n+1} + (3^{n+1} - 1) S_{n+1}]\ ,$$
$$\Psi^{(n)}(2/3) = \tfrac{1}{2}\ (-1)^{n+1} n! [-3^{n+1} \sigma_{n+1} + (3^{n+1} - 1)\ S_{n+1}]\ ,$$

where we employ the abbreviations,

$$S_r = \sum_{m=1}^{\infty} 1/(m)^r\ ,\ T_r = \sum_{m=1}^{\infty} (-1)^{m+1}/(2m-1)^r\ ,$$
$$\sigma_r = 1 - 1/2^r + 1/4^r - 1/5^r + 1/7^r - 1/8^r$$
$$+ 1/10^r - 1/11^r + \cdots\ .$$

9. *Computation of the Tables.* The computation of the principal tables was accomplished in the following manner. Tables of the second, third, fourth and fifth powers of the reciprocals of the first thousand integers to 16 significant figures for the first and to 18 significant figures for the last three powers were first computed

by Edward Morris. These computations were checked in groups of fifty by the asymptotic formula of Bernoulli,

$$\sum_{p+1}^{p+m} x^{-n} = \varphi(p+m) - \varphi(p) \quad ,$$

where we abbreviate,

$$\varphi(p-1) = \sum_{1}^{p-1} x^{-n} = \text{Const.} + p^{1-n}/(1-n) - \tfrac{1}{2}p^{-n} - B_1\, n\, p^{-n-1}/2! \\ + B_2\, n(n+1)(n+2)\, p^{-n-3}/4! + \cdots \;.$$

Values of $\Psi^{(n)}(x)$ were next computed from 20.00 to 21.00 at intervals of .02 by means of the asymptotic formulas of section 6. Tables were then constructed for $n = 3$ and $n = 4$ both forward by intervals of .1 and backward by intervals of .02, the difference formula (2.1) being successively applied with the help of the reciprocal tables of Morris. The tables from 20.0 to 100.0 were checked by comparing the last ten values with values directly computed by means of the asymptotic formulas. A final check of the correctness of the values was obtained by differencing.

The tables of the pentagamma and the hexagamma functions over the range 1.00 to 2.00 at intervals of .02 were first checked by means of the formula,

$$\Psi^{(m)}(nx) = (1/n^{m+1})\,[\Psi^{(m)}(x) + \Psi^{(m)}(x+1/n) + \cdots \\ + \Psi^{(m)}(x+1-1/n)] \;, \qquad (9.1)$$

where n was set equal to 50. The final check was effected by means of differencing. From the differences thus obtained the values at odd intervals were computed by subtabulating one value between each of the computations corresponding to the even intervals. The final result was then first checked by means of formula (9.1) and the total set of values checked by means of differencing.

A slightly different procedure was followed in the case of the trigamma and tetragamma functions. The values of $\Psi''(x)$ for the range 1.00 to 2.00 (Table 13) were directly computed by an evaluation of the derivative $\Psi'(x)$ from the original table of Gauss (Table 12, vol. 1). The difference table for $\Psi'(x)$ was made by

Esther Kantz and the work of constructing $\Psi'(x)$ from these values by means of the formula (see vol. 1, p. 73),

$$f'(x) = (1/d)\left[\Delta f(x) - \frac{1}{2}\Delta^2 f(x) + \frac{1}{3}\Delta^3 f(x) - \frac{1}{4}\Delta^4 f(x) + \cdots\right] ,$$

was the work of Kathryn Withers.

In the case of the tetragamma function, values for the range 1.00 to 2.00 were computed by numerical integration from the difference table of $\Psi^{(5)}(x)$. In this computation the following formula (see vol. 1, p. 85) was employed:

$$(1/d) \int_x^{x+d} \Psi^{(3)}(t)\,dt = (1/d)\left[\Psi^{(2)}(x+d) - \Psi^{(2)}(x)\right]$$

$$= \frac{1}{2}\left[\Psi^{(3)}(x) + \Psi^{(3)}(x+d)\right] - \frac{1}{24}\left[\Delta^2 \Psi^{(3)}(x-d) + \Delta^2 \Psi^{(3)}(x)\right] - \frac{19}{720}\left[\Delta^3 \Psi^{(3)}(x-2d) - \Delta^3 \Psi^{(3)}(x)\right] - \cdots .$$

From these basic tables of the trigamma and the tetragamma functions the values were then constructed over the range from 2.00 to 4.00 at intervals of .01 and from 4.00 to 20.00 at intervals of .02 by means of the difference formulas and the tables of the reciprocal powers. Checks were effected both by differencing and by comparing the end computations with those directly obtained by means of the asymptotic formulas.

The values of the trigamma and tetragamma functions over the range from 20.0 to 100.0 were independently computed from values found by means of the asymptotic formulas applied to the range between 90.0 and 100.0. Values obtained from these by a successive application of the difference formula (2.1) were checked against the values at the lower end of the range which had been previously computed by the asymptotic formulas.

While the computation of these tables was in progress the tables of the British Association [See Bibliography, British Association (1)] appeared and these were very serviceable in checking the present values.

Because of their essential importance in extending the polygamma functions to negative values, tables of the following functions (see section 1) have been appended to this section:

$$A(x) = \pi^2 \csc^2 \pi x \ ,$$
$$B(x) = 2\pi^3 \csc^2 \pi x \cot \pi x \ ,$$
$$C(x) = 2\pi^4 \csc^2 \pi x \ (3 \csc^2 \pi x - 2)$$
$$D(x) = 8\pi^5 \csc^2 \pi x \cot \pi x \ (3 \csc^2 \pi x - 1)$$

This work was done by Esther Kantz who first computed the values directly from the trigonometric tables of Andoyer (See Bibliography) and then checked them against the values obtained for the range from 0.00 to 1.00 at intervals of .01 by means of formula (2.1). The accuracy of the end values was rechecked by Kathryn Withers.

10. Origin of the Tables. The computation of tables of the derivatives of the psi function is comparatively a recent project.

Eleanor Pairman (See Bibliography) in 1919 published values of $\Psi''(x+1)$ to 8 decimal places with central differences from $x =$ .00 to $x = 20.00$ at intervals of .02.

Tables 12, 13 and 14 of the first volume of the *Mathematical Tables* published by the British Association in 1931 (See Bibliography) contain values of the trigamma function $\Psi''(x+1)$ from $x = 0.00$ to $x = 1.00$ at intervals of .01 and from $x = 10.0$ to $x = 60.0$ at intervals of .1 to 12 decimal places, values of the tetragamma function $\Psi'''(x+1)$ over the same range to 12 decimal places, and values of the pentagamma function $\Psi''''(x+1)$ over the range from $x = 0.00$ to $x = 1.00$ at intervals of .01 to 10 decimal places and over the range $x = 10.0$ to $x = 60.0$ at intervals of .1 to 12 decimal places. Central differences are provided for each table. The computation for the first two functions was done by A. Lodge and for the pentagamma function by A. Lodge and J. Wishart.

One also finds in the *Funktionentafeln* of Jahnke and Emde (See Bibliography) tables of both $\Psi''(x+1)$ and $1/\Psi''(x+1)$ to four decimal places over the range from $x = 0.00$ to $x = 1.00$ at intervals of .05.

The present work contains 16 tables which we may describe as follows:

x	$A(x)$	x	x	$B(x)$	x
.01	10003.290517 629385	.99	.01	1999999.87008 05098	.99
.02	2503.292467 338106	.98	.02	249999.73991 66040	.98
.03	1114.406832 041007	.97	.03	74073.68333 69268	.97
.04	628.300284 538418	.96	.04	31249.47787 17892	.96
.05	403.306166 786491	.95	.05	15999.34549 28744	.95
.06	281.091156 599870	.94	.06	9258.47113 06965	.94
.07	207.403567 018149	.93	.07	5829.98053 76739	.93
.08	159.581849 765036	.92	.08	3905.18985 50835	.92
.09	126.799934 193941	.91	.09	2742.28514 51350	.91
.10	103.355839 110271	.90	.10	1998.65966 03346	.90
.11	86.014587 686615	.89	.11	1501.14538 48283	.89
.12	72.829977 630929	.88	.12	1155.77639 16790	.88
.13	62.574188 327835	.87	.13	908.55121 85097	.87
.14	54.441574 301887	.86	.14	726.92831 43891	.86
.15	47.885741 381053	.85	.15	590.50040 96800	.85
.16	42.525524 068727	.84	.16	486.02726 13301	.84
.17	38.088468 389939	.83	.17	404.66278 67567	.83
.18	34.375647 746977	.82	.18	340.34351 88379	.82
.19	31.239081 825455	.81	.19	288.81863 24622	.81
.20	28.566851 342926	.80	.20	247.04792 04060	.80
.21	26.273050 854713	.79	.21	212.81785 16431	.79
.22	24.290862 561344	.78	.22	184.49071 25005	.78
.23	22.567690 416108	.77	.23	160.83710 35590	.77
.24	21.061682 807845	.76	.24	140.92188 23485	.76
.25	19.739208 802179	.75	.25	124.02510 67212	.75
.26	18.573000 454427	.74	.26	109.58634 40271	.74
.27	17.540767 670797	.73	.27	97.16485 45906	.73
.28	16.624153 134047	.72	.28	86.41073 43836	.72
.29	15.807935 184768	.71	.29	77.04373 60215	.71
.30	15.079413 702803	.70	.30	68.83754 37145	.70
.31	14.427932 576346	.69	.31	61.60796 93362	.69
.32	13.844505 190218	.68	.32	55.20400 42739	.68
.33	13.321518 378651	.67	.33	49.50096 96477	.67
.34	12.852496 692147	.66	.34	44.39522 71386	.66
.35	12.431913 430155	.65	.35	39.80005 99637	.65
.36	12.055038 234299	.64	.36	35.64243 93205	.64
.37	11.717813 489873	.63	.37	31.86046 62928	.63
.38	11.416753 600451	.62	.38	28.40133 28907	.62
.39	11.148862 558764	.61	.39	25.21968 48241	.61
.40	10.911566 261433	.60	.40	22.27629 70864	.60
.41	10.702656 793967	.59	.41	19.53699 44321	.59
.42	10.520246 509872	.58	.42	16.97176 44427	.58
.43	10.362730 189402	.57	.43	14.55402 25610	.57
.44	10.228753 923848	.56	.44	12.25999 72646	.56
.45	10.117189 654588	.55	.45	10.06821 01980	.55
.46	10.027114 521726	.54	.46	7.95903 11211	.54
.47	9.957794 358473	.53	.47	5.91429 13550	.53
.48	9.908670 815569	.52	.48	3.91694 22902	.52
.49	9.879351 723041	.51	.49	1.95074 76774	.51
.50	9.869604 401090	.50	.50	0.00000 00000	.50
x	$A(x)$	x	x	$-B(x)$	x

x	$C(x)$	x	x	$D(x)$	x
.01	600000013.00009 11394	.99	.01	2399999 99997.55668 9196	.99
.02	37500013.03677 88101	.98	.02	74999 99995.10323 9110	.98
.03	7407420.50550 15866	.97	.03	9876 54313.61709 1866	.97
.04	2343763.18429 24023	.96	.04	2343 74990.12492 5672	.96
.05	960013.29573 32696	.95	.05	767 99987.57912 26862	.95
.06	462976.39584 67748	.94	.06	308 64182.51201 22107	.94
.07	249909.47303 97356	.93	.07	142 79746.70362 78260	.93
.08	146498.16173 94836	.92	.08	73 24198.33317 13000	.92
.09	91463.47911 52127	.91	.09	40 64397.83317 98416	.91
.10	60014.25189 08840	.90	.10	23 99973.83475 70233 3	.90
.11	40995.33595 66311	.89	.11	14 90181.97161 69545 0	.89
.12	28950.02157 93003	.88	.12	9 64473.80198 00790 5	.88
.13	21022.84433 12640	.87	.13	6 46354.09621 08538 4	.87
.14	15634.04286 94792	.86	.14	4 46203.48251 56480 6	.86
.15	11867.81207 01432	.85	.15	3 16006.57707 12059 5	.85
.16	9171.68078 58898	.84	.16	2 28835.18080 71161 39	.84
.17	7200.71608 47398	.83	.17	1 68980.38265 90876 12	.83
.18	5733.01477 09701	.82	.18	1 26958.12343 28276 01	.82
.19	4622.01188 19025	.81	.19	96867.04412 21642 41	.81
.20	3768.61588 69352	.80	.20	74935.51363 17332 69	.80
.21	3104.42073 39034	.79	.21	58694.77884 72835 80	.79
.22	2581.31120 76780	.78	.22	46493.86110 21168 74	.78
.23	2164.86719 76907	.77	.23	37207.06918 59398 63	.77
.24	1830.08498 68510	.76	.24	30053.25091 95626 36	.76
.25	1558.54545 65440	.75	.25	24481.57478 28225 163	.75
.26	1336.50540 71751	.74	.26	20697.87115 57373 245	.74
.27	1153.58943 16766	.73	.27	16616.23897 56909 093	.73
.28	1001.87954 48007	.72	.28	13826.70431 23222 500	.72
.29	875.27262 21482	.71	.29	11573.26385 26089 360	.71
.30	769.02091 43335	.70	.30	9738.76030 18539 632	.70
.31	679.39948 31491	.69	.31	8234.32239 17736 195	.69
.32	603.46278 63468	.68	.32	6991.89875 01368 419	.68
.33	538.86464 57998	.67	.33	5958.91697 15003 560	.67
.34	483.72379 56587	.66	.34	5094.42080 27371 018	.66
.35	436.52255 91925	.65	.35	4366.24741 18791 319	.65
.36	396.02984 73323	.64	.36	3748.94452 13340 580	.64
.37	361.24218 35376	.63	.37	3222.21922 72594 546	.63
.38	331.33821 03157	.62	.38	2769.77255 40747 069	.62
.39	305.64336 62168	.61	.39	2378.41634 92145 147	.61
.40	283.60230 00793	.60	.40	2037.39854 15587 2830	.60
.41	264.75722 03084	.59	.41	1737.88332 93871 3322	.59
.42	248.73083 47532	.58	.42	1472.54734 35835 8263	.58
.43	235.21287 19315	.57	.43	1235.26312 71522 1539	.57
.44	223.94942 20299	.56	.44	1020.84861 96810 8199	.56
.45	214.73452 08751	.55	.45	824.86689 79396 5088	.55
.46	207.40353 93355	.54	.46	643.46344 40702 7982	.54
.47	201.82804 68117	.53	.47	473.23270 11245 8211	.53
.48	197.91189 96266	.52	.48	311.10561 76195 4097	.52
.49	195.58836 98227	.51	.49	154.25303 76865 6823	.51
.50	194.81818 20680	.50	.50	0.00000 00000 0000	.50
x	$C(x)$	x	x	$-D(x)$	x

Table 13. $\Psi'(x)$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1; $\mathrm{Log}_{10}\Psi'(x)$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1; $\Psi'(x)$ to 12 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01; $Log_{10}\Psi'(x)$ to 10 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

Table 14. $\Psi'(x)$ to 12 decimal places with second, fourth and sixth central differences from $x = 1.00$ to $x = 4.00$ by increments of .01.

Table 15. $\Psi'(x)$ to 10 decimal places with central differences from $x = 4.00$ to $x = 20.00$ at intervals of .02.

Table 16. $\Psi'(x)$ to 15 decimal places with central differences from $x = 20.0$ to $x = 100.0$ by increments of .1.

Table 17. $\Psi''(x)$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1; $\mathrm{Log}_{10}|\Psi''(x)|$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1; $\Psi''(x)$ to 12 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01; $\mathrm{Log}_{10}|\Psi''(x)|$ to 10 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

Table 18. $\Psi''(x)$ to 12 decimal places with central differences from $x = 1.00$ to $x = 4.00$ at intervals of .01.

Table 19. $\Psi''(x)$ to 10 decimal places with central differences from $x = 4.00$ to $x = 20.00$ at intervals of .02.

Table 20. $\Psi''(x)$ to 17 decimal places with central differences from $x = 20.0$ to $x = 100.00$ at intervals of .1.

Table 21. $\Psi^{(3)}(x)$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1; $\mathrm{Log}_{10}\Psi^{(3)}(x)$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1; $\Psi^{(3)}(x)$ to 12 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01; $\mathrm{Log}_{10}\Psi^{(3)}(x)$ to 10 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

Table 22. $\Psi^{(3)}(x)$ to 15 decimal places with central differences from $x = 1.00$ to $x = 4.00$ by increments of .01.

Table 23. $\Psi^{(3)}(x)$ to 15 decimal places with central differences from $x = 4.00$ to $x = 20.00$ by increments of .02.

Table 24. $\Psi^{(3)}(x)$ to 17 and 18 decimal places with central differences from $x = 20.0$ to $x = 100.0$ by increments of .1.

Table 25. $\Psi^{(4)}(x)$ to 12 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1; $\text{Log}_{10}|\Psi^{(4)}(x)|$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1; $\Psi^{(4)}(x)$ to 12 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01; $\text{Log}_{10}|\Psi^{(4)}(x)|$ to 10 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

Table 26. $\Psi^{(4)}(x)$ to 14 decimal places with central differences from $x = 1.00$ to $x = 4.00$ by increments of .01.

Table 27. $\Psi^{(4)}(x)$ to 16 decimal places with central differences from $x = 4.00$ to $x = 20.00$ by increments of .02.

Table 28. $\Psi^{(4)}(x)$ to 19 decimal places with central differences from $x = 20.0$ to $x = 100.0$ by increments of .1.

TABLE 13

THE TRIGAMMA FUNCTION

Description: $\Psi'(x)$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1.

$\text{Log}_{10}\, \Psi'(x)$ to 10 decimal places from $x = 0.0$ to $x = -10.00$ by increments of .1.

$\Psi'(x)$ to 12 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

$\text{Log}_{10}\, \Psi'(x)$ to 10 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

x	$\Psi'(-x)$	$\mathrm{Log}_{10}\Psi'(-x)$	x	$\Psi'(-x)$	$\mathrm{Log}_{10}\Psi'(-x)$
0.0	∞	∞	5.0	∞	∞
0.1	101.92253 99595	2.00827 02376	5.1	103.17773 70337	2.01358 59980
0.2	27.29947 41375	1.43615 42813	5.2	28.39185 79931	1.45319 38141
0.3	13.94516 02678	1.14442 35097	5.3	14.90742 26714	1.17340 25650
0.4	9.88620 96709	0.99502 98169	5.4	10.74247 64993	1.03110 44118
0.5	8.93480 22005	0.95108 49425	5.5	9.70331 98653	0.98692 03479
0.6	10.05313 43683	1.00230 14871	5.6	10.74799 55905	1.03132 74796
0.7	14.28618 08726	1.15491 61441	5.7	14.91846 99179	1.17372 42827
0.8	27.82987 72054	1.44451 12900	5.8	28.40845 75725	1.45344 76544
0.9	102.66786 70520	2.01143 45392	5.9	103.19990 43361	2.01367 92947
1.0	∞	∞	6.0	∞	∞
1.1	102.74898 62405	2.01177 75454	6.1	103.20461 15298	2.01369 91030
1.2	27.99391 85819	1.44706 36952	6.2	28.41787 25613	1.45359 15623
1.3	14.53687 62441	1.16247 10931	6.3	14.93261 79347	1.17413 59532
1.4	10.39641 37525	1.01688 35546	6.4	10.76689 05618	1.03209 02986
1.5	9.37924 66450	0.97216 79566	6.5	9.72698 85044	0.98797 84026
1.6	10.44375 93683	1.01885 68565	6.6	10.77095 24317	1.03225 41078
1.7	14.63220 16339	1.16530 96770	6.7	14.94074 65942	1.17437 22997
1.8	28.13851 91807	1.44930 12384	6.8	28.43007 78701	1.45377 80492
1.9	102.94487 53623	2.01260 47316	6.9	103.22090 83268	2.01376 76763
2.0	∞	∞	7.0	∞	∞
2.1	102.97574 36101	2.01273 49363	7.1	103.22444 88637	2.01378 25722
2.2	28.20053 01522	1.45025 72728	7.2	28.43716 26847	1.45388 62625
2.3	14.72591 21610	1.16808 22055	7.3	14.95138 31814	1.17468 13717
2.4	10.57002 48636	1.02407 60088	7.4	10.78515 20666	1.03282 62726
2.5	9.53924 66450	0.97951 40780	7.5	9.74476 62822	0.98877 14275
2.6	10.59168 83624	1.02496 51937	7.6	10.78826 54511	1.03295 16236
2.7	14.76937 58451	1.16936 21422	7.7	14.95761 28449	1.17486 22878
2.8	28.26607 02012	1.45126 54332	7.8	28.44651 44250	1.45402 90595
2.9	103.06328 14265	2.01310 39653	7.9	103.23693 14000	2.01383 50867
3.0	∞	∞	8.0	∞	∞
3.1	103.07980 18827	2.01317 35752	8.1	103.23969 04427	2.01384 66936
3.2	28.29818 64022	1.45175 86030	8.2	28.45203 47847	1.45411 33309
3.3	14.81773 95255	1.17078 19560	8.3	14.96589 90763	1.17510 28121
3.4	10.65653 00540	1.02761 58136	8.4	10.79932 44022	1.03339 65868
3.5	9.62087 92981	0.98321 47661	8.5	9.75860 71126	0.98938 78334
3.6	10.66884 88562	1.02811 75623	8.6	10.80178 62731	1.03349 55796
3.7	14.84242 18641	1.17150 47711	8.7	14.97082 46298	1.17524 57229
3.8	28.33532 22787	1.45232 81565	8.8	28.45942 76482	1.45422 61616
3.9	103.12952 76461	2.01338 30285	8.9	103.24955 60686	2.01388 81929
4.0	∞	∞	9.0	∞	∞
4.1	103.13929 02825	2.01342 41385	9.1	103.25166 62790	2.01389 70687
4.2	28.35487 57446	1.45262 77484	9.2	28.46384 95295	1.45429 36348
4.3	14.87182 28138	1.17236 42024	9.3	14.97746 11066	1.17543 82003
4.4	10.70818 29465	1.02971 57821	9.4	10.81064 17403	1.03385 14748
4.5	9.67026 20141	0.98543 82414	9.5	9.76968 74450	0.98988 06699
4.6	10.71610 78354	1.03003 70750	9.6	10.81263 69676	1.03393 16214
4.7	14.88769 12168	1.17282 73524	9.7	14.98145 27518	1.17555 39286
4.8	28.37872 50565	1.45299 28804	9.8	28.46983 99764	1.45438 50260
4.9	103.17117 69589	2.01355 83847	9.9	103.25975 91091	2.01393 11075
5.0	∞	∞	10.0	∞	∞

x	$\Psi'(x)$	$\mathrm{Log}_{10}\Psi'(x)$	x	$\Psi'(x)$	$\mathrm{Log}_{10}\Psi'(x)$
0.00	∞	∞	0.50	4.934802 200545	0.69326 97498
0.01	10001.621213 528313	4.00002 69776	0.51	4.771259 239159	0.67863 30139
0.02	2501.598118 191868	3.39821 75415	0.52	4.616728 378554	0.66433 43239
0.03	1112.686736 026479	3.04637 29108	0.53	4.470542 884596	0.65036 02653
0.04	626.553711 642606	2.79695 83070	0.54	4.332096 979611	0.63669 81703
0.05	401.552357 342115	2.60372 05485	0.55	4.200839 261056	0.62333 60642
0.06	279.289319 727769	2.44605 43282	0.56	4.076266 935081	0.61026 26161
0.07	205.572878 969465	2.31296 58180	0.57	3.957920 751666	0.59746 70942
0.08	157.721452 155554	2.19789 07669	0.58	3.845380 545395	0.58493 93248
0.09	124.908932 122266	2.09659 34954	0.59	3.738261 300509	0.57266 96548
0.10	101.433299 150792	2.00618 05508	0.60	3.636209 670903	0.56064 89175
0.11	84.077927 249967	1.92468 19966	0 61	3.538900 895927	0.54886 84007
0.12	70.841396 665712	1.85028 71149	0.62	3.446036 061363	0.53731 98178
0.13	60.551015 890480	1.78212 14339	0.63	3.357339 662074	0.52599 52810
0.14	52.382699 930283	1.71918 78793	0.64	3.272557 428965	0.51488 72765
0.15	45.790003 796480	1.66077 06795	0.65	3.191454 387950	0.50398 86419
0.16	40.391708 175237	1.60629 22200	0.66	3.113813 123066	0.49329 25446
0.17	35.915302 056813	1.55527 95233	0.67	3.039432 219512	0.48279 24628
0.18	32.161798 355694	1.50734 03247	0.68	2.968124 865642	0.47248 21672
0.19	28.983152 560743	1.46214 56228	0.69	2.899717 595607	0.46235 57039
0.20	26.267377 205424	1.41941 67106	0.70	2.834049 156695	0.45240 73788
0.21	23.928494 370468	1.37891 53727	0.71	2.770969 487433	0.44263 17435
0.22	21.899609 160829	1.34043 63640	0.72	2.710338 794235	0.43302 35813
0.23	20.128043 452599	1.30380 15613	0.73	2.652026 715860	0.42357 78946
0.24	18.571858 181768	1.26885 53586	0.74	2.595911 566288	0.41428 98934
0.25	17.197329 154507	1.23546 10035	0.75	2.541879 647672	0.40515 49838
0.26	15.977088 888138	1.20349 76513	0.76	2.489824 626076	0.39616 87580
0.27	14.888740 954937	1.17285 79736	0.77	2.439646 963509	0.38732 69849
0.28	13.913814 339811	1.14344 62038	0.78	2.391253 400515	0.37862 56005
0.29	13.036965 697335	1.11517 65228	0.79	2.344556 484245	0.37006 07000
0.30	12.245364 546108	1.08797 17183	0.80	2.299474 137502	0.36162 85293
0.31	11.528214 980739	1.06176 20664	0.81	2.255929 264712	0.35332 54780
0.32	10.876380 324576	1.03648 43851	0.82	2.213849 391283	0.34514 80723
0.33	10.282086 159139	1.01208 12385	0.83	2.173166 333126	0.33709 29682
0.34	9.738683 569081	0.98850 02549	0.84	2.133815 893490	0.32915 69455
0.35	9.240459 042205	0.96569 35463	0.85	2.095737 584573	0.32133 69019
0.36	8.782480 805334	0.94361 72093	0.86	2.058874 371605	0.31362 98475
0.37	8.360473 827799	0.92223 08917	0.87	2.023172 437355	0.30603 28996
0.38	7.970717 539088	0.90149 74191	0.88	1.988580 965217	0.29854 32779
0.39	7.609961 662837	0.88138 24689	0.89	1.955051 939223	0.29115 82995
0.40	7.275356 590530	0.86185 42844	0.90	1.922539 959477	0.28387 53751
0.41	6.964395 493458	0.84288 34258	0.91	1.891002 071675	0.27669 20046
0.42	6.674865 964476	0.82444 25492	0.92	1.860397 609482	0.26960 57727
0.43	6.404809 437736	0.80650 62127	0.93	1.830688 048684	0.26261 43462
0.44	6.152486 988766	0.78905 07040	0.94	1.801836 872101	0.25571 54698
0.45	5.916350 393532	0.77205 38870	0.95	1.773809 444376	0.24890 69629
0.46	5.695017 542115	0.75549 50661	0.96	1.746572 895813	0.24218 67161
0.47	5.487251 473877	0.73935 48639	0.97	1.720096 014528	0.23555 26896
0.48	5.291942 437015	0.72361 51115	0.98	1.694349 146238	0.22900 29080
0.49	5.108092 483882	0.70825 87517	0.99	1.669304 101072	0.22253 54603
0.50	4.934802 200545	0.69326 97498	1.00	1.644934 066848	0.21614 84948

TABLE 14

THE TRIGAMMA FUNCTION

Description: $\Psi'(x)$ to 12 decimal places with second, fourth and sixth central differences from $x = 1.00$ to $x = 4.00$ by increments of .01.

x	$\Psi'(x)$	δ^2	δ^4	δ^6
1.00	1.644934 066848	649 495689	1 221654	5075
1.01	1.621213 528313	625 202090	1 151454	4683
1.02	1.598118 191868	602 059945	1 085936	4324
1.03	1.575624 915368	580 003737	1 024745	4004
1.04	1.553711 642605	558 972273	967558	3706
1.05	1.532357 342115	538 908366	914077	3435
1.06	1.511541 949991	519 758537	864030	3182
1.07	1.491246 316404	501 472737	817166	2956
1.08	1.471452 155554	484 004105	773257	2741
1.09	1.452141 998809	467 308729	732089	2549
1.10	1.433299 150793	451 345441	693470	2368
1.11	1.414907 648218	436 075625	657218	2204
1.12	1.396952 221268	421 463026	623171	2051
1.13	1.379418 257344	407 473597	591175	1911
1.14	1.362291 767017	394 075346	561090	1781
1.15	1.345559 352036	381 238182	532787	1661
1.16	1.329208 175237	368 933808	506145	1551
1.17	1.313225 932246	357 135575	481053	1448
1.18	1.297600 824830	345 818398	457410	1353
1.19	1.282321 535812	334 958630	435119	1265
1.20	1.267377 205424	324 533981	414094	1183
1.21	1.252757 409017	314 523426	394252	1108
1.22	1.238452 136036	304 907124	375518	1037
1.23	1.224451 770179	295 666336	357820	972
1.24	1.210747 070658	286 783370	341094	911
1.25	1.197329 154507	278 241499	325280	855
1.26	1.184189 479855	270 024905	310320	802
1.27	1.171319 830108	262 118635	296163	753
1.28	1.158712 298996	254 508524	282759	708
1.29	1.146359 276408	247 181177	270062	665
1.30	1.134253 434997	240 123886	258030	626
1.31	1.122387 717472	233 324629	246624	588
1.32	1.110755 324576	226 771995	235806	554
1.33	1.099349 703675	220 455165	225542	521
1.34	1.088164 537939	214 363880	215799	491
1.35	1.077193 736083	208 488391	206547	463
1.36	1.066431 422618	202 819450	197759	437
1.37	1.055871 928603	197 348267	189407	412
1.38	1.045509 782855	192 066493	181467	389
1.39	1.035339 703600	186 966185	173916	367
1.40	1.025356 590530	182 039793	166732	347
1.41	1.015555 517253	177 280137	159895	327
1.42	1.005931 724113	172 680370	153386	310
1.43	0.996480 611343	168 233995	147186	293
1.44	0.987197 732568	163 934801	141279	277
1.45	0.978078 788594	159 776890	135649	262
1.46	0.969119 621510	155 754625	130282	248
1.47	0.960316 209051	151 862645	125163	235
1.48	0.951664 659237	148 095825	120279	223
1.49	0.943161 205248	144 449286	115618	211
1.50	0.934802 200545	140 918365	111169	200

	$\Psi'(x)$	δ^2	δ^4	
1.50	0.934802 200545	140 918365	111169	200
1.51	0.926584 114207	137 498614	106920	190
1.52	0.918503 526483	134 185780	102861	181
1.53	0.910557 124539	130 975809	98982	171
1.54	0.902741 698404	127 864820	94274	163
1.55	0.895054 137089	124 849104	91730	154
1.56	0.887491 424878	121 925118	88339	147
1.57	0.880050 637785	119 089472	85096	140
1.58	0.872728 940164	116 338920	81991	133
1.59	0.865523 581463	113 670363	79020	126
1.60	0.858431 893125	111 080820	76175	120
1.61	0.851451 285607	108 567457	73449	114
1.62	0.844579 245546	106 127539	70838	109
1.63	0.837813 333024	103 758463	68336	104
1.64	0.831151 178965	101 457719	65937	99
1.65	0.824590 482625	99 222915	63637	94
1.66	0.818129 009200	97 051748	61432	90
1.67	0.811764 587523	94 942011	59315	85
1.68	0.805495 107857	92 891592	57285	82
1.69	0.799318 519783	90 898455	55336	78
1.70	0.793232 830164	88 960655	53464	74
1.71	0.787236 101200	87 076320	51667	71
1.72	0.781326 448556	85 243649	49941	68
1.73	0.775502 039561	83 460923	48282	65
1.74	0.769761 091489	81 726477	46689	62
1.75	0.764101 869894	80 038719	45156	59
1.76	0.758522 687018	78 396119	43683	56
1.77	0.753021 900261	76 797202	42267	54
1.78	0.747597 910706	75 240550	40904	52
1.79	0.742249 161701	73 724806	39593	49
1.80	0.736974 137502	72 248650	38331	48
1.81	0.731771 361953	70 810828	37117	45
1.82	0.726639 397232	69 410123	35947	44
1.83	0.721576 842634	68 045363	34821	41
1.84	0.716582 333399	66 715426	33737	40
1.85	0.711654 539590	65 419226	32692	38
1.86	0.706792 165007	64 155717	31685	37
1.87	0.701993 946141	62 923892	30715	35
1.88	0.697258 651167	61 722785	29779	33
1.89	0.692585 078978	60 551454	28877	32
1.90	0.687972 058243	59 409001	28007	31
1.91	0.683418 446509	58 294556	27168	30
1.92	0.678923 129331	57 207279	26359	28
1.93	0.674485 019432	56 146362	25578	27
1.94	0.670103 055895	55 111021	24824	26
1.95	0.665776 203379	54 100506	24096	25
1.96	0.661503 451369	53 114085	23394	24
1.97	0.657283 813444	52 151061	22715	23
1.98	0.653116 326580	51 210749	22060	22
1.99	0.649000 050465	50 292498	21427	22
2.00	0.644934 066848	49 395675	20816	19

x	$\Psi'(x)$	δ^2	δ^4	δ^6
2.00	0.644934 066848	49 395674	20816	19
2.01	0.640917 478906	48 519666	20224	19
2.02	0.636949 410630	47 663881	19652	17
2.03	0.633029 006234	46 827748	19101	18
2.04	0.629155 429587	46 010716	18566	17
2.05	0.625327 863657	45 212250	18050	15
2.06	0.621545 509977	44 431834	17550	15
2.07	0.617807 588130	43 668968	17066	18
2.08	0.614113 335252	42 923168	16600	13
2.09	0.610462 005542	42 193968	16146	16
2.10	0.606852 869801	41 480915	15709	14
2.11	0.603285 214974	40 783570	15285	14
2.12	0.599758 343717	40 101510	14874	14
2.13	0.596271 573970	39 434324	14477	12
2.14	0.592824 238547	38 781615	14092	13
2.15	0.589415 684739	38 142998	13719	11
2.16	0.586045 273929	37 518100	13357	13
2.17	0.582712 381219	36 906559	13008	9
2.18	0.579416 395068	36 308026	12668	12
2.19	0.576156 716943	35 722161	12340	10
2.20	0.572932 760979	35 148636	12021	9
2.21	0.569743 953652	34 587132	11712	10
2.22	0.566589 733456	34 037339	11412	8
2.23	0.563469 550600	33 498959	11121	9
2.24	0.560382 866703	32 971700	10840	8
2.25	0.557329 154507	32 455281	10566	8
2.26	0.554307 897592	31 949428	10301	7
2.27	0.551318 590105	31 453877	10043	8
2.28	0.548360 736496	30 968368	9794	7
2.29	0.545433 851254	30 492653	9551	7
2.30	0.542537 458665	30 026490	9315	8
2.31	0.539671 092566	29 569642	9087	6
2.32	0.536834 296109	29 121881	8865	6
2.33	0.534026 621533	28 682985	8649	7
2.34	0.531247 629942	28 252739	8441	5
2.35	0.528496 891090	27 830933	8237	6
2.36	0.525773 983172	27 417365	8039	5
2.37	0.523078 492618	27 011835	7847	5
2.38	0.520410 013899	26 614153	7661	6
2.39	0.517768 149332	26 224131	7480	4
2.40	0.515152 508897	25 841589	7303	7
2.41	0.512562 710051	25 466350	7132	3
2.42	0.509998 377554	25 098243	6965	6
2.43	0.507459 143301	24 737101	6803	4
2.44	0.504944 646148	24 382762	6645	5
2.45	0.502454 531757	24 035069	6493	3
2.46	0.499988 452435	23 693868	6343	5
2.47	0.497546 066981	23 359010	6198	4
2.48	0.495127 040537	23 030351	6057	3
2.49	0.492731 044444	22 707749	5920	4
2.50	0.490357 756100	22 391067	5786	4

x	$\Psi'(x)$	δ^2	δ^4	δ^6
2.50	0.490357 756100	22 391067	5786	4
2.51	0.488006 858824	22 080171	5657	2
2.52	0.485678 041718	21 774932	5529	4
2.53	0.483370 999544	21 475222	5406	4
2.54	0.481085 432592	21 180918	5287	2
2.55	0.478821 046558	20 891901	5169	4
2.56	0.476577 552425	20 608053	5056	2
2.57	0.474354 666346	20 329261	4945	3
2.58	0.472152 109527	20 055415	4837	3
2.59	0.469969 608124	19 786404	4732	2
2.60	0.467806 893125	19 522126	4629	3
2.61	0.465663 700252	19 262477	4529	3
2.62	0.463539 769856	19 007358	4432	2
2.63	0.461434 846819	18 756671	4337	3
2.64	0.459348 680452	18 510321	4245	3
2.65	0.457281 024406	18 268216	4155	1
2.66	0.455231 636577	18 030266	4067	3
2.67	0.453200 279014	17 796383	3982	2
2.68	0.451186 717834	17 566482	3898	3
2.69	0.449190 723137	17 340479	3817	1
2.70	0.447212 068918	17 118292	3737	3
2.71	0.445250 532991	16 899842	3660	0
2.72	0.443305 896906	16 685052	3584	3
2.73	0.441377 945873	16 473846	3511	1
2.74	0.439466 468686	16 266150	3439	2
2.75	0.437571 257649	16 061893	3368	2
2.76	0.435692 108506	15 861005	3300	1
2.77	0.433828 820367	15 663416	3233	2
2.78	0.431981 195644	15 469060	3168	2
2.79	0.430149 039982	15 277873	3105	0
2.80	0.428332 162193	15 089789	3042	3
2.81	0.426530 374193	14 904748	2982	1
2.82	0.424743 490940	14 722688	2922	1
2.83	0.422971 330376	14 543550	2864	2
2.84	0.421213 713361	14 367276	2808	1
2.85	0.419470 463622	14 193811	2753	1
2.86	0.417741 407694	14 023098	2699	2
2.87	0.416026 374863	13 855083	2646	1
2.88	0.414325 197116	13 689715	2595	2
2.89	0.412637 709083	13 526942	2545	1
2.90	0.410963 747993	13 366713	2496	1
2.91	0.409303 153616	13 208980	2447	1
2.92	0.407655 768220	13 053694	2400	2
2.93	0.406021 436517	12 900809	2355	1
2.94	0.404400 005624	12 750279	2310	1
2.95	0.402791 325010	12 602059	2266	1
2.96	0.401195 246454	12 456105	2224	1
2.97	0.399611 624004	12 312375	2181	1
2.98	0.398040 313928	12 170826	2141	1
2.99	0.396481 174679	12 031419	2101	1
3.00	0.394934 066848	11 894112	2062	1

x	$\Psi'(x)$	δ^2	δ^4	x	$\Psi'(x)$	δ^2	δ^4
3.00	0.394934 066848	11 894112	2062	3.50	0.330357 756100	7 030657	870
3.01	0.393398 853130	11 758867	2023	3.51	0.329279 219580	6 963093	857
3.02	0.391875 398278	11 625646	1986	3.52	0.328207 646152	6 896386	843
3.03	0.390363 569073	11 494411	1950	3.53	0.327142 969111	6 830522	830
3.04	0.388863 234278	11 365125	1914	3.54	0.326085 122592	6 765488	817
3.05	0.387374 264609	11 237754	1879	3.55	0.325034 041560	6 701272	804
3.06	0.385896 532693	11 112261	1845	3.56	0.323989 661800	6 637859	792
3.07	0.384429 913039	10 988613	1811	3.57	0.322951 919900	6 575239	780
3.08	0.382974 281998	10 866776	1779	3.58	0.321920 753239	6 513399	768
3.09	0.381529 517733	10 746718	1746	3.59	0.320896 099977	6 452327	756
3.10	0.380095 500186	10 628406	1715	3.60	0.319877 899042	6 392010	744
3.11	0.378672 111046	10 511809	1684	3.61	0.318866 090117	6 332438	733
3.12	0.377259 233714	10 396896	1654	3.62	0.317860 613630	6 273599	722
3.13	0.375856 753277	10 283637	1625	3.63	0.316861 410742	6 215482	717
3.14	0.374464 556479	10 172004	1596	3.64	0.315868 423335	6 158075	700
3.15	0.373082 531684	10 061966	1568	3.65	0.314881 594004	6 101369	690
3.16	0.371710 568854	9 953496	1540	3.66	0.313900 866042	6 045352	679
3.17	0.370348 559520	9 846566	1514	3.67	0.312926 183431	5 990014	669
3.18	0.368996 396751	9 741149	1486	3.68	0.311957 490835	5 935346	659
3.19	0.367653 975132	9 637219	1461	3.69	0.310994 733585	5 881336	649
3.20	0.366321 190731	9 534750	1436	3.70	0.310037 857670	5 827975	639
3.21	0.364997 941080	9 433716	1410	3.71	0.309086 809730	5 775254	630
3.22	0.363684 125145	9 334093	1386	3.72	0.308141 537045	5 723163	620
3.23	0.362379 643303	9 235856	1362	3.73	0.307201 987521	5 671692	612
3.24	0.361084 397316	9 138981	1339	3.74	0.306268 109690	5 620832	602
3.25	0.359798 290309	9 043445	1316	3.75	0.305339 852690	5 570576	594
3.26	0.358521 226749	8 949226	1294	3.76	0.304417 166267	5 520912	585
3.27	0.357253 112413	8 856300	1272	3.77	0.303500 000755	5 471834	576
3.28	0.355993 854378	8 764646	1250	3.78	0.302588 307078	5 423332	568
3.29	0.354743 360988	8 674242	1229	3.79	0.301682 036732	5 375398	560
3.30	0.353501 541841	8 585068	1208	3.80	0.300781 141785	5 328024	551
3.31	0.352268 307761	8 497101	1188	3.81	0.299885 574862	5 281202	544
3.32	0.351043 570782	8 410323	1169	3.82	0.298995 289140	5 234923	536
3.33	0.349827 244127	8 324714	1149	3.83	0.298110 238341	5 189180	528
3.34	0.348619 242185	8 240253	1130	3.84	0.297230 376721	5 143965	521
3.35	0.347419 480497	8 156922	1111	3.85	0.296355 659067	5 099271	513
3.36	0.346227 875731	8 074703	1093	3.86	0.295486 040683	5 055090	506
3.37	0.345044 345668	7 993576	1075	3.87	0.294621 477390	5 011415	499
3.38	0.343868 809181	7 913524	1057	3.88	0.293761 925511	4 968239	491
3.39	0.342701 186219	7 834529	1040	3.89	0.292907 341870	4 925554	485
3.40	0.341541 397786	7 756575	1022	3.90	0.292057 683784	4 883354	478
3.41	0.340389 365928	7 679643	1007	3.91	0.291212 909052	4 841632	471
3.42	0.339245 013713	7 603718	990	3.92	0.290372 975951	4 800381	464
3.43	0.338108 265216	7 528783	974	3.93	0.289537 843231	4 759595	458
3.44	0.336979 045503	7 454822	958	3.94	0.288707 470106	4 719267	452
3.45	0.335857 280612	7 381820	943	3.95	0.287881 816248	4 679390	445
3.46	0.334742 897540	7 309761	928	3.96	0.287060 841779	4 639959	440
3.47	0.333635 824230	7 238629	913	3.97	0.286244 507269	4 600967	433
3.48	0.332535 989548	7 168411	899	3.98	0.285432 773727	4 562409	427
3.49	0.331443 323278	7 099092	884	3.99	0.284625 602593	4 524277	422
3.50	0.330357 756100	7 030657	870	4.00	0.283822 955737	4 486568	417

TABLE 15

THE TRIGAMMA FUNCTION

Description: $\Psi'(x)$ to ten decimal places with central differences from $x = 4.00$ to $x = 20.00$ at intervals of .02.

x	$\Psi'(x)$	δ^2	δ^4	x	$\Psi'(x)$	δ^2	δ^4
4.00	.28382 29557	179467	67	5.00	.22132 29557	85713	20
4.02	.28223 10844	176500	65	5.02	.22035 14280	84598	19
4.04	.28065 68631	173597	63	5.04	.21938 83600	83501	19
4.06	.27910 00014	170757	61	5.06	.21843 36422	82424	18
4.08	.27756 02155	167979	60	5.08	.21748 71667	81366	18
4.10	.27603 72276	165260	58	5.10	.21654 88278	80324	18
4.12	.27453 07656	162600	57	5.12	.21561 85213	79301	17
4.14	.27304 05636	159996	55	5.14	.21469 61448	78295	17
4.16	.27156 63612	157446	54	5.16	.21378 15979	77305	17
4.18	.27010 79034	154951	52	5.18	.21287 47815	76333	16
4.20	.26866 49407	152508	51	5.20	.21197 55983	75376	16
4.22	.26723 72288	150115	50	5.22	.21108 39528	74436	16
4.24	.26582 45284	147772	48	5.24	.21019 97509	73511	15
4.26	.26442 66052	145477	47	5.26	.20932 29000	72601	15
4.28	.26304 32297	143230	46	5.28	.20845 33092	71706	15
4.30	.26167 41773	141028	45	5.30	.20759 08890	70825	14
4.32	.26031 92276	138871	44	5.32	.20673 55514	69959	14
4.34	.25897 81651	136757	43	5.34	.20588 72096	69107	14
4.36	.25765 07782	134686	41	5.36	.20504 57786	68269	13
4.38	.25633 68600	132657	40	5.38	.20421 11746	67444	13
4.40	.25503 62075	130667	39	5.40	.20338 33149	66633	13
4.42	.25374 86217	128717	38	5.42	.20256 21185	65834	13
4.44	.25247 39076	126806	38	5.44	.20174 75055	65048	12
4.46	.25121 18741	124932	37	5.46	.20093 93973	64274	12
4.48	.24996 23338	123095	36	5.48	.20013 77165	63513	12
4.50	.24872 51030	121293	35	5.50	.19934 23870	62763	12
4.52	.24750 00015	119526	34	5.52	.19855 33338	62026	11
4.54	.24628 68526	117793	33	5.54	.19777 04832	61299	11
4.56	.24508 54830	116094	32	5.56	.19699 37625	60584	11
4.58	.24389 57228	114426	32	5.58	.19622 31002	59880	11
4.60	.24271 74052	112791	31	5.60	.19545 84260	59187	11
4.62	.24155 03667	111186	30	5.62	.19469 96705	58505	10
4.64	.24039 44468	109611	29	5.64	.19394 67654	57833	10
4.66	.23924 94880	108066	29	5.66	.19319 96436	57171	10
4.68	.23811 53358	106550	28	5.68	.19245 82389	56519	10
4.70	.23699 18387	105062	27	5.70	.19172 24860	55877	10
4.72	.23587 88477	103601	27	5.72	.19099 23209	55245	9
4.74	.23477 62169	102167	26	5.74	.19026 76801	54622	9
4.76	.23368 38028	100760	26	5.76	.18954 85016	54008	9
4.78	.23260 14646	99378	25	5.78	.18883 47238	53404	9
4.80	.23152 90642	98021	25	5.80	.18812 62864	52808	9
4.82	.23046 64659	96688	24	5.82	.18742 31299	52222	9
4.84	.22941 35364	95380	23	5.84	.18672 51955	51644	8
4.86	.22837 01450	94095	23	5.86	.18603 24254	51074	8
4.88	.22733 61629	92833	22	5.88	.18534 47628	50513	8
4.90	.22631 14642	91593	22	5.90	.18466 21514	49960	8
4.92	.22529 59247	90375	21	5.92	.18398 45360	49415	8
4.94	.22428 94227	89178	21	5.94	.18331 18621	48878	8
4.96	.22329 18386	88003	20	5.96	.18264 40759	48348	8
4.98	.22230 30548	86848	20	5.98	.18198 11245	47826	7
5.00	.22132 29557	85713	20	6.00	.18132 29557	47312	7

x	$\Psi'(x)$	δ^2	δ^4	x	$\Psi'(x)$	δ^2
6.00	.18132 29557	47312	7	7.00	.15354 51780	28793
6.02	.18066 95182	46805	7	7.02	.15307 60038	28531
6.04	.18002 07611	46305	7	7.04	.15260 96826	28272
6.06	.17937 66345	45812	7	7.06	.15214 61887	28016
6.08	.17873 70892	45327	7	7.08	.15168 54964	27763
6.10	.17810 20765	44848	7	7.10	.15122 75804	27514
6.12	.17747 15486	44375	7	7.12	.15077 24158	27267
6.14	.17684 54582	43910	7	7.14	.15031 99779	27023
6.16	.17622 37588	43450	6	7.16	.14987 02422	26782
6.18	.17560 64044	42998	6	7.18	.14942 31847	26544
6.20	.17499 33498	42551	6	7.20	.14897 87816	26309
6.22	.17438 45503	42111	6	7.22	.14853 70094	26076
6.24	.17377 99618	41676	6	7.24	.14809 78448	25846
6.26	.17317 95410	41248	6	7.26	.14766 12648	25619
6.28	.17258 32450	40825	6	7.28	.14722 72467	25395
6.30	.17199 10314	40408	6	7.30	.14679 57681	25173
6.32	.17140 28587	39997	6	7.32	.14636 68068	24954
6.34	.17081 86857	39591	5	7.34	.14594 03408	24737
6.36	.17023 84719	39191	5	7.36	.14551 63486	24523
6.38	.16966 21772	38796	5	7.38	.14509 48085	24311
6.40	.16908 97621	38407	5	7.40	.14467 56996	24101
6.42	.16852 11877	38022	5	7.42	.14425 90008	23895
6.44	.16795 64155	37643	5	7.44	.14384 46915	23690
6.46	.16739 54077	37269	5	7.46	.14343 27511	23488
6.48	.16683 81268	36900	5	7.48	.14302 31595	23288
6.50	.16628 45357	36535	5	7.50	.14261 58967	23090
6.52	.16573 45983	36175	5	7.52	.14221 09429	22894
6.54	.16518 82783	35820	5	7.54	.14180 82785	22701
6.56	.16464 55404	35470	5	7.56	.14140 78842	22510
6.58	.16410 63494	35124	4	7.58	.14100 97409	22321
6.60	.16357 06709	34783	4	7.60	.14061 38298	22134
6.62	.16303 84707	34446	4	7.62	.14022 01320	21949
6.64	.16250 97150	34113	4	7.64	.13982 86292	21767
6.66	.16198 43706	33785	4	7.66	.13943 93030	21586
6.68	.16146 24047	33461	4	7.68	.13905 21354	21407
6.70	.16094 37849	33140	4	7.70	.13866 71086	21230
6.72	.16042 84791	32824	4	7.72	.13828 42047	21055
6.74	.15991 64558	32512	4	7.74	.13790 34064	20883
6.76	.15940 76837	32204	4	7.76	.13752 46964	20711
6.78	.15890 21320	31900	4	7.78	.13714 80575	20542
6.80	.15839 97704	31600	4	7.80	.13677 34728	20375
6.82	.15790 05687	31303	4	7.82	.13640 09256	20209
6.84	.15740 44974	31010	4	7.84	.13603 03994	20046
6.86	.15691 15271	30721	4	7.86	.13566 18777	19884
6.88	.15642 16289	30435	4	7.88	.13529 53444	19723
6.90	.15593 47742	30153	3	7.90	.13493 07835	19565
6.92	.15545 09348	29874	3	7.92	.13456 81790	19408
6.94	.15497 00829	29599	3	7.94	.13420 75153	19253
6.96	.15449 21908	29327	3	7.96	.13384 87769	19099
6.98	.15401 72315	29058	3	7.98	.13349 19484	18947
7.00	.15354 51780	28793	3	8.00	.13313 70147	18797

x	$\Psi'(x)$	δ^2	x	$\Psi'(x)$	δ^2
8.00	.13313 70147	18797	9.00	.11751 20147	12938
8.02	.13278 39607	18648	9.02	.11723 67937	12847
8.04	.13243 27715	18501	9.04	.11696 28574	12757
8.06	.13208 34324	18356	9.06	.11669 01968	12669
8.08	.13173 59289	18212	9.08	.11641 88032	12581
8.10	.13139 02466	18069	9.10	.11614 86675	12494
8.12	.13104 63711	17928	9.12	.11587 97813	12407
8.14	.13070 42884	17788	9.14	.11561 21358	12322
8.16	.13036 39846	17650	9.16	.11534 57224	12237
8.18	.13002 54457	17513	9.18	.11508 05327	12153
8.20	.12968 86582	17378	9.20	.11481 65583	12069
8.22	.12935 36084	17244	9.22	.11455 37908	11987
8.24	.12902 02831	17111	9.24	.11429 22220	11905
8.26	.12868 86688	16980	9.26	.11403 18437	11824
8.28	.12835 87526	16850	9.28	.11377 26479	11744
8.30	.12803 05214	16722	9.30	.11351 46265	11664
8.32	.12770 39623	16594	9.32	.11325 77715	11586
8.34	12737 90626	16468	9.34	.11300 20750	11507
8.36	.12705 58098	16344	9.36	.11274 75293	11430
8.38	.12673 41913	16220	9.38	.11249 41266	11353
8.40	12641 41949	16098	9.40	.11224 18593	11277
8.42	.12609 58082	15977	9.42	.11199 07196	11202
8.44	.12577 90191	15857	9.44	.11174 07001	11127
8.46	.12546 38158	15738	9.46	.11149 17934	11053
8.48	.12515 01863	15621	9.48	.11124 39919	10980
8.50	.12483 81189	15505	9.50	.11099 72885	10907
8.52	.12452 76020	15390	9.52	.11075 16757	10835
8.54	.12421 86240	15276	9.54	.11050 71464	10763
8.56	.12391 11736	15163	9.56	.11026 36935	10693
8.58	.12360 52394	15051	9.58	.11002 13098	10622
8.60	.12330 08104	14940	9.60	.10977 99883	10553
8.62	.12299 78753	14831	9.62	.10953 97221	10484
8.64	.12269 64234	14722	9.64	.10930 05043	10415
8.66	.12239 64436	14615	9.66	.10906 23280	10348
8.68	.12209 79254	14508	9.68	.10882 51865	10280
8.70	.12180 08579	14403	9.70	.10858 90730	10214
8.72	.12150 52308	14299	9.72	.10835 39809	10148
8.74	.12121 10335	14195	9.74	.10811 99035	10082
8.76	.12091 82557	14093	9.76	.10788 68344	10017
8.78	.12062 68873	13991	9.78	.10765 47669	9953
8.80	.12033 69179	13891	9.80	.10742 36948	9889
8.82	.12004 83377	13792	9.82	.10719 36115	9826
8.84	.11976 11366	13693	9.84	.10696 45108	9763
8.86	.11947 53048	13595	9.86	.10673 63864	9701
8.88	.11919 08326	13499	9.88	.10650 92321	9639
8.90	.11890 77102	13403	9.90	.10628 30416	9578
8.92	.11862 59282	13308	9.92	.10605 78090	9517
8.94	.11834 54770	13214	9.94	.10583 35280	9457
8.96	.11806 63472	13121	9.96	.10561 01928	9397
8.98	.11778 85295	13029	9.98	.10538 77973	9338
9.00	.11751 20147	12938	10.00	.10516 63357	9280

x	$\Psi'(x)$	δ^2	x	$\Psi'(x)$	δ^2
10.00	.10516 63357	9280	11.00	.09516 63357	6880
10.02	.10494 58020	9221	11.02	.09498 56823	6841
10.04	.10472 61905	9164	11.04	.09480 57130	6802
10.06	.10450 74953	9107	11.06	.09462 64239	6763
10.08	.10428 97108	9050	11.08	.09444 78111	6725
10.10	.10407 28313	8994	11.10	.09426 98708	6687
10.12	.10385 68512	8938	11.12	.09409 25993	6650
10.14	.10364 17648	8883	11.14	.09391 59927	6612
10.16	.10342 75667	8828	11.16	.09374 00474	6575
10.18	.10321 42514	8773	11.18	.09356 47596	6539
10.20	.10300 18135	8719	11.20	.09339 01256	6502
10.22	.10279 02474	8666	11.22	.09321 61419	6466
10.24	.10257 95479	8613	11.24	.09304 28048	6430
10.26	.10236 97098	8560	11.26	.09287 01106	6394
10.28	.10216 07276	8508	11.28	.09269 80559	6359
10.30	.10195 25962	8456	11.30	.09252 66371	6324
10.32	.10174 53104	8405	11.32	.09235 58506	6289
10.34	.10153 88650	8354	11.34	.09218 56930	6254
10.36	.10133 32551	8303	11.36	.09201 61608	6220
10.38	.10112 84754	8253	11.38	.09184 72506	6186
10.40	.10092 45211	8203	11.40	.09167 89590	6152
10.42	.10072 13871	8154	11.42	.09151 12825	6118
10.44	.10051 90684	8105	11.44	.09134 42178	6085
10.46	.10031 75603	8056	11.46	.09117 77616	6051
10.48	.10011 68578	8008	11.48	.09101 19105	6019
10.50	.09991 69561	7960	11.50	.09084 66613	5986
10.52	.09971 78504	7913	11.52	.09068 20106	5953
10.54	.09951 95360	7866	11.54	.09051 79554	5921
10.56	.09932 20083	7819	11.56	.09035 44922	5889
10.58	.09912 52624	7773	11.58	.09019 16180	5857
10.60	.09892 92939	7727	11.60	.09002 93295	5826
10.62	.09873 40980	7681	11.62	.08986 76236	5795
10.64	.09853 96703	7636	11.64	.08970 64972	5764
10.66	.09834 60062	7591	11.66	.08954 59471	5733
10.68	.09815 31013	7547	11.68	.08938 59703	5702
10.70	.09796 09510	7503	11.70	.08922 65637	5672
10.72	.09776 95510	7459	11.72	.08906 77243	5642
10.74	.09757 88969	7415	11.74	.08890 94491	5612
10.76	.09738 89843	7372	11.76	.08875 17350	5582
10.78	.09719 98090	7329	11.78	.08859 45791	5552
10.80	.09701 13666	7287	11.80	.08843 79784	5523
10.82	.09682 36529	7245	11.82	.08828 19300	5494
10.84	.09663 66636	7203	11.84	.08812 64309	5465
10.86	.09645 03947	7161	11.86	.08797 14784	5436
10.88	.09626 48419	7120	11.88	.08781 70694	5407
10.90	.09608 00011	7079	11.90	.08766 32012	5379
10.92	.09589 58683	7039	11.92	.08750 98709	5351
10.94	.09571 24393	6999	11.94	.08735 70757	5323
10.96	.09552 97102	6959	11.96	.08720 48128	5295
10.98	.09534 76770	6919	11.98	.08705 30795	5268
11.00	.09516 63357	6880	12.00	.08690 18729	5240

x	$\Psi'(x)$	δ^2	x	$\Psi'(x)$	δ^2
12.00	.08690 18729	5240	13.00	.07995 74284	4083
12.02	.08675 11903	5213	13.02	.07982 98363	4063
12.04	.08660 10291	5186	13.04	.07970 26505	4044
12.06	.08645 13865	5159	13.06	.07957 58691	4025
12.08	.08630 22599	5133	13.08	.07944 94902	4006
12.10	.08615 36465	5106	13.10	.07932 35119	3987
12.12	.08600 55437	5080	13.12	.07919 79323	3968
12.14	.08585 79490	5054	13.14	.07907 27494	3949
12.16	.08571 08597	5028	13.16	.07894 79615	3930
12.18	.08556 42731	5002	13.18	.07882 35666	3912
12.20	.08541 81869	4977	13.20	.07869 95628	3893
12.22	.08527 25983	4952	13.22	.07857 59485	3875
12.24	.08512 75048	4926	13.24	.07845 27216	3857
12.26	.08498 29040	4901	13.26	.07832 98805	3839
12.28	.08483 87933	4876	13.28	.07820 74232	3821
12.30	.08469 51702	4852	13.30	.07808 53481	3803
12.32	.08455 20324	4827	13.32	.07796 36532	3785
12.34	.08440 93772	4803	13.34	.07784 23369	3768
12.36	.08426 72023	4779	13.36	.07772 13974	3750
12.38	.08412 55053	4755	13.38	.07760 08329	3733
12.40	.08398 42837	4731	13.40	.07748 06416	3716
12.42	.08384 35352	4707	13.42	.07736 08220	3698
12.44	.08370 32574	4683	13.44	.07724 13721	3681
12.46	.08356 34479	4660	13.46	.07712 22904	3664
12.48	.08342 41044	4637	13.48	.07700 35751	3647
12.50	.08328 52246	4614	13.50	.07688 52246	3631
12.52	.08314 68062	4591	13.52	.07676 72371	3614
12.54	.08300 88468	4568	13.54	.07664 96110	3597
12.56	.08287 13442	4545	13.56	.07653 23447	3581
12.58	.08273 42962	4523	13.58	.07641 54364	3564
12.60	.08259 77005	4501	13.60	.07629 88846	3548
12.62	.08246 15547	4478	13.62	.07618 26877	3532
12.64	.08232 58569	4456	13.64	.07606 68439	3516
12.66	.08219 06046	4434	13.66	.07595 13517	3500
12.68	.08205 57958	4413	13.68	.07583 62096	3484
12.70	.08192 14282	4391	13.70	.07572 14158	3468
12.72	.08178 74998	4369	13.72	.07560 69689	3453
12.74	.08165 40082	4348	13.74	.07549 28673	3437
12.76	.08152 09515	4327	13.76	.07537 91094	3422
12.78	.08138 83275	4306	13.78	.07526 56936	3406
12.80	.08125 61341	4285	13.80	.07515 26185	3391
12.82	.08112 43691	4264	13.82	.07503 98824	3376
12.84	.08099 30306	4244	13.84	.07492 74839	3361
12.86	.08086 21165	4223	13.86	.07481 54215	3345
12.88	.08073 16246	4203	13.88	.07470 36936	3331
12.90	.08060 15530	4182	13.90	.07459 22988	3316
12.92	.08047 18996	4162	13.92	.07448 12355	3301
12.94	.08034 26625	4142	13.94	.07437 05023	3286
12.96	.08021 38396	4122	13.96	.07426 00977	3272
12.98	.08008 54289	4103	13.98	.07415 00204	3257
13.00	.07995 74284	4083	14.00	.07404 02687	3243

x	$\Psi'(x)$	δ^2	x	$\Psi'(x)$	δ^2
14.00	.07404 02687	3243	15.00	.06893 82278	2618
14.02	.07393 08412	3228	15.02	.06884 33465	2607
14.04	.07382 17367	3214	15.04	.06874 87259	2596
14.06	.07371 29535	3200	15.06	.06865 43649	2586
14.08	.07360 44903	3186	15.08	.06856 02625	2575
14.10	.07349 63457	3172	15.10	.06846 64176	2565
14.12	.07338 85182	3158	15.12	.06837 28292	2554
14.14	.07328 10066	3144	15.14	.06827 94961	2544
14.16	.07317 38093	3130	15.16	.06818 64175	2533
14.18	.07306 69251	3117	15.18	.06809 35921	2523
14.20	.07296 03526	3103	15.20	.06800 10191	2513
14.22	.07285 40903	3089	15.22	.06790 86973	2502
14.24	.07274 81370	3076	15.24	.06781 66258	2492
14.26	.07264 24912	3063	15.26	.06772 48035	2482
14.28	.07253 71518	3049	15.28	.06763 32294	2472
14.30	.07243 21172	3036	15.30	.06754 19025	2462
14.32	.07232 73863	3023	15.32	.06745 08219	2452
14.34	.07222 29577	3010	15.34	.06735 99865	2442
14.36	.07211 88300	2997	15.36	.06726 93953	2433
14.38	.07201 50021	2984	15.38	.06717 90474	2423
14.40	.07191 14726	2971	15.40	.06708 89417	2413
14.42	.07180 82401	2958	15.42	.06699 90773	2403
14.44	.07170 53035	2946	15.44	.06690 94533	2394
14.46	.07160 26615	2933	15.46	.06682 00686	2384
14.48	.07150 03128	2921	15.48	.06673 09224	2375
14.50	.07139 82562	2908	15.50	.06664 20136	2365
14.52	.07129 64903	2896	15.52	.06655 33413	2356
14.54	.07119 50140	2883	15.54	.06646 49046	2346
14.56	.07109 38261	2871	15.56	.06637 67025	2337
14.58	.07099 29252	2859	15.58	.06628 87341	2328
14.60	.07089 23102	2847	15.60	.06620 09985	2319
14.62	.07079 19799	2835	15.62	.06611 34948	2309
14.64	.07069 19331	2823	15.64	.06602 62220	2300
14.66	.07059 21686	2811	15.66	.06593 91792	2291
14.68	.07049 26851	2799	15.68	.06585 23655	2282
14.70	.07039 34815	2787	15.70	.06576 57800	2273
14.72	.07029 45566	2775	15.72	.06567 94219	2264
14.74	.07019 59092	2764	15.74	.06559 32901	2255
14.76	.07009 75382	2752	15.76	.06550 73839	2246
14.78	.06999 94425	2741	15.78	.06542 17024	2238
14.80	.06990 16208	2729	15.80	.06533 62446	2229
14.82	.06980 40720	2718	15.82	.06525 10097	2220
14.84	.06970 67949	2706	15.84	.06516 59968	2212
14.86	.06960 97886	2695	15.86	.06508 12050	2203
14.88	.06951 30517	2684	15.88	.06499 66336	2194
14.90	.06941 65832	2673	15.90	.06491 22816	2186
14.92	.06932 03820	2662	15.92	.06482 81482	2177
14.94	.06922 44470	2651	15.94	.06474 42325	2169
14.96	.06912 87770	2640	15.96	.06466 05337	2161
14.98	.06903 33710	2629	15.98	.06457 70509	2152
15.00	.06893 82278	2618	16.00	.06449 37834	2144

x	$\Psi'(x)$	δ^2	x	$\Psi'(x)$	δ^2
16.00	.06449 37834	2144	17.00	.06058 75334	1778
16.02	.06441 07303	2136	17.02	.06051 42276	1771
16.04	.06432 78907	2127	17.04	.06044 10989	1765
16.06	.06424 52638	2119	17.06	.06036 81467	1758
16.08	.06416 28489	2111	17.08	.06029 53703	1752
16.10	.06408 06450	2103	17.10	.06022 27692	1746
16.12	.06399 86515	2095	17.12	.06015 03426	1739
16.14	.06391 68675	2087	17.14	.06007 80900	1733
16.16	.06383 52921	2079	17.16	.06000 60106	1727
16.18	.06375 39246	2071	17.18	.05993 41040	1721
16.20	.06367 27642	2063	17.20	.05986 23695	1715
16.22	.06359 18102	2055	17.22	.05979 08064	1708
16.24	.06351 10616	2047	17.24	.05971 94142	1702
16.26	.06343 05178	2040	17.26	.05964 81922	1696
16.28	.06335 01780	2032	17.28	.05957 71398	1690
16.30	.06327 00413	2024	17.30	.05950 62564	1684
16.32	.06319 01071	2017	17.32	.05943 55415	1678
16.34	.06311 03745	2009	17.34	.05936 49944	1672
16.36	.06303 08428	2001	17.36	.05929 46145	1666
16.38	.06295 15112	1994	17.38	.05922 44013	1660
16.40	.06287 23790	1986	17.40	.05915 43540	1655
16.42	.06279 34455	1979	17.42	.05908 44723	1649
16.44	.06271 47098	1971	17.44	.05901 47554	1643
16.46	.06263 61713	1964	17.46	.05894 52028	1637
16.48	.06255 78291	1957	17.48	.05887 58139	1631
16.50	.06247 96827	1949	17.50	.05880 65881	1626
16.52	.06240 17311	1942	17.52	.05873 75249	1620
16.54	.06232 39738	1935	17.54	.05866 86236	1614
16.56	.06224 64100	1928	17.56	.05859 98838	1608
16.58	.06216 90389	1920	17.58	.05853 13048	1603
16.60	.06209 18598	1913	17.60	.05846 28861	1597
16.62	.06201 48721	1906	17.62	.05839 46271	1592
16.64	.06193 80750	1899	17.64	.05832 65273	1586
16.66	.06186 14678	1892	17.66	.05825 85861	1581
16.68	.06178 50498	1885	17.68	.05819 08029	1575
16.70	.06170 88203	1878	17.70	.05812 31772	1570
16.72	.06163 27787	1871	17.72	.05805 57085	1564
16.74	.06155 69241	1864	17.74	.05798 83962	1559
16.76	.06148 12560	1857	17.76	.05792 12398	1553
16.78	.06140 57736	1851	17.78	.05785 42387	1548
16.80	.06133 04763	1844	17.80	.05778 73924	1542
16.82	.06125 53633	1837	17.82	.05772 07003	1537
16.84	.06118 04341	1830	17.84	.05765 41619	1532
16.86	.06110 56879	1824	17.86	.05758 77768	1527
16.88	.06103 11240	1817	17.88	.05752 15443	1521
16.90	.06095 67419	1810	17.90	.05745 54639	1516
16.92	.06088 25407	1804	17.92	.05738 95351	1511
16.94	.06080 85200	1797	17.94	.05732 37575	1506
16.96	.06073 46789	1791	17.96	.05725 81304	1501
16.98	.06066 10170	1784	17.98	.05719 26533	1495
17.00	.06058 75334	1778	18.00	.05712 73258	1490

x	$\Psi'(x)$	δ^2	x	$\Psi'(x)$	δ^2
18.00	.05712 73258	1490	19.00	.05404 09060	1262
18.02	.05706 21473	1485	19.02	.05398 25749	1258
18.04	.05699 71173	1480	19.04	.05392 43694	1254
18.06	.05693 22354	1475	19.06	.05386 62893	1249
18.08	.05686 75009	1470	19.08	.05380 83342	1245
18.10	.05680 29135	1465	19.10	.05375 05036	1241
18.12	.05673 84726	1460	19.12	.05369 27972	1237
18.14	.05667 41776	1455	19.14	.05363 52145	1233
18.16	.05661 00282	1450	19.16	.05357 77551	1230
18.18	.05654 60238	1445	19.18	.05352 04187	1226
18.20	.05648 21640	1440	19.20	.05346 32049	1222
18.22	.05641 84481	1436	19.22	.05340 61132	1218
18.24	.05635 48759	1431	19.24	.05334 91433	1214
18.26	.05629 14466	1426	19.26	.05329 22948	1210
18.28	.05622 81600	1421	19.28	.05323 55673	1206
18.30	.05616 50155	1416	19.30	.05317 89604	1202
18.32	.05610 20126	1412	19.32	.05312 24737	1198
18.34	.05603 91509	1407	19.34	.05306 61069	1195
18.36	.05597 64298	1402	19.36	.05300 98595	1191
18.38	.05591 38489	1397	19.38	.05295 37312	1187
18.40	.05585 14078	1393	19.40	.05289 77216	1183
18.42	.05578 91060	1388	19.42	.05284 18304	1180
18.44	.05572 69429	1383	19.44	.05278 60571	1176
18.46	.05566 49182	1379	19.46	.05273 04014	1172
18.48	.05560 30314	1374	19.48	.05267 48629	1168
18.50	.05554 12820	1370	19.50	.05261 94412	1165
18.52	.05547 96695	1365	19.52	.05256 41360	1161
18.54	.05541 81936	1361	19.54	.05250 89470	1157
18.56	.05535 68537	1356	19.56	.05245 38736	1154
18.58	.05529 56494	1352	19.58	.05239 89157	1150
18.60	.05523 45803	1347	19.60	.05234 40727	1147
18.62	.05517 36459	1343	19.62	.05228 93445	1143
18.64	.05511 28457	1338	19.64	.05223 47305	1139
18.66	.05505 21794	1334	19.66	.05218 02304	1136
18.68	.05499 16465	1329	19.68	.05212 58440	1132
18.70	.05493 12464	1325	19.70	.05207 15707	1129
18.72	.05487 09789	1321	19.72	.05201 74104	1125
18.74	.05481 08435	1316	19.74	.05196 33625	1122
18.76	.05475 08397	1312	19.76	.05190 94269	1118
18.78	.05469 09671	1308	19.78	.05185 56030	1115
18.80	.05463 12252	1303	19.80	.05180 18907	1111
18.82	.05457 16137	1299	19.82	.05174 82895	1108
18.84	.05451 21321	1295	19.84	.05169 47990	1104
18.86	.05445 27801	1291	19.86	.05164 14190	1101
18.88	.05439 35570	1287	19.88	.05158 81491	1098
18.90	.05433 44627	1282	19.90	.05153 49890	1094
18.92	.05427 54966	1278	19.92	.05148 19383	1091
18.94	.05421 66582	1274	19.94	.05142 89966	1087
18.96	.05415 79473	1270	19.96	.05137 61638	1084
18.98	.05409 93634	1266	19.98	.05132 34393	1081
19.00	.05404 09060	1262	20.00	.05127 08229	1078

TABLE 16

The Trigamma Function

Description: $\Psi'(x)$ to 15 decimal places with central differences from $x = 20.0$ to $x = 100.0$ by increments of .1.

x	$\Psi'(x)$	δ^2	δ^4	δ^6
20.0	.05127 08229 35203	26938 12917	848538	668
20.1	.05100 93507 00253	26528 25777	827138	644
20.2	.05075 05312 91080	26126 65773	806382	622
20.3	.05049 43245 47679	25733 12152	786248	600
20.4	.05024 06911 16431	25347 44778	766714	579
20.5	.04998 95924 29961	24969 44118	747759	560
20.6	.04974 09906 87609	24598 91217	729364	540
20.7	.04949 48488 36473	24235 67679	711509	522
20.8	.04925 11305 53017	23879 55651	694176	504
20.9	.04900 98002 25211	23530 37798	677347	487
21.0	.04877 08229 35203	23187 97292	661005	471
21.1	.04853 41644 42487	22852 17791	645134	455
21.2	.04829 97911 67562	22522 83424	629718	440
21.3	.04806 76701 76062	22199 78775	614742	426
21.4	.04783 77691 63336	21882 88868	600191	411
21.5	.04761 00564 39479	21571 99152	586051	398
21.6	.04738 45009 14774	21266 95488	572310	384
21.7	.04716 10720 85557	20967 64132	558953	373
21.8	.04693 97400 20472	20673 91730	545968	360
21.9	.04672 04753 47118	20385 65295	533343	349
22.0	.04650 32492 39058	20102 72204	521067	338
22.1	.04628 80334 03202	19825 00180	509129	327
22.2	.04607 48000 67527	19552 37285	497517	317
22.3	.04586 35219 69136	19284 71908	486222	306
22.4	.04565 41723 42653	19021 92752	475233	297
22.5	.04544 67249 08922	18763 88829	464540	288
22.6	.04524 11538 64020	18510 49446	454135	278
22.7	.04503 74338 68564	18261 64198	444008	270
22.8	.04483 55400 37306	18017 22959	434151	261
22.9	.04463 54479 29007	17777 15871	424556	253
23.0	.04443 71335 36579	17541 33338	415213	246
23.1	.04424 05732 77488	17309 66019	406117	238
23.2	.04404 57439 84416	17082 04816	397258	231
23.3	.04385 26228 96161	16858 40871	388630	224
23.4	.04366 11876 48776	16638 65556	380226	217
23.5	.04347 14162 66947	16422 70467	372039	211
23.6	.04328 32871 55585	16210 47418	364063	204
23.7	.04309 67790 91642	16001 88432	356291	198
23.8	.04291 18712 16130	15796 85736	348717	192
23.9	.04272 85430 26355	15595 31758	341336	187
24.0	.04254 67743 68337	15397 19116	334141	181
24.1	.04236 65454 29434	15202 40614	327128	176
24.2	.04218 78367 31147	15010 89241	320290	171
24.3	.04201 06291 22100	14822 58157	313623	166
24.4	.04183 49037 71210	14637 40697	307122	161
24.5	.04166 06421 61016	14455 30358	300782	156
24.6	.04148 78260 81182	14276 20802	294598	152
24.7	.04131 64376 22148	14100 05844	288567	148
24.8	.04114 64591 68958	13926 79452	282682	143
24.9	.04097 78733 95221	13756 35743	276942	139
25.0	.04081 06632 57226	13588 68975	271340	135

x	$\Psi'(x)$	δ^2	δ^4	δ^6
25.0	.04081 06632 57226	13588 68975	271340	135
25.1	.04064 48119 88206	13423 73548	265874	132
25.2	.04048 03030 92734	13261 43995	260540	130
25.3	.04031 71203 41257	13101 74981	255333	124
25.4	.04015 52477 64760	12944 61300	250251	121
25.5	.03999 46696 49563	12789 97870	245289	118
25.6	.03983 53705 32236	12637 79729	240446	114
25.7	.03967 73351 94637	12488 02034	235716	111
25.8	.03952 05486 59073	12340 60054	231098	108
25.9	.03936 49961 83563	12195 49173	226588	105
26.0	.03921 06632 57226	12052 64879	222183	102
26.1	.03905 75355 95768	11912 02768	217880	99
26.2	.03890 55991 37078	11773 58537	213677	97
26.3	.03875 48400 36924	11637 27983	209571	94
26.4	.03860 52446 64754	11503 06999	205559	92
26.5	.03845 67995 99582	11370 91575	201639	90
26.6	.03830 94916 25986	11240 77790	197809	87
26.7	.03816 33077 30179	11112 61813	194065	85
26.8	.03801 82350 96186	10986 39901	190406	82
26.9	.03787 42611 02094	10862 08395	186829	80
27.0	.03773 13733 16397	10739 63718	183333	78
27.1	.03758 95594 94419	10619 02374	179915	76
27.2	.03744 88075 74814	10500 20946	176573	74
27.3	.03730 91056 76156	10383 16090	173306	72
27.4	.03717 04420 93587	10267 84540	170110	70
27.5	.03703 28052 95559	10154 23101	166985	69
27.6	.03689 61839 20632	10042 28647	163929	66
27.7	.03676 05667 74352	9931 98122	160940	65
27.8	.03662 59428 26193	9823 28536	158016	63
27.9	.03649 23012 06570	9716 16967	155155	62
28.0	.03635 96312 03914	9610 60552	152356	60
28.1	.03622 79222 61810	9506 56493	149618	59
28.2	.03609 71639 76198	9404 02051	146938	58
28.3	.03596 73460 92638	9302 94548	144316	56
28.4	.03583 84585 03625	9203 31360	141749	55
28.5	.03571 04912 45973	9105 09922	139238	53
28.6	.03558 34344 98242	9008 27722	136779	52
28.7	.03545 72785 78233	8912 82301	134373	51
28.8	.03533 20139 40524	8818 71252	132017	49
28.9	.03520 76311 74068	8725 92221	129710	48
29.0	.03508 41209 99833	8634 42900	127452	47
29.1	.03496 14742 68497	8544 21030	125241	46
29.2	.03483 96819 58191	8455 24401	123075	45
29.3	.03471 87351 72287	8367 50847	120954	44
29.4	.03459 86251 37230	8280 98248	118877	43
29.5	.03447 93432 00420	8195 64525	116843	42
29.6	.03436 08808 28136	8111 47646	114850	40
29.7	.03424 32296 03497	8028 45616	112898	40
29.8	.03412 63812 24475	7946 56485	110986	39
29.9	.03401 03275 01938	7865 78339	109112	38
30.0	.03389 50603 57740	7786 09306	107276	37

x	$\Psi'(x)$	δ^2	δ^4	δ^6
30.0	.03389 50603 57740	7786 09306	107276	37
30.1	.03378 05718 22848	7707 47548	105477	36
30.2	.03366 68540 35504	7629 91268	103714	35
30.3	.03355 38992 39428	7553 38702	101987	34
30.4	.03344 16997 82054	7477 88122	100294	34
30.5	.03333 02481 12802	7403 37837	98634	33
30.6	.03321 95367 81386	7329 86185	97007	32
30.7	.03310 95584 36156	7257 31540	95413	31
30.8	.03300 03058 22466	7185 72308	93849	31
30.9	.03289 17717 81084	7115 06926	92317	30
31.0	.03278 39492 46629	7045 33860	90814	30
31.1	.03267 68312 46033	6976 51608	89341	28
31.2	.03257 04108 97045	6908 58696	87896	29
31.3	.03246 46814 06754	6841 53680	86479	27
31.4	.03235 96360 70142	6775 35143	85089	27
31.5	.03225 52682 68674	6710 01695	83726	26
31.6	.03215 15714 68900	6645 51972	82389	25
31.7	.03204 85392 21098	6581 84639	81078	25
31.8	.03194 61651 57936	6518 98385	79792	24
31.9	.03184 44429 93158	6456 91922	78530	24
32.0	.03174 33665 20302	6395 63989	77292	23
32.1	.03164 29296 11435	6335 13347	76077	23
32.2	.03154 31262 15914	6275 38783	74885	23
32.3	.03144 39503 59177	6216 39104	73716	21
32.4	.03134 53961 41545	6158 13142	72568	22
32.5	.03124 74577 37054	6100 59747	71442	21
32.6	.03115 01293 92309	6043 77794	70337	20
32.7	.03105 34054 25359	5987 66178	69252	20
32.8	.03095 72802 24587	5932 23813	68187	20
32.9	.03086 17482 47627	5877 49635	67141	19
33.0	.03076 68040 20302	5823 42598	66115	19
33.1	.03067 24421 35575	5770 01677	65108	18
33.2	.03057 86572 52526	5717 25863	64119	18
33.3	.03048 54440 95339	5665 14168	63148	17
33.4	.03039 27974 52320	5613 65621	62194	17
33.5	.03030 07121 74923	5562 79269	61258	17
33.6	.03020 91831 76795	5512 54174	60339	16
33.7	.03011 82054 32841	5462 89418	59436	16
33.8	.03002 77739 78305	5413 84098	58549	16
33.9	.02993 78839 07867	5365 37327	57678	14
34.0	.02984 85303 74756	5317 48235	56822	18
34.1	.02975 97085 89879	5270 15965	55984	12
34.2	.02967 14138 20968	5223 39678	55157	16
34.3	.02958 36413 91734	5177 18548	54347	14
34.4	.02949 63866 81049	5131 51765	53550	14
34.5	.02940 96451 22128	5086 38532	52767	14
34.6	.02932 34122 01738	5041 78066	51999	13
34.7	.02923 76834 59415	4997 69599	51243	13
34.8	.02915 24544 86690	4954 12375	50501	13
34.9	.02906 77209 26340	4911 05651	49771	12
35.0	.02898 34784 71642	4868 48700	49054	13

	$\Psi'(x)$	δ^2	δ^4	
35.0	.02898 34784 71642	4868 48700	49054	13
35.1	.02889 97228 65643	4826 40802	48350	12
35.2	.02881 64499 00446	4784 81254	47657	12
35.3	.02873 36554 16503	4743 69364	46977	12
35.4	.02865 13353 01925	4703 04451	46308	12
35.5	.02856 94854 91798	4662 85846	45651	11
35.6	.02848 81019 67517	4623 12892	45004	11
35.7	.02840 71807 56129	4583 84942	44369	11
35.8	.02832 67179 29681	4545 01360	43744	10
35.9	.02824 67096 04594	4506 61523	43130	10
36.0	.02816 71519 41029	4468 64815	42526	10
36.1	.02808 80411 42280	4431 10634	41932	10
36.2	.02800 93734 54165	4393 98385	41349	10
36.3	.02793 11451 64435	4357 27485	40775	9
36.4	.02785 33526 02190	4320 97360	40210	9
36.5	.02777 59921 37305	4285 07444	39655	9
36.6	.02769 90601 79864	4249 57184	39109	9
36.7	.02762 25531 79608	4214 46032	38572	9
36.8	.02754 64676 25384	4179 73453	38044	9
36.9	.02747 08000 44612	4145 38917	37524	9
37.0	.02739 55470 02758	4111 41905	37013	8
37.1	.02732 07051 02808	4077 81906	36510	
37.2	.02724 62709 84765	4044 58418	36016	
37.3	.02717 22413 25140	4011 70946	35529	
37.4	.02709 86128 36461	3979 19003	35051	
37.5	.02702 53822 66785	3947 02111	34580	
37.6	.02695 25463 99220	3915 19798	34116	7
37.7	.02688 01020 51453	3883 71602	33660	7
37.8	.02680 80460 75289	3852 57067	33212	7
37.9	.02673 63753 56192	3821 75743	32770	7
38.0	.02666 50868 12838	3791 27190	32336	7
38.1	.02659 41773 96674	3761 10972	31908	7
38.2	.02652 36440 91482	3731 26663	31486	7
38.3	.02645 34839 12954	3701 73842	31073	7
38.4	.02638 36939 08267	3672 52093	30666	6
38.5	.02631 42711 55674	3643 61011	30264	6
38.6	.02624 52127 64091	3615 00192	29870	6
38.7	.02617 65158 72700	3586 69244	29481	6
38.8	.02610 81776 50553	3558 67775	29098	6
38.9	.02604 01952 96181	3530 95405	28721	6
39.0	.02597 25660 37215	3503 51756	28350	6
39.1	.02590 52871 30005	3476 36458	27985	6
39.2	.02583 83558 59252	3449 49144	27625	6
39.3	.02577 17695 37643	3422 89455	27271	6
39.4	.02570 55255 05490	3396 57038	26923	5
39.5	.02563 96211 30375	3370 51544	26579	5
39.6	.02557 40538 06803	3344 72629	26241	5
39.7	.02550 88209 55862	3319 19956	25908	5
39.8	.02544 39200 24875	3293 93191	25581	5
39.9	.02537 93484 87080	3268 92006	25258	5
40.0	.02531 51038 41291	3244 16080	24940	5

x	$\Psi'(x)$	δ^2	δ^4	δ^6
40.0	.02531 51038 41291	3244 16080	24940	5
40.1	.02525 11836 11582	3219 65093	24627	5
40.2	.02518 75853 46965	3195 38733	24318	5
40.3	.02512 43066 21082	3171 36690	24014	5
40.4	.02506 13450 31888	3147 58662	23715	5
40.5	.02499 86982 01357	3124 04349	23420	5
40.6	.02493 63637 75174	3100 73455	23130	5
40.7	.02487 43394 22447	3077 65691	22843	5
40.8	.02481 26228 35411	3054 80771	22561	4
40.9	.02475 12117 29145	3032 18411	22284	4
41.0	.02469 01038 41291	3009 78336	22010	4
41.1	.02462 92969 31773	2987 60270	21740	4
41.2	.02456 87887 82524	2965 63944	21474	4
41.3	.02450 85771 97220	2943 89093	21213	4
41.4	.02444 86600 01009	2922 35455	20955	4
41.5	.02438 90350 40253	2901 02772	20700	4
41.6	.02432 97001 82270	2879 90789	20450	4
41.7	.02427 06533 15075	2858 99256	20203	4
41.8	.02421 18923 47137	2838 27927	19960	4
41.9	.02415 34152 07125	2817 76557	19720	4
42.0	.02409 52198 43671	2797 44907	19484	4
42.1	.02403 73042 25123	2777 32741	19251	4
42.2	.02397 96663 39315	2757 39825	19021	3
42.3	.02392 23041 93333	2737 65930	18795	3
42.4	.02386 52158 13280	2718 10829	18571	3
42.5	.02380 83992 44056	2698 74301	18351	3
42.6	.02375 18525 49133	2679 56123	18135	3
42.7	.02369 55738 10333	2660 56080	17921	3
42.8	.02363 95611 27613	2641 73958	17710	3
42.9	.02358 38126 18851	2623 09545	17502	3
43.0	.02352 83264 19634	2604 62635	17297	3
43.1	.02347 31006 83053	2586 33023	17095	3
43.2	.02341 81335 79494	2568 20505	16896	3
43.3	.02336 34232 96441	2550 24884	16700	3
43.4	.02330 89680 38271	2532 45962	16506	3
43.5	.02325 47660 26063	2514 83546	16315	3
43.6	.02320 08154 97402	2497 37446	16127	3
43.7	.02314 71147 06187	2480 07472	15941	3
43.8	.02309 36619 22443	2462 93439	15758	3
43.9	.02304 04554 32137	2445 95163	15577	3
44.0	.02298 74935 36995	2429 12465	15399	2
44.1	.02293 47745 54318	2412 45165	15223	2
44.2	.02288 22968 16806	2395 93089	15050	2
44.3	.02283 00586 72383	2379 56063	14879	2
44.4	.02277 80584 84023	2363 33915	14710	2
44.5	.02272 62946 29578	2347 26478	14544	2
44.6	.02267 47655 01611	2331 33584	14380	2
44.7	.02262 34695 07228	2315 55070	14218	2
44.8	.02257 24050 67915	2299 90774	14058	2
44.9	.02252 15706 19377	2284 40536	13901	2
45.0	.02247 09646 11375	2269 04199	13745	2

x	$\Psi'(x)$	δ^2	δ^4	δ^6
45.0	.02247 09646 11375	2269 04199	13745	2
45.1	.02242 05855 07573	2253 81607	13592	2
45.2	.02237 04317 85377	2238 72606	13440	2
45.3	.02232 05019 35788	2223 77046	13291	2
45.4	.02227 07944 63245	2208 94777	13144	2
45.5	.02222 13078 85480	2194 25652	12998	2
45.6	.02217 20407 33367	2179 69526	12855	2
45.7	.02212 29915 50779	2165 26254	12713	2
45.8	.02207 41588 94446	2150 95696	12574	2
45.9	.02202 55413 33808	2136 77711	12436	2
46.0	.02197 71374 50881	2122 72163	12300	2
46.1	.02192 89458 40118	2108 78914	12166	2
46.2	.02188 09651 08268	2094 97832	12033	2
46.3	.02183 31938 74251	2081 28782	11902	2
46.4	.02178 56307 69015	2067 71635	11773	2
46.5	.02173 82744 35413	2054 26261	11646	2
46.6	.02169 11235 28073	2040 92533	11520	2
46.7	.02164 41767 13265	2027 70325	11396	2
46.8	.02159 74326 68783	2014 59514	11274	2
46.9	.02155 08900 83814	2001 59976	11153	2
47.0	.02150 45476 58821	1988 71591	11033	2
47.1	.02145 84041 05419	1975 94239	10915	1
47.2	.02141 24581 46255	1963 27802	10799	1
47.3	.02136 67085 14894	1950 72165	10684	1
47.4	.02132 11539 55697	1938 27212	10571	1
47.5	.02127 57932 23713	1925 92830	10459	1
47.6	.02123 06250 84557	1913 68906	10348	1
47.7	.02118 56483 14309	1901 55331	10239	1
47.8	.02114 08616 99391	1889 51996	10131	1
47.9	.02109 62640 36469	1877 58791	10025	1
48.0	.02105 18541 32338	1865 75612	9920	1
48.1	.02100 76308 03820	1854 02353	9816	1
48.2	.02096 35928 77654	1842 38910	9714	1
48.3	.02091 97391 90398	1830 85181	9613	1
48.4	.02087 60685 88324	1819 41065	9513	1
48.5	.02083 25799 27314	1808 06461	9414	1
48.6	.02078 92720 72764	1796 81271	9317	1
48.7	.02074 61438 99487	1785 65399	9220	1
48.8	.02070 31942 91608	1774 58746	9125	1
48.9	.02066 04221 42475	1763 61219	9032	1
49.0	.02061 78263 54561	1752 72724	8939	1
49.1	.02057 54058 39370	1741 93167	8847	1
49.2	.02053 31595 17347	1731 22458	8757	1
49.3	.02049 10863 17781	1720 60505	8668	1
49.4	.02044 91851 78721	1710 07220	8579	1
49.5	.02040 74550 46880	1699 62514	8492	1
49.6	.02036 58948 77554	1689 26301	8406	1
49.7	.02032 45036 34528	1678 98493	8321	1
49.8	.02028 32802 89995	1668 79006	8237	1
49.9	.02024 22238 24468	1658 67756	8154	1
50.0	.02020 13332 26697	1648 64660	8072	1

x	$\Psi'(x)$	δ^2	δ^4	x	$\Psi'(x)$	δ^2	δ^4
50.0	.02020 13332 26697	1648 64660	8072	55.0	.01834 81091 24868	1235 28981	4989
50.1	.02016 06074 93586	1638 69636	7991	55.1	.01831 45064 09456	1228 51567	4944
50.2	.02012 00456 30110	1628 82602	7911	55.2	.01828 10265 45611	1221 79096	4899
50.3	.02007 96466 49237	1619 03479	7832	55.3	.01824 76688 60861	1215 11525	4854
50.4	.02003 94095 71843	1609 32188	7754	55.4	.01821 44326 87637	1208 48808	4810
50.5	.01999 93334 26637	1599 68651	7676	55.5	.01818 13173 63222	1201 90902	4767
50.6	.01995 94172 50082	1590 12790	7600	55.6	.01814 83222 29708	1195 37762	4724
50.7	.01991 96600 86317	1580 64529	7525	55.7	.01811 54466 33955	1188 89346	4681
50.8	.01988 00609 87081	1571 23792	7450	55.8	.01808 26899 27549	1182 45611	4639
50.9	.01984 06190 11636	1561 90506	7377	55.9	.01805 00514 66753	1176 06515	4597
51.0	.01980 13332 26697	1552 64596	7304	56.0	.01801 75306 12472	1169 72016	4556
51.1	.01976 22027 06354	1543 45990	7232	56.1	.01798 51267 30206	1163 42073	4515
51.2	.01972 32265 32001	1534 34616	7161	56.2	.01795 28391 90013	1157 16645	4475
51.3	.01968 44037 92263	1525 30403	7091	56.3	.01792 06673 66465	1150 95692	4435
51.4	.01964 57335 82929	1516 33280	7021	56.4	.01788 86106 38608	1144 79173	4395
51.5	.01960 72150 06874	1507 43179	6953	56.5	.01785 66683 89924	1138 67050	4356
51.6	.01956 88471 73999	1498 60031	6885	56.6	.01782 48400 08291	1132 59283	4318
51.7	.01953 06292 01155	1489 83768	6818	56.7	.01779 31248 85940	1126 55834	4279
51.8	.01949 25602 12079	1481 14323	6752	56.8	.01776 15224 19423	1120 56663	4241
51.9	.01945 46393 37326	1472 51630	6687	56.9	.01773 00320 09570	1114 61735	4204
52.0	.01941 68657 14202	1463 95623	6622	57.0	.01769 86530 61451	1108 71010	4167
52.1	.01937 92384 86701	1455 46238	6558	57.1	.01766 73849 84342	1102 84451	4130
52.2	.01934 17568 05438	1447 03411	6495	57.2	.01763 62271 91685	1097 02024	4094
52.3	.01930 44198 27587	1438 67079	6432	57.3	.01760 51791 01051	1091 23689	4058
52.4	.01926 72267 16814	1430 37179	6371	57.4	.01757 42401 34106	1085 49413	4022
52.5	.01923 01766 43221	1422 13650	6310	57.5	.01754 34097 16575	1079 79160	3987
52.6	.01919 32687 83278	1413 96430	6249	57.6	.01751 26872 78203	1074 12893	3953
52.7	.01915 65023 19765	1405 85460	6190	57.7	.01748 20722 52724	1068 50579	3918
52.8	.01911 98764 41712	1397 80679	6131	57.8	.01745 15640 77824	1062 92183	3884
52.9	.01908 33903 44339	1389 82030	6073	57.9	.01742 11621 95108	1057 37671	3850
53.0	.01904 70432 28995	1381 89452	6015	58.0	.01739 08660 50063	1051 87010	3817
53.1	.01901 08343 03103	1374 02890	5958	58.1	.01736 06750 92028	1046 40165	3784
53.2	.01897 47627 80101	1366 22285	5902	58.2	.01733 05887 74158	1040 97104	3751
53.3	.01893 88278 79384	1358 47582	5846	58.3	.01730 06065 53393	1035 57795	3719
53.4	.01890 30288 26248	1350 78725	5791	58.4	.01727 07278 90422	1030 22204	3687
53.5	.01886 73648 51838	1343 15659	5736	58.5	.01724 09522 49656	1024 90301	3655
53.6	.01883 18351 93085	1335 58329	5683	58.6	.01721 12790 99190	1019 62052	3624
53.7	.01879 64390 92662	1328 06682	5629	58.7	.01718 17079 10777	1014 37428	3593
53.8	.01876 11757 98920	1320 60664	5577	58.8	.01715 22381 59792	1009 16396	3562
53.9	.01872 60445 65843	1313 20224	5525	58.9	.01712 28693 25203	1003 98927	3532
54.0	.01869 10446 52989	1305 85308	5473	59.0	.01709 36008 89540	998 84989	3502
54.1	.01865 61753 25443	1298 55866	5423	59.1	.01706 44323 38866	993 74553	3472
54.2	.01862 14358 53763	1291 31846	5372	59.2	.01703 53631 62746	988 67589	3442
54.3	.01858 68255 13928	1284 13198	5323	59.3	.01700 63928 54214	983 64067	3413
54.4	.01855 23435 87291	1276 99873	5273	59.4	.01697 75209 09750	978 63959	3384
54.5	.01851 79893 60528	1269 91822	5225	59.5	.01694 87468 29245	973 67235	3356
54.6	.01848 37621 25587	1262 88995	5177	59.6	.01692 00701 15976	968 73867	3328
54.7	.01844 96611 79641	1255 91345	5129	59.7	.01689 14902 76573	963 83827	3300
54.8	.01841 56858 25039	1248 98824	5082	59.8	.01686 30068 20998	958 97086	3272
54.9	.01838 18353 69262	1242 11385	5036	59.9	.01683 46192 62508	954 13617	3244
55.0	.01834 81091 24868	1235 28981	4989	60.0	.01680 63271 17635	949 33392	3217

x	$\Psi'(x)$	δ^2	δ^4	x	$\Psi'(x)$	δ^2	δ^4
60.0	.01680 63271 17635	949 33392	3217	65.0	.01550 35654 39339	745 24609	2149
60.1	.01677 81299 06155	944 56385	3190	65.1	.01547 95670 71114	741 79082	2133
60.2	.01675 00271 51059	939 82567	3164	65.2	.01545 56428 81971	738 35688	2116
60.3	.01672 20183 78530	935 11914	3137	65.3	.01543 17925 28516	734 94410	2100
60.4	.01669 41031 17915	930 44398	3111	65.4	.01540 80156 69471	731 55232	2084
60.5	.01666 62809 01698	925 79993	3085	65.5	.01538 43119 65658	728 18137	2068
60.6	.01663 85512 65473	921 18673	3060	65.6	.01536 06810 79981	724 83111	2052
60.7	.01661 09137 47922	916 60413	3034	65.7	.01533 71226 77416	721 50136	2036
60.8	.01658 33678 90785	912 05188	3009	65.8	.01531 36364 24987	718 19198	2021
60.9	.01655 59132 38835	907 52972	2985	65.9	.01529 02219 91756	714 90281	2005
61.0	.01652 85493 39858	903 03741	2960	66.0	.01526 68790 48806	711 63369	1990
61.1	.01650 12757 44621	898 57470	2936	66.1	.01524 36072 69226	708 38447	1975
61.2	.01647 40920 06855	894 14134	2912	66.2	.01522 04063 28093	705 15501	1960
61.3	.01644 69976 83223	889 73710	2888	66.3	.01519 72759 02460	701 94514	1945
61.4	.01641 99923 33300	885 36174	2864	66.4	.01517 42156 71342	698 75472	1930
61.5	.01639 30755 19552	881 01502	2841	66.5	.01515 12253 15696	695 58361	1916
61.6	.01636 62468 07305	876 69670	2818	66.6	.01512 83045 18411	692 43166	1902
61.7	.01633 95057 64728	872 40656	2795	66.7	.01510 54529 64293	689 29873	1887
61.8	.01631 28519 62807	868 14436	2772	66.8	.01508 26703 40047	686 18466	1873
61.9	.01628 62849 75322	863 90989	2749	66.9	.01505 99563 34267	683 08932	1859
62.0	.01625 98043 78826	859 70290	2727	67.0	.01503 73106 37420	680 01258	1845
62.1	.01623 34097 52620	855 52319	2705	67.1	.01501 47329 41830	676 95428	1831
62.2	.01620 71006 78733	851 37053	2683	67.2	.01499 22229 41669	673 91429	1817
62.3	.01618 08767 41900	847 24470	2662	67.3	.01496 97803 32936	670 89248	1804
62.4	.01615 47375 29536	843 14548	2640	67.4	.01494 74048 13452	667 88871	1790
62.5	.01612 86826 31721	839 07267	2619	67.5	.01492 50960 82839	664 90284	1777
62.6	.01610 27116 41172	835 02605	2598	67.6	.01490 28538 42510	661 93474	1764
62.7	.01607 68241 53229	831 00540	2577	67.7	.01488 06777 95655	658 98429	1751
62.8	.01605 10197 65825	827 01052	2556	67.8	.01485 85676 47229	656 05134	1738
62.9	.01602 52980 79474	823 04121	2536	67.9	.01483 65231 03936	653 13577	1725
63.0	.01599 96586 97244	819 09726	2516	68.0	.01481 45438 74221	650 23745	1712
63.1	.01597 41012 24740	815 17847	2496	68.1	.01479 26296 68250	647 35625	1700
63.2	.01594 86252 70084	811 28464	2476	68.2	.01477 07801 97904	644 49205	1687
63.3	.01592 32304 43890	807 41556	2456	68.3	.01474 89951 76764	641 64472	1675
63.4	.01589 79163 59253	803 57105	2437	68.4	.01472 72743 20096	638 81414	1662
63.5	.01587 26826 31721	799 75090	2417	68.5	.01470 56173 44842	636 00019	1650
63.6	.01584 75288 79278	795 95493	2399	68.6	.01468 40239 69606	633 20273	1638
63.7	.01582 24547 22329	792 18295	2379	68.7	.01466 24939 14644	630 42166	1626
63.8	.01579 74597 83676	788 43476	2361	68.8	.01464 10269 01847	627 65685	1614
63.9	.01577 25436 88498	784 71018	2342	68.9	.01461 96226 54736	624 90818	1603
64.0	.01574 77060 64339	781 00902	2324	69.0	.01459 82808 98442	622 17554	1591
64.1	.01572 29465 41082	777 33111	2306	69.1	.01457 70013 59703	619 45881	1579
64.2	.01569 82647 50936	773 67624	2288	69.2	.01455 57837 66845	616 75787	1568
64.3	.01567 36603 28414	770 04426	2270	69.3	.01453 46278 49774	614 07261	1557
64.4	.01564 91329 10318	766 43497	2252	69.4	.01451 35333 39965	611 40292	1545
64.5	.01562 46821 35720	762 84820	2234	69.5	.01449 24999 70448	608 74868	1534
64.6	.01560 03076 45941	759 28378	2217	69.6	.01447 15274 75798	606 10978	1523
64.7	.01557 60090 84540	755 74153	2200	69.7	.01445 06155 92127	603 48611	1512
64.8	.01555 17860 97293	752 22128	2183	69.8	.01442 97640 57066	600 87756	1501
64.9	.01552 76383 32173	748 72286	2166	69.9	.01440 89726 09762	598 28402	1490
65.0	.01550 35654 39339	745 24609	2149	70.0	.01438 82409 90860	595 70539	1480

x	$\Psi'(x)$	δ^2	δ^4	x	$\Psi'(x)$	δ^2	δ^4
70.0	.01438 82409 90860	595 70539	1480	75.0	.01342 26172 69906	483 64070	1046
70.1	.01436 75689 42497	593 14156	1469	75.1	.01340 46250 24515	481 69848	1039
70.2	.01434 69562 08290	590 59241	1459	75.2	.01338 66809 48973	479 76665	1032
70.3	.01432 64025 33324	588 05785	1448	75.3	.01336 87848 50097	477 84514	1025
70.4	.01430 59076 64144	585 53778	1438	75.4	.01335 09365 35734	475 93387	1018
70.5	.01428 54713 48741	583 03208	1428	75.5	.01333 31358 14758	474 03278	1011
70.6	.01426 50933 36546	580 54066	1418	75.6	.01331 53824 97061	472 14180	1004
70.7	.01424 47733 78417	578 06341	1407	75.7	.01329 76763 93544	470 26087	998
70.8	.01422 45112 26630	575 60024	1397	75.8	.01328 00173 16115	468 38992	991
70.9	.01420 43066 34866	573 15104	1388	75.9	.01326 24050 77678	466 52888	985
71.0	.01418 41593 58207	570 71572	1378	76.0	.01324 48394 92128	464 67768	978
71.1	.01416 40691 53119	568 29417	1368	76.1	.01322 73203 74346	462 83626	972
71.2	.01414 40357 77449	565 88631	1358	76.2	.01320 98475 40191	461 00456	965
71.3	.01412 40589 90410	563 49203	1349	76.3	.01319 24208 06493	459 18252	959
71.4	.01410 41385 52573	561 11123	1339	76.4	.01317 50399 91046	457 37006	953
71.5	.01408 42742 25860	558 74383	1330	76.5	.01315 77049 12605	455 56713	946
71.6	.01406 44657 73529	556 38973	1321	76.6	.01314 04153 90877	453 77366	940
71.7	.01404 47129 60171	554 04883	1311	76.7	.01312 31712 46515	451 98959	934
71.8	.01402 50155 51696	551 72105	1302	76.8	.01310 59723 01112	450 21487	928
71.9	.01400 53733 15326	549 40628	1293	76.9	.01308 88183 77196	448 44942	922
72.0	.01398 57860 19584	547 10445	1284	77.0	.01307 17092 98222	446 69319	916
72.1	.01396 62534 34287	544 81546	1275	77.1	.01305 46448 88567	444 94612	910
72.2	.01394 67753 30536	542 53922	1266	77.2	.01303 76249 73524	443 20815	904
72.3	.01392 73514 80707	540 27564	1257	77.3	.01302 06493 79295	441 47921	898
72.4	.01390 79816 58443	538 02464	1249	77.4	.01300 37179 32988	439 75926	892
72.5	.01388 86656 38643	535 78612	1240	77.5	.01298 68304 62607	438 04823	886
72.6	.01386 94031 97455	533 56001	1232	77.6	.01296 99867 97049	436 34607	881
72.7	.01385 01941 12267	531 34621	1223	77.7	.01295 31867 66098	434 65271	875
72.8	.01383 10381 61701	529 14464	1215	77.8	.01293 64302 00418	432 96810	869
72.9	.01381 19351 25597	526 95521	1206	77.9	.01291 97169 31548	431 29219	864
73.0	.01379 28847 85016	524 77785	1198	78.0	.01290 30467 91897	429 62492	858
73.1	.01377 38869 22219	522 61246	1190	78.1	.01288 64196 14738	427 96623	853
73.2	.01375 49413 20668	520 45898	1182	78.2	.01286 98352 34202	426 31606	847
73.3	.01373 60477 65015	518 31731	1173	78.3	.01285 32934 85272	424 67437	842
73.4	.01371 72060 41093	516 18737	1165	78.4	.01283 67942 03778	423 04110	836
73.5	.01369 84159 35908	514 06908	1157	78.5	.01282 03372 26395	421 41619	831
73.6	.01367 96772 37631	511 96238	1150	78.6	.01280 39223 90630	419 79959	826
73.7	.01366 09897 35592	509 86716	1142	78.7	.01278 75495 34824	418 19125	820
73.8	.01364 23532 20268	507 78337	1134	78.8	.01277 12184 98142	416 59111	815
73.9	.01362 37674 83282	505 71091	1126	78.9	.01275 49291 20573	414 99913	810
74.0	.01360 52323 17386	503 64971	1119	79.0	.01273 86812 42916	413 41525	805
74.1	.01358 67475 16461	501 59970	1111	79.1	.01272 24747 06785	411 83942	800
74.2	.01356 83128 75507	499 56081	1104	79.2	.01270 63093 54596	410 27159	795
74.3	.01354 99281 90634	497 53294	1096	79.3	.01269 01850 29566	408 71171	790
74.4	.01353 15932 59055	495 51604	1089	79.4	.01267 41015 75707	407 15972	785
74.5	.01351 33078 79080	493 51003	1081	79.5	.01265 80588 37819	405 61558	780
74.6	.01349 50718 50107	491 51482	1074	79.6	.01264 20566 61489	404 07924	775
74.7	.01347 68849 72617	489 53036	1067	79.7	.01262 60948 93083	402 55065	770
74.8	.01345 87470 48163	487 55657	1060	79.8	.01261 01733 79742	401 02975	765
74.9	.01344 06578 79366	485 59337	1053	79.9	.01259 42919 69376	399 51651	760
75.0	.01342 26172 69906	483 64070	1046	80.0	.01257 84505 10662	398 01088	756

x	$\Psi'(x)$	δ^2	δ^4	x	$\Psi'(x)$	δ^2	δ^4
80.0	.01257 84505 10662	398 01088	756	85.0	.01183 41814 15923	331 45919	557
80.1	.01256 26488 53036	396 51280	751	85.1	.01182 01933 47722	330 28525	554
80.2	.01254 68868 46689	395 02223	746	85.2	.01180 62383 08047	329 11684	550
80.3	.01253 11643 42565	393 53912	742	85.3	.01179 23161 80055	327 95394	547
80.4	.01251 54811 92353	392 06342	737	85.4	.01177 84268 47457	326 79651	544
80.5	.01249 98372 48483	390 59510	732	85.5	.01176 45701 94509	325 64452	541
80.6	.01248 42323 64123	389 13409	728	85.6	.01175 07461 06014	324 49793	538
80.7	.01246 86663 93172	387 68037	723	85.7	.01173 69544 67311	323 35673	534
80.8	.01245 31391 90258	386 23387	719	85.8	.01172 31951 64281	322 22086	531
80.9	.01243 76506 10732	384 79457	714	85.9	.01170 94680 83337	321 09032	528
81.0	.01242 22005 10662	383 36240	710	86.0	.01169 57731 11424	319 96505	525
81.1	.01240 67887 46832	381 93734	706	86.1	.01168 21101 36017	318 84504	522
81.2	.01239 14151 76737	380 51933	701	86.2	.01166 84790 45114	317 73025	519
81.3	.01237 60796 58573	379 10832	697	86.3	.01165 48797 27235	316 62064	516
81.4	.01236 07820 51243	377 70429	692	86.4	.01164 13120 71420	315 51620	513
81.5	.01234 55222 14341	376 30718	688	86.5	.01162 77759 67226	314 41689	510
81.6	.01233 03000 08158	374 91695	684	86.6	.01161 42713 04721	313 32268	507
81.7	.01231 51152 93670	373 53357	680	86.7	.01160 07979 74484	312 23355	504
81.8	.01229 99679 32538	372 15698	676	86.8	.01158 73558 67602	311 14945	501
81.9	.01228 48577 87105	370 78714	672	86.9	.01157 39448 75665	310 07037	498
82.0	.01226 97847 20386	369 42402	667	87.0	.01156 05648 90765	308 99627	496
82.1	.01225 47485 96069	368 06758	663	87.1	.01154 72158 05491	307 92712	493
82.2	.01223 97492 78511	366 71776	659	87.2	.01153 38975 12930	306 86291	490
82.3	.01222 47866 32728	365 37454	655	87.3	.01152 06099 06660	305 80359	487
82.4	.01220 98605 24400	364 03787	651	87.4	.01150 73528 80748	304 74914	484
82.5	.01219 49708 19859	362 70771	647	87.5	.01149 41263 29751	303 69953	481
82.6	.01218 01173 86089	361 38403	643	87.6	.01148 09301 48706	302 65474	479
82.7	.01216 53000 90722	360 06678	639	87.7	.01146 77642 33136	301 61474	476
82.8	.01215 05188 02034	358 75592	636	87.8	.01145 46284 79040	300 57949	473
82.9	.01213 57733 88936	357 45142	632	87.9	.01144 15227 82893	299 54898	471
83.0	.01212 10637 20981	356 15323	628	88.0	.01142 84470 41643	298 52317	468
83.1	.01210 63896 68348	354 86132	624	88.1	.01141 54011 52711	297 50204	465
83.2	.01209 17511 01848	353 57566	620	88.2	.01140 23850 13982	296 48556	462
83.3	.01207 71478 92913	352 29619	617	88.3	.01138 93985 23810	295 47371	460
83.4	.01206 25799 13598	351 02289	613	88.4	.01137 64415 81008	294 46646	457
83.5	.01204 80470 36571	349 75572	609	88.5	.01136 35140 84853	293 46378	455
83.6	.01203 35491 35117	348 49465	606	88.6	.01135 06159 35074	292 46564	452
83.7	.01201 90860 83128	347 23962	602	88.7	.01133 77470 31861	291 47203	450
83.8	.01200 46577 55101	345 99062	598	88.8	.01132 49072 75850	290 48291	447
83.9	.01199 02640 26137	344 74760	595	88.9	.01131 20965 68131	289 49827	445
84.0	.01197 59047 71932	343 51053	591	89.0	.01129 93148 10238	288 51807	442
84.1	.01196 15798 68780	342 27936	588	89.1	.01128 65619 04153	287 54229	440
84.2	.01194 72891 93564	341 05408	584	89.2	.01127 38377 52296	286 57090	437
84.3	.01193 30326 23756	339 83464	581	89.3	.01126 11422 57529	285 60389	435
84.4	.01191 88100 37412	338 62100	577	89.4	.01124 84753 23151	284 64122	432
84.5	.01190 46213 13168	337 41313	574	89.5	.01123 58368 52895	283 68287	430
84.6	.01189 04663 30237	336 21100	570	89.6	.01122 32267 50926	282 72882	427
84.7	.01187 63449 68406	335 01458	567	89.7	.01121 06449 21838	281 77904	425
84.8	.01186 22571 08033	333 82383	564	89.8	.01119 80912 70655	280 83351	423
84.9	.01184 82026 30042	332 63871	560	89.9	.01118 55657 02824	279 89221	420
85.0	.01183 41814 15923	331 45919	557	90.0	.01117 30681 24214	278 95511	418

x	$\Psi'(x)$	δ^2	δ^4	x	$\Psi'(x)$	δ^2	δ^4
90.0	.01117 30681 24214	278 95511	418	95.0	.01058 19118 39013	236 97948	318
90.1	.01116 05984 41115	278 02219	416	95.1	.01057 07260 94114	236 22878	317
90.2	.01114 81565 60235	277 09342	413	95.2	.01055 95639 72094	235 48125	315
90.3	.01113 57423 88697	276 16879	411	95.3	.01054 84253 98199	234 73687	313
90.4	.01112 33558 34039	275 24827	409	95.4	.01053 73102 97991	233 99562	312
90.5	.01111 09968 04207	274 33183	406	95.5	.01052 62185 97344	233 25749	310
90.6	.01109 86652 07558	273 41945	404	95.6	.01051 51502 22447	232 52246	309
90.7	.01108 63609 52855	272 51112	402	95.7	.01050 41050 99797	231 79052	307
90.8	.01107 40839 49263	271 60681	400	95.8	.01049 30831 56198	231 06165	305
90.9	.01106 18341 06352	270 70649	397	95.9	.01048 20843 18764	230 33583	304
91.0	.01104 96113 34090	269 81015	395	96.0	.01047 11085 14913	229 61304	302
91.1	.01103 74155 42843	268 91776	393	96.1	.01046 01556 72366	228 89328	300
91.2	.01102 52466 43371	268 02930	391	96.2	.01044 92257 19146	228 17652	299
91.3	.01101 31045 46829	267 14475	389	96.3	.01043 83185 83578	227 46275	297
91.4	.01100 09891 64762	266 26409	387	96.4	.01042 74341 94285	226 75195	296
91.5	.01098 89004 09103	265 38729	384	96.5	.01041 65724 80188	226 04412	294
91.6	.01097 68381 92174	264 51434	383	96.6	.01040 57333 70501	225 33922	293
91.7	.01096 48024 26679	263 64522	380	96.7	.01039 49167 94738	224 63726	291
91.8	.01095 27930 25706	262 77990	378	96.8	.01038 41226 82700	223 93820	290
91.9	.01094 08099 02722	261 91836	376	96.9	.01037 33509 64482	223 24205	288
92.0	.01092 88529 71574	261 06058	374	97.0	.01036 26015 70469	222 54877	287
92.1	.01091 69221 46483	260 20654	372	97.1	.01035 18744 31333	221 85837	285
92.2	.01090 50173 42048	259 35623	370	97.2	.01034 11694 78034	221 17081	284
92.3	.01089 31384 73235	258 50962	368	97.3	.01033 04866 41816	220 48610	282
92.4	.01088 12854 55383	257 66668	366	97.4	.01031 98258 54208	219 80421	281
92.5	.01086 94582 04200	256 82741	364	97.5	.01030 91870 47022	219 12513	279
92.6	.01085 76566 35758	255 99178	362	97.6	.01029 85701 52347	218 44884	278
92.7	.01084 58806 66494	255 15977	360	97.7	.01028 79751 02557	217 77533	277
92.8	.01083 41302 13207	254 33136	358	97.8	.01027 74018 30299	217 10459	275
92.9	.01082 24051 93056	253 50653	356	97.9	.01026 68502 68500	216 43660	274
93.0	.01081 07055 23559	252 68527	354	98.0	.01025 63203 50360	215 77134	272
93.1	.01079 90311 22588	251 86755	352	98.1	.01024 58120 09355	215 10881	271
93.2	.01078 73819 08373	251 05336	351	98.2	.01023 53251 79231	214 44899	270
93.3	.01077 57577 99493	250 24267	349	98.3	.01022 48597 94006	213 79187	268
93.4	.01076 41587 14880	249 43546	347	98.4	.01021 44157 87969	213 13743	267
93.5	.01075 25845 73813	248 63173	345	98.5	.01020 39930 95674	212 48565	265
93.6	.01074 10352 95919	247 83144	343	98.6	.01019 35916 51944	211 83653	264
93.7	.01072 95108 01169	247 03459	341	98.7	.01018 32113 91868	211 19006	263
93.8	.01071 80110 09878	246 24114	339	98.8	.01017 28522 50797	210 54621	261
93.9	.01070 65358 42701	245 45110	338	98.9	.01016 25141 64347	209 90497	260
94.0	.01069 50852 20633	244 66442	336	99.0	.01015 21970 68394	209 26634	259
94.1	.01068 36590 65008	243 88111	334	99.1	.01014 19008 99075	208 63029	257
94.2	.01067 22572 97494	243 10114	332	99.2	.01013 16255 92785	207 99682	256
94.3	.01066 08798 40093	242 32448	330	99.3	.01012 13710 86177	207 36591	255
94.4	.01064 95266 15141	241 55114	329	99.4	.01011 11373 16160	206 73755	254
94.5	.01063 81975 45302	240 78108	327	99.5	.01010 09242 19897	206 11173	252
94.6	.01062 68925 53571	240 01429	325	99.6	.01009 07317 34808	205 48843	251
94.7	.01061 56115 63269	239 25075	323	99.7	.01008 05597 98561	204 86764	250
94.8	.01060 43544 98042	238 49045	322	99.8	.01007 04083 49078	204 24934	249
94.9	.01059 31212 81859	237 73336	320	99.9	.01006 02773 24529	203 63354	248
95.0	.01058 19118 39013	236 97948	318	100.0	.01005 01666 63334	203 02021	247

TABLE 17

THE TETRAGAMMA FUNCTION

Description : $\Psi''(x)$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1.

$\text{Log}_{10}|\Psi''(x)|$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1.

$\Psi''(x)$ to 12 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

$\text{Log}_{10}|\Psi''(x)|$ to 10 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

x	$\Psi''(-x)$	$\text{Log}_{10}\lvert\Psi''(-x)\rvert$	x	$\Psi''(-x)$	$\text{Log}_{10}\lvert\Psi''(-x)\rvert$
0.0	∞	∞	5.0	∞	∞
0.1	+1996.79820 29562	3.30033 41771	5.1	+1998.62802 22160	3.30073 19722
0.2	+ 245.56988 42916	2.39017 51056	5.2	+ 247.01737 44222	2.39272 75012
0.3	+ 67.63908 11999	1.83019 76996	5.3	+ 68.80803 44038	1.83763 91518
0.4	+ 21.28716 84629	1.32811 78971	5.4	+ 22.24777 26720	1.34728 65382
0.5	− 0.82879 66442	9.91844 79839*	5.5	− .02758 79107	8.44071 88116*
0.6	− 22.97986 93643	1.36134 75555	5.6	− 22.30299 37732	1.34836 31631
0.7	− 69.44163 27986	1.84161 99240	5.7	− 68.86339 15901	1.83798 84083
0.8	− 247.57178 61144	2.39370 11499	5.8	− 247.07295 92392	2.39282 52016
0.9	−1999.11797 31534	3.30083 84237	5.9	−1998.68392 74527	3.30074 41200
1.0	∞	∞	6.0	∞	∞
1.1	+1998.30083 25580	3.30066 08693	6.1	+1998.63683 35262	3.30073 38868
1.2	+ 246.72729 16990	2.39221 71915	6.2	+ 247.02576 62183	2.39274 22550
1.3	+ 68.54941 34712	1.83600 37431	6.3	+ 68.81603 29001	1.83768 96328
1.4	+ 22.01603 14366	1.34273 90368	6.4	+ 22.25540 20665	1.34743 54445
1.5	− .23620 40516	9.37328 73428*	6.5	− .02030 52525	8.30760 83950*
1.6	− 22.49158 81143	1.35202 01216	6.6	− 22.29603 71546	1.34822 76794
1.7	− 69.03454 95501	1.83906 64954	6.7	− 68.85674 18360	1.83794 64689
1.8	− 247.22885 05863	2.39309 91497	6.8	− 247.06659 85634	2.39281 40361
1.9	−1998.82638 54584	3.30077 50736	6.9	−1998.67783 93394	3.30074 27971
2.0	∞	∞	7.0	∞	∞
2.1	+1998.51679 19576	3.30070 78016	7.1	+1998.64242 15076	3.30073 51010
2.2	+ 246.91512 03992	2.39254 76857	7.2	+ 247.03112 45860	2.39275 16755
2.3	+ 68.71379 25293	1.83704 39193	7.3	+ 68.82117 40636	1.83772 20772
2.4	+ 22.16070 73625	1.34558 36187	7.4	+ 22.26033 76084	1.34753 17466
2.5	− .10820 40516	9.03424 35225*	7.5	− .01556 45118	8.19213 55027*
2.6	− 22.37779 65804	1.34981 73216	7.6	− 22.29148 10969	1.34813 89249
2.7	− 68.93293 90233	1.83842 67954	7.7	− 68.85236 09917	1.83791 88371
2.8	− 247.13774 27146	2.39293 90755	7.8	− 247.06238 40622	2.39280 66278
2.9	−1998.74438 12762	3.30075 72558	7.9	−1998.67378 28652	3.30074 19157
3.0	∞	∞	8.0	∞	∞
3.1	+1998.58392 63271	3.30072 23902	8.1	+1998.64618 48604	3.30073 59188
3.2	+ 246.97615 55555	2.39265 50260	8.2	+ 247.03475 19274	2.39275 80524
3.3	+ 68.76944 54775	1.83739 55224	8.3	+ 68.82467 18696	1.83774 41495
3.4	+ 22.21159 27686	1.34657 97023	8.4	+ 22.26371 19740	1.34759 75748
3.5	− .06155 68213	8.78927 61856*	8.5	− .01230 78458	8.09018 20466*
3.6	− 22.33492 96394	1.34898 45886	8.6	− 22.28833 67197	1.34807 76601
3.7	− 68.89345 46887	1.83817 79632	8.7	− 68.84932 37997	1.83789 96792
3.8	− 247.10129 42527	2.39287 50201	8.8	− 247.05944 92387	2.39280 14689
3.9	−1998.71066 52661	3.30074 99299	8.9	−1998.67094 58610	3.30074 12992
4.0	∞	∞	9.0	∞	∞
4.1	+1998.61294 50587	3.30072 86960	9.1	+1998.64883 88904	3.30073 64955
4.2	+ 247.00315 04804	2.39270 24926	9.2	+ 247.03732 03502	2.39276 25678
4.3	+ 68.79460 04953	1.83755 43530	9.3	+ 68.82715 83277	1.83775 98391
4.4	+ 22.23507 13561	1.34703 85275	9.4	+ 22.26611 99183	1.34764 45436
4.5	− .03960 89475	8.59779 33026*	9.5	− .00997 51443	7.99891 91846*
4.6	− 22.31438 22571	1.34858 48684	9.6	− 22.28607 61583	1.34803 36103
4.7	− 68.87419 11344	1.83805 65115	9.7	− 68.84713 24344	1.83788 58561
4.8	− 247.08320 97620	2.39284 32344	9.8	− 247.05732 42738	2.39279 77335
4.9	−1998.69366 55466	3.30074 62361	9.9	−1998.66888 46407	3.30074 08513
5.0	∞	∞	10.0	∞	∞

*Ten should be subtracted from these logarithms

x	$\Psi''(x)$	$\text{Log}_{10}\lvert\Psi''(x)\rvert$	x	$\Psi''(x)$	$\text{Log}_{10}\lvert\Psi''(x)\rvert$
0.00	$-\infty$	∞	0.50	−16.828796 644234	1.22605 30623
0.01	−2000002.340398 677085	6.30103 05039	0.51	−15.892034 964823	1.20117 95118
0.02	−250002.279054 205238	5.39794 39677	0.52	−15.025236 878663	1.17682 13274
0.03	−74076.294042 670340	4.86967 92469	0.53	−14.221947 270392	1.15295 90640
0.04	−31252.163036 385488	4.49488 00812	0.54	−13.476414 185030	1.12957 43500
0.05	−16002.108158 021945	4.20417 72015	0.55	−12.783501 004806	1.10664 98094
0.06	−9261.314498 742591	3.96667 26324	0.56	−12.138610 944348	1.08416 89917
0.07	−5832.907982 006837	3.76588 51252	0.57	−11.537621 953863	1.06211 63040
0.08	−3908.204931 321315	3.59197 73284	0.58	−10.976830 443589	1.04047 69554
0.09	−2745.391603 181102	3.43860 43010	0.59	−10.452902 507014	1.01923 68998
0.10	−2001.861457 378344	3.30143 40179	0.60	−9.962831 537143	0.99838 27868
0.11	−1504.446699 376300	3.17737 68055	0.61	−9.503901 308252	0.97790 19178
0.12	−1159.181638 710867	3.06415 14931	0.62	−9.073653 742760	0.95778 22022
0.13	−912.065065 864255	2.96002 58216	0.63	−8.669860 704748	0.93801 21199
0.14	−730.555700 495088	2.86365 33339	0.64	−8.290499 262825	0.91858 06850
0.15	−594.246562 760270	2.77396 66784	0.65	−7.933729 949539	0.89947 74138
0.16	−489.897720 220613	2.69010 54185	0.66	−7.597877 614981	0.88069 22938
0.17	−408.663423 497455	2.61136 57689	0.67	−7.281414 531423	0.86221 57562
0.18	−344.480563 211014	2.53716 47225	0.68	−6.982945 455515	0.84403 86497
0.19	−293.098698 671560	2.46701 38902	0.69	−6.701194 396483	0.82615 22166
0.20	−251.478036 114436	2.40050 00601	0.70	−6.434992 874190	0.80854 80703
0.21	−217.405489 609334	2.33727 05059	0.71	−6.183269 480910	0.79121 81744
0.22	−189.243825 085568	2.27702 17175	0.72	−5.945040 586161	0.77415 48239
0.23	−165.764160 402980	2.21949 06381	0.73	−5.719402 045672	0.75735 06263
0.24	−146.031911 556704	2.16444 77700	0.74	−5.505521 794014	0.74079 84862
0.25	−129.327739 937537	2.11169 16879	0.75	−5.302633 216338	0.72449 15879
0.26	−115.091865 821125	2.06104 46305	0.76	−5.110029 208173	0.70842 33825
0.27	−102.884256 636266	2.01234 89235	0.77	−4.927056 843972	0.69258 75728
0.28	−92.355774 969739	1.96546 40569	0.78	−4.753112 585042	0.67697 81014
0.29	−83.227005 582439	1.92026 42694	0.79	−4.587637 966270	0.66158 91382
0.30	−75.272536 588726	1.87663 65515	0.80	−4.430115 708422	0.64641 50696
0.31	−68.309163 732719	1.83447 89686	0.81	−4.280066 209368	0.63145 04873
0.32	−62.186949 729449	1.79369 92552	0.82	−4.137044 373152	0.61669 01791
0.33	−56.782384 179113	1.75421 36238	0.83	−4.000636 740740	0.60212 91190
0.34	−51.993104 753609	1.71594 57520	0.84	−3.870458 890509	0.58776 24590
0.35	−47.733789 913236	1.67882 59174	0.85	−3.746153 080292	0.57358 55201
0.36	−43.932938 583316	1.64279 02533	0.86	−3.627386 105974	0.55959 37852
0.37	−40.530326 997577	1.60778 01076	0.87	−3.513847 354538	0.54578 28913
0.38	−37.474986 633506	1.57374 14866	0.88	−3.405247 031886	0.53214 86230
0.39	−34.723586 132350	1.54062 45712	0.89	−3.301314 547988	0.51868 69057
0.40	−32.239128 623578	1.50838 32950	0.90	−3.201797 043795	0.50539 37993
0.41	−29.989896 939141	1.47697 49733	0.91	−3.106458 046076	0.49226 54926
0.42	−27.948594 886281	1.44635 99786	0.92	−3.015076 237791	0.47929 82979
0.43	−26.091644 514832	1.41650 14528	0.93	−2.927444 332961	0.46648 86454
0.44	−24.398608 208966	1.38736 50531	0.94	−2.843368 046124	0.45383 30784
0.45	−22.851711 202802	1.35891 87268	0.95	−2.762665 147556	0.44132 82488
0.46	−21.435445 306082	1.33113 25100	0.96	−2.685164 596254	0.42897 09123
0.47	−20.136238 625402	1.30397 83491	0.97	−2.610705 743589	0.41675 79246
0.48	−18.942179 168840	1.27742 99401	0.98	−2.539137 601190	0.40468 62368
0.49	−17.842782 642228	1.25146 25849	0.99	−2.470318 167288	0.39275 28923
0.50	−16.828796 644234	1.22605 30623	1.00	−2.404113 806319	0.38095 50224

TABLE 18

THE TETRAGAMMA FUNCTION

Description: $\Psi''(x)$ to 12 decimal places with central differences from $x = 1.00$ to $x = 4.00$ by increments of .01.

x	$\Psi'(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
1.00	—2.404113 806319	2489 231735	7 266849	40507
1.01	—2.340398 677085	2370 657387	6 779836	37008
1.02	—2.279054 205238	2258 862875	6 329831	33896
1.03	—2.219968 596266	2153 398194	5 913722	31041
1.04	—2.163036 385488	2053 847235	5 528654	28443
1.05	—2.108158 021945	1959 824930	5 172029	26119
1.06	—2.055239 483332	1870 974654	4 841523	23974
1.07	—2.004191 919373	1786 965901	4 534991	22031
1.08	—1.954931 321315	1707 492139	4 250490	20268
1.09	—1.907378 215396	1632 268867	3 986257	18655
1.10	—1.861457 378344	1561 031852	3 740679	17181
1.11	—1.817097 573144	1493 535516	3 512282	15846
1.12	—1.774231 303460	1429 551462	3 299731	14609
1.13	—1.732794 585238	1368 867139	3 101789	13496
1.14	—1.692726 734155	1311 284605	2 917343	12463
1.15	—1.653970 167677	1256 619414	2 745360	11521
1.16	—1.616470 220613	1204 699583	2 584898	10666
1.17	—1.580174 973132	1155 364650	2 435102	9865
1.18	—1.545035 090301	1108 464819	2 295171	9143
1.19	—1.511003 672289	1063 860159	2 164383	8480
1.20	—1.478036 114436	1021 419882	2 042075	7855
1.21	—1.446089 976465	981 021680	1 927622	7297
1.22	—1.415124 860174	942 551100	1 820466	6782
1.23	—1.385102 294983	905 900986	1 720092	6292
1.24	—1.355985 630778	870 970964	1 626010	5857
1.25	—1.327739 937537	837 666952	1 537785	5444
1.26	—1.300331 911248	805 900725	1 455004	5084
1.27	—1.273729 785684	775 589502	1 377307	4702
1.28	—1.247903 249622	746 655586	1 304312	4431
1.29	—1.222823 369146	719 025982	1 235748	4079
1.30	—1.198462 514652	692 632126	1 171263	3849
1.31	—1.174794 292284	667 409533	1 110627	3564
1.32	—1.151793 479449	643 297567	1 053555	3351
1.33	—1.129435 964181	620 239156	999834	3109
1.34	—1.107698 688069	598 180579	949222	2929
1.35	—1.086559 592536	577 071224	901539	2724
1.36	—1.065997 568227	556 863408	856580	2554
1.37	—1.045992 407326	537 512172	814175	2397
1.38	—1.026524 758597	518 975111	774167	2240
1.39	—1.007576 084979	501 212217	736399	2094
1.40	—0.989128 623578	484 185722	700725	1979
1.41	—0.971165 347899	467 859952	667030	1843
1.42	—0.953669 932172	452 201212	635178	1735
1.43	—0.936626 717657	437 177650	605061	1630
1.44	—0.920020 680792	422 759149	576574	1525
1.45	—0.903837 403076	408 917222	549612	1449
1.46	—0.888063 042582	395 624907	524099	1336
1.47	—0.872684 306995	382 856691	499922	1285
1.48	—0.857688 428099	370 588397	477030	1195
1.49	—0.843063 137600	358 797133	455333	1115
1.50	—0.828796 644234	347 461202	434751	1078

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
1.50	−0.828796 644234	347 461202	434751	1078
1.51	−0.814877 612070	336 560022	415247	986
1.52	−0.801295 139928	326 074089	396729	950
1.53	−0.788038 741875	315 984885	379161	886
1.54	−0.775098 328707	306 274842	362479	835
1.55	−0.762464 190381	296 927278	346632	799
1.56	−0.750126 979333	287 926346	331584	736
1.57	−0.738077 694631	279 256998	317272	715
1.58	−0.726307 666927	270 904922	303675	661
1.59	−0.714808 544145	262 856521	290739	626
1.60	−0.703572 277884	255 098859	278429	607
1.61	−0.692591 110482	247 619626	266726	548
1.62	−0.681857 562706	240 407119	255571	543
1.63	−0.671364 422049	233 450183	244959	500
1.64	−0.661104 731575	226 738206	234847	478
1.65	−0.651071 779307	220 261076	225213	452
1.66	−0.641259 088115	214 009159	216031	426
1.67	−0.631660 406082	207 973273	207275	407
1.68	−0.622269 697322	202 144662	198926	389
1.69	−0.613081 133224	196 514977	190966	353
1.70	−0.604089 084103	191 076258	183359	361
1.71	−0.595288 111240	185 820898	176113	315
1.72	−0.586672 959275	180 741651	169182	320
1.73	−0.578238 548961	175 831586	162571	290
1.74	−0.569979 970233	171 084092	156250	292
1.75	−0.561892 475597	166 492848	150221	244
1.76	−0.553971 473809	162 051825	144436	282
1.77	−0.546212 523846	157 755238	138933	223
1.78	−0.538611 329121	153 597584	133653	231
1.79	−0.531163 731980	149 573583	128604	225
1.80	−0.523865 708422	145 678186	123780	202
1.81	−0.516713 363050	141 906569	119158	197
1.82	−0.509702 924247	138 254110	114733	195
1.83	−0.502830 739554	134 716384	110503	167
1.84	−0.496093 271245	131 289161	106440	183
1.85	−0.489487 092097	127 968378	102560	155
1.86	−0.483008 881327	124 750155	98835	157
1.87	−0.476655 420712	121 630767	95267	145
1.88	−0.470423 590864	118 606646	91844	147
1.89	−0.464310 367662	115 674369	88568	128
1.90	−0.458312 818829	112 830660	85420	131
1.91	−0.452428 100656	110 072371	82403	118
1.92	−0.446653 454854	107 396485	79504	127
1.93	−0.440986 205537	104 800103	76732	97
1.94	−0.435423 756323	102 280453	74057	120
1.95	−0.429963 587562	99 834860	71502	89
1.96	−0.424603 253661	97 460769	69036	109
1.97	−0.419340 380529	95 155714	66679	84
1.98	−0.414172 663111	92 917338	64406	92
1.99	−0.409097 863031	90 743368	62225	81
2.00	−0.404113 806319	88 631623	60125	94

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
2.00	−0.404113 806319	88 631623	60125
2.01	−0.399218 381230	86 580003	58119
2.02	−0.394409 536144	84 586502	56171
2.03	−0.389685 277560	82 649171	54310
2.04	−0.385043 668146	80 766149	52521
2.05	−0.380482 824882	78 935649	50789
2.06	−0.376000 917267	77 155939	49132
2.07	−0.371596 165591	75 425359	47535
2.08	−0.367266 839275	73 742316	45996
2.09	−0.363011 255274	72 105268	44515
2.10	−0.358827 776541	70 512734	43089
2.11	−0.354714 810542	68 963290	41713
2.12	−0.350670 807833	67 455558	40392
2.13	−0.346694 260683	65 988219	39114
2.14	−0.342783 701751	64 559994	37887
2.15	−0.338937 702813	63 169655	36702
2.16	−0.335154 873530	61 816019	35557
2.17	−0.331433 860266	60 497940	34460
2.18	−0.327773 344942	59 214320	33394
2.19	−0.324172 043938	57 964095	32370
2.20	−0.320628 707029	56 746238	31384
2.21	−0.317142 116357	55 559766	30428
2.22	−0.313711 085453	54 403722	29505
2.23	−0.310334 458270	53 277183	28621
2.24	−0.307011 108271	52 179265	27761
2.25	−0.303739 937537	51 109108	26933
2.26	−0.300519 875911	50 065884	26130
2.27	−0.297349 880168	49 048790	25366
2.28	−0.294228 933216	48 057062	24606
2.29	−0.291156 043325	47 089939	23900
2.30	−0.288130 243373	46 146717	23192
2.31	−0.285150 590138	45 226687	22525
2.32	−0.282216 163591	44 329181	21869
2.33	−0.279326 066224	43 453544	21245
2.34	−0.276479 422401	42 599153	20631
2.35	−0.273675 377731	41 765392	20047
2.36	−0.270913 098453	40 951677	19476
2.37	−0.268191 770852	40 157439	18923
2.38	−0.265510 600690	39 382124	18393
2.39	−0.262868 812652	38 625203	17879
2.40	−0.260265 649817	37 886161	17375
2.41	−0.257700 373143	37 164494	16896
2.42	−0.255172 260963	36 459723	16428
2.43	−0.252680 608507	35 771381	15975
2.44	−0.250224 727431	35 099014	15538
2.45	−0.247803 945370	34 442185	15110
2.46	−0.245417 605493	33 800466	14706
2.47	−0.243065 066083	33 173454	14300
2.48	−0.240745 700126	32 560741	13919
2.49	−0.238458 894911	31 961946	13548
2.50	−0.236204 051641	31 376700	13177

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
2.50	—0.236204 051641	31 376700	13177
2.51	—0.233980 585072	30 804630	12834
2.52	—0.231787 923133	30 245395	12489
2.53	—0.229625 506588	29 698648	12161
2.54	—0.227492 788691	29 164063	11841
2.55	—0.225389 234858	28 641319	11528
2.56	—0.223314 322343	28 130103	11232
2.57	—0.221267 539932	27 630120	10935
2.58	—0.219248 387641	27 141072	10656
2.59	—0.217256 376422	26 662680	10381
2.60	—0.215291 027884	26 194670	10110
2.61	—0.213351 874016	25 736769	9859
2.62	—0.211438 456916	25 288728	9601
2.63	—0.209550 328545	24 850288	9361
2.64	—0.207687 050462	24 421209	9124
2.65	—0.205848 193588	24 001254	8894
2.66	—0.204033 337967	23 590193	8672
2.67	—0.202242 072539	23 187804	8454
2.68	—0.200473 994914	22 793869	8243
2.69	—0.198728 711158	22 408177	8043
2.70	—0.197005 835579	22 030528	7837
2.71	—0.195304 990528	21 660717	7652
2.72	—0.193625 806194	21 298559	7459
2.73	—0.191967 920420	20 943860	7280
2.74	—0.190330 978505	20 596441	7100
2.75	—0.188714 633031	20 256123	6937
2.76	—0.187118 543681	19 922742	6755
2.77	—0.185542 377073	19 596115	6606
2.78	—0.183985 806580	19 276095	6443
2.79	—0.182448 512182	18 962517	6287
2.80	—0.180930 180301	18 655226	6141
2.81	—0.179430 503647	18 354077	5995
2.82	—0.177949 181070	18 058922	5851
2.83	—0.176485 917414	17 769619	5719
2.84	—0.175040 423378	17 486034	5578
2.85	—0.173612 415375	17 208027	5453
2.86	—0.172201 615399	16 935473	5325
2.87	—0.170807 750897	16 668245	5202
2.88	—0.169430 554639	16 406218	5080
2.89	—0.168069 764599	16 149270	4967
2.90	—0.166725 123830	15 897290	4851
2.91	—0.165396 380350	15 650160	4741
2.92	—0.164083 287030	15 407770	4630
2.93	—0.162785 601480	15 170010	4532
2.94	—0.161503 085941	14 936782	4422
2.95	—0.160235 507183	14 707976	4329
2.96	—0.158982 636401	14 483498	4226
2.97	—0.157744 249117	14 263246	4137
2.98	—0.156520 125079	14 047130	4042
2.99	—0.155310 048171	13 835056	3952
3.00	—0.154113 806319	13 626935	3863

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
3.00	−0.154113 806319	13 626935	3863
3.01	−0.152931 191402	13 422676	3785
3.02	−0.151761 999162	13 222203	3694
3.03	−0.150606 029124	13 025423	3615
3.04	−0.149463 084509	12 832258	3541
3.05	−0.148332 972152	12 642634	3457
3.06	−0.147215 502429	12 456468	3385
3.07	−0.146110 489174	12 273687	3314
3.08	−0.145017 749607	12 094221	3241
3.09	−0.143937 104260	11 917995	3172
3.10	−0.142868 376908	11 744940	3105
3.11	−0.141811 394496	11 574991	3037
3.12	−0.140765 987075	11 408078	2975
3.13	−0.139731 987732	11 244140	2909
3.14	−0.138709 232528	11 083111	2850
3.15	−0.137697 560436	10 924932	2790
3.16	−0.136696 813275	10 769544	2729
3.17	−0.135706 835659	10 616885	2675
3.18	−0.134727 474927	10 466902	2618
3.19	−0.133758 581097	10 319536	2563
3.20	−0.132800 006803	10 174734	2513
3.21	−0.131851 607243	10 032445	2459
3.22	−0.130913 240128	9 892615	2407
3.23	−0.129984 765627	9 755192	2361
3.24	−0.129066 046318	9 620130	2312
3.25	−0.128156 947139	9 487381	2265
3.26	−0.127257 335341	9 356896	2216
3.27	−0.126367 080440	9 228627	2180
3.28	−0.125486 054165	9 102538	2122
3.29	−0.124614 130429	8 978572	2095
3.30	−0.123751 185265	8 856700	2041
3.31	−0.122897 096801	8 736869	2007
3.32	−0.122051 745205	8 619046	1962
3.33	−0.121215 012657	8 503186	1929
3.34	−0.120386 783293	8 389254	1885
3.35	−0.119566 943184	8 277208	1853
3.36	−0.118755 380283	8 167014	1815
3.37	−0.117951 984397	8 058636	1777
3.38	−0.117156 647146	7 952034	1745
3.39	−0.116369 261929	7 847178	1713
3.40	−0.115589 723891	7 744035	1674
3.41	−0.114817 929888	7 642566	1646
3.42	−0.114053 778450	7 542742	1613
3.43	−0.113297 169754	7 444532	1582
3.44	−0.112548 005591	7 347905	1553
3.45	−0.111806 189333	7 252830	1519
3.46	−0.111071 625904	7 159275	1498
3.47	−0.110344 221750	7 067217	1461
3.48	−0.109623 884813	6 976620	1438
3.49	−0.108910 524496	6 887462	1414
3.50	−0.108204 051641	6 799717	1379

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
3.50	—0.108204 051641	6 799717	1379
3.51	—0.107504 378503	6 713351	1361
3.52	—0.106811 418715	6 628345	1331
3.53	—0.106125 087273	6 544671	1308
3.54	—0.105445 300502	6 462305	1284
3.55	—0.104771 976035	6 381223	1257
3.56	—0.104105 032792	6 301398	1239
3.57	—0.103444 390947	6 222812	1211
3.58	—0.102789 971913	6 145437	1193
3.59	—0.102141 698316	6 069254	1170
3.60	—0.101499 493974	5 994242	1145
3.61	—0.100863 283874	5 920375	1131
3.62	—0.100232 994148	5 847638	1104
3.63	—0.099608 552061	5 776006	1087
3.64	—0.098989 885980	5 705461	1067
3.65	—0.098376 925359	5 635983	1048
3.66	—0.097769 600722	5 567553	1030
3.67	—0.097167 843638	5 500152	1010
3.68	—0.096571 586706	5 433762	992
3.69	—0.095980 763535	5 368363	978
3.70	—0.095395 308728	5 303942	953
3.71	—0.094815 157863	5 240475	944
3.72	—0.094240 247472	5 177952	922
3.73	—0.093670 515034	5 116351	909
3.74	—0.093105 898946	5 055659	890
3.75	—0.092546 338516	4 995856	883
3.76	—0.091991 773943	4 936937	852
3.77	—0.091442 146306	4 878869	851
3.78	—0.090897 397539	4 821653	832
3.79	—0.090357 470426	4 765269	815
3.80	—0.089822 308581	4 709699	805
3.81	—0.089291 856436	4 654934	789
3.82	—0.088766 059225	4 600958	774
3.83	—0.088244 862973	4 547757	766
3.84	—0.087728 214477	4 495320	746
3.85	—0.087216 061301	4 443630	739
3.86	—0.086708 351755	4 392678	724
3.87	—0.086205 034887	4 342450	712
3.88	—0.085706 060469	2 292934	698
3.89	—0.085211 378985	4 244116	690
3.90	—0.084720 941616	4 195988	676
3.91	—0.084234 700237	4 148537	666
3.92	—0.083752 607394	4 101751	651
3.93	—0.083274 616302	4 055616	647
3.94	—0.082800 680827	4 010129	629
3.95	—0.082330 755480	3 965271	625
3.96	—0.081864 795405	3 921038	609
3.97	—0.081402 756367	3 877413	604
3.98	—0.080944 594743	3 834393	591
3.99	—0.080490 267512	3 791964	579
4.00	—0.080039 732245	3 750114	566

TABLE 19

THE TETRAGAMMA FUNCTION

Description: $\Psi''(x)$ to 10 decimal places with central differences from $x = 4.00$ to $x = 20.00$ at intervals of .02.

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
4.00	—.08003 97322	150010	91	5.00	—.04878 97322	56255	21
4.02	—.07914 98708	146731	89	5.02	—.04836 39721	55284	20
4.04	—.07827 46824	143540	86	5.04	—.04794 37403	54334	20
4.06	—.07741 38480	140435	83	5.06	—.04752 89420	53405	19
4.08	—.07656 70571	137413	80	5.08	—.04711 94841	52495	19
4.10	—.07573 40075	134471	78	5.10	—.04671 52759	51605	18
4.12	—.07491 44049	131608	75	5.12	—.04631 62281	50733	18
4.14	—.07410 79632	128819	73	5.14	—.04592 22536	49880	17
4.16	—.07331 44034	126104	71	5.16	—.04553 32671	49044	17
4.18	—.07253 34540	123460	69	5.18	—.04514 91851	48226	17
4.20	—.07176 48505	120884	67	5.20	—.04476 99256	47424	16
4.22	—.07100 83356	118375	64	5.22	—.04439 54086	46640	16
4.24	—.07026 36581	115930	63	5.24	—.04402 55555	45871	15
4.26	—.06953 05736	113548	61	5.26	—.04366 02895	45118	15
4.28	—.06880 88440	111227	60	5.28	—.04329 95354	44380	15
4.30	—.06809 82370	108964	57	5.30	—.04294 32192	43658	14
4.32	—.06739 85264	106758	55	5.32	—.04259 12689	42950	14
4.34	—.06670 94916	104607	54	5.34	—.04224 36135	42256	14
4.36	—.06603 09175	102510	52	5.36	—.04190 01838	41576	13
4.38	—.06536 25944	100465	50	5.38	—.04156 09115	40910	13
4.40	—.06470 43178	98471	49	5.40	—.04122 57303	40257	13
4.42	—.06405 58884	96526	48	5.42	—.04089 45747	39616	12
4.44	—.06341 71114	94628	46	5.44	—.04056 73808	38989	12
4.46	—.06278 77973	92776	45	5.46	—.04024 40857	38374	12
4.48	—.06216 77608	90970	44	5.48	—.03992 46281	37771	12
4.50	—.06155 68213	89207	42	5.50	—.03960 89475	37180	11
4.52	—.06095 48024	87486	41	5.52	—.03929 69849	36600	11
4.54	—.06036 15321	85806	40	5.54	—.03898 86822	36031	11
4.56	—.05977 68425	84166	39	5.56	—.03868 39826	35473	10
4.58	—.05920 05694	82565	38	5.58	—.03838 28303	34926	10
4.60	—.05863 25530	81002	37	5.60	—.03808 51707	34390	10
4.62	—.05807 26367	79476	35	5.62	—.03779 09500	33864	10
4.64	—.05752 06681	77985	35	5.64	—.03750 01158	33347	9
4.66	—.05697 64979	76529	34	5.66	—.03721 26163	32841	9
4.68	—.05643 99807	75106	33	5.68	—.03692 84008	32344	9
4.70	—.05591 09741	73717	32	5.70	—.03664 74198	31857	9
4.72	—.05538 93392	72359	31	5.72	—.03636 96245	31378	9
4.74	—.05487 49402	71032	30	5.74	—.03609 49669	30909	9
4.76	—.05436 76444	69735	29	5.76	—.03582 34002	30448	8
4.78	—.05386 73222	68468	28	5.78	—.03555 48783	29996	8
4.80	—.05337 38467	67229	28	5.80	—.03528 93560	29552	8
4.82	—.05288 70942	66018	27	5.82	—.03502 67888	29116	8
4.84	—.05240 69435	64834	26	5.84	—.03476 71332	28689	8
4.86	—.05193 32762	63677	26	5.86	—.03451 03464	28268	7
4.88	—.05146 59767	62545	25	5.88	—.03425 63864	27856	7
4.90	—.05100 49316	61438	24	5.90	—.03400 52121	27451	7
4.92	—.05055 00302	60355	24	5.92	—.03375 67828	27053	7
4.94	—.05010 11644	59296	23	5.94	—.03351 10588	26663	7
4.96	—.04965 82281	58260	22	5.96	—.03326 80012	26279	7
4.98	—.04922 11179	57246	22	5.98	—.03302 75716	25903	6
5.00	—.04878 97322	56255	21	6.00	—.03278 97322	25533	6

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
6.00	—.03278 97322	25533	6	7.00	—.02353 04730	13187	2
6.02	—.03255 44462	25170	6	7.02	—.02338 71657	13028	2
6.04	—.03232 16772	24813	6	7.04	—.02324 51612	12870	2
6.06	—.03209 13896	24463	6	7.06	—.02310 44437	12716	2
6.08	—.03186 35481	24118	6	7.08	—.02296 49979	12563	2
6.10	—.03163 81185	23780	6	7.10	—.02282 68083	12413	2
6.12	—.03141 50669	23448	6	7.12	—.02268 98601	12265	2
6.14	—.03119 43600	23120	6	7.14	—.02255 41384	12119	2
6.16	—.03097 59652	22800	6	7.16	—.02241 96286	11976	2
6.18	—.03075 98503	22484	5	7.18	—.02228 63164	11834	2
6.20	—.03054 59839	22174	5	7.20	—.02215 41877	11695	2
6.22	—.03033 43348	21870	5	7.22	—.02202 32284	11558	2
6.24	—.03012 48726	21570	5	7.24	—.02189 34249	11422	2
6.26	—.02991 75675	21275	5	7.26	—.02176 47636	11289	2
6.28	—.02971 23898	20986	5	7.28	—.02163 72311	11157	2
6.30	—.02950 93107	20701	5	7.30	—.02151 08144	11028	2
6.32	—.02930 83017	20421	5	7.32	—.02138 55005	10900	2
6.34	—.02910 93349	20146	5	7.34	—.02126 12766	10774	2
6.36	—.02891 23827	19876	4	7.36	—.02113 81301	10650	2
6.38	—.02871 74181	19610	4	7.38	—.02101 60486	10528	2
6.40	—.02852 44144	19348	4	7.40	—.02089 50200	10407	2
6.42	—.02833 33456	19091	4	7.42	—.02077 50320	10289	2
6.44	—.02814 41859	18838	4	7.44	—.02065 60729	10171	2
6.46	—.02795 69101	18590	4	7.46	—.02053 81309	10056	2
6.48	—.02777 14932	18345	4	7.48	—.02042 11946	9942	2
6.50	—.02758 79107	18104	4	7.50	—.02030 52525	9830	2
6.52	—.02740 61387	17867	4	7.52	—.02019 02934	9719	2
6.54	—.02722 61533	17634	4	7.54	—.02007 63063	9610	2
6.56	—.02704 79315	17405	4	7.56	—.01996 32802	9503	2
6.58	—.02687 14501	17180	4	7.58	—.01985 12044	9397	1
6.60	—.02669 66867	16958	3	7.60	—.01974 00682	9292	1
6.62	—.02652 36191	16740	3	7.62	—.01962 98612	9189	1
6.64	—.02635 22255	16525	3	7.64	—.01952 05732	9087	1
6.66	—.02618 24844	16314	3	7.66	—.01941 21939	8987	1
6.68	—.02601 43747	16106	3	7.68	—.01930 47133	8888	1
6.70	—.02584 78756	15901	3	7.70	—.01919 81214	8790	1
6.72	—0.2568 29665	15700	3	7.72	—.01909 24086	8694	1
6.74	—.02551 96274	15501	3	7.74	—.01898 75653	8599	1
6.76	—.02535 78384	15306	3	7.76	—.01888 35819	8506	1
6.78	—.02519 75801	15114	3	7.78	—.01878 04490	8413	1
6.80	—.02503 88332	14925	3	7.80	—.01867 81574	8322	1
6.82	—.02488 15788	14739	3	7.82	—.01857 66981	8232	1
6.84	—.02472 57982	14556	3	7.84	—.01847 60620	8143	1
6.86	—.02457 14733	14375	3	7.86	—.01837 62402	8056	1
6.88	—.02441 85858	14198	3	7.88	—.01827 72240	7970	1
6.90	—.02426 71181	14023	3	7.90	—.01817 90048	7885	1
6.92	—.02411 70526	13850	3	7.92	—.01808 15741	7800	1
6.94	—.02396 83723	13681	2	7.94	—.01798 49234	7718	1
6.96	—.02382 10599	13514	2	7.96	—.01788 90444	7636	1
6.98	—.02367 50990	13349	2	7.98	—.01779 39291	7555	1
7.00	—.02353 04730	13187	2	8.00	—.01769 95692	7475	1

x	$\Psi''(x)$	$-\delta^2$	x	$\Psi''(x)$	$-\delta^2$
8.00	—.01769 95692	7475	9.00	—.01379 33192	4546
8.02	—.01760 59568	7397	9.02	—.01372 88578	4503
8.04	—.01751 30841	7319	9.04	—.01366 48468	4461
8.06	—.01742 09434	7242	9.06	—.01360 12819	4420
8.08	—.01732 95268	7167	9.08	—.01353 81590	4379
8.10	—.01723 88269	7092	9.10	—.01347 54741	4339
8.12	—.01714 88363	7019	9.12	—.01341 32230	4299
8.14	—.01705 95475	6946	9.14	—.01335 14019	4260
8.16	—.01697 09533	6874	9.16	—.01329 00067	4221
8.18	—.01688 30465	6803	9.18	—.01322 90335	4182
8.20	—.01679 58200	6733	9.20	—.01316 84786	4144
8.22	—.01670 92669	6664	9.22	—.01310 83380	4106
8.24	—.01662 33802	6596	9.24	—.01304 86081	4069
8.26	—.01653 81531	6529	9.26	—.01298 92850	4032
8.28	—.01645 35789	6462	9.28	—.01293 03652	3996
8.30	—.01636 96509	6397	9.30	—.01287 18449	3960
8.32	—.01628 63627	6332	9.32	—.01281 37206	3924
8.34	—.01620 37076	6268	9.34	—.01275 59887	3889
8.36	—.01612 16793	6205	9.36	—.01269 86457	3854
8.38	—.01604 02716	6143	9.38	—.01264 16881	3820
8.40	—.01595 94781	6081	9.40	—.01258 51125	3786
8.42	—.01587 92927	6020	9.42	—.01252 89154	3752
8.44	—.01579 97093	5960	9.44	—.01247 30935	3719
8.46	—.01572 07220	5901	9.46	—.01241 76434	3686
8.48	—.01564 23248	5842	9.48	—.01236 25620	3653
8.50	—.01556 45118	5785	9.50	—.01230 78458	3621
8.52	—.01548 72773	5727	9.52	—.01225 34917	3590
8.54	—.01541 06155	5671	9.54	—.01219 94966	3558
8.56	—.01533 45208	5615	9.56	—.01214 58572	3526
8.58	—.01525 89876	5560	9.58	—.01209 25704	3496
8.60	—.01518 40105	5506	9.60	—.01203 96332	3465
8.62	—.01510 95839	5452	9.62	—.01198 70426	3435
8.64	—.01503 57026	5399	9.64	—.01193 47954	3405
8.66	—.01496 23611	5347	9.66	—.01188 28887	3376
8.68	—.01488 95544	5295	9.68	—.01183 13196	3346
8.70	—.01481 72771	5244	9.70	—.01178 00852	3318
8.72	—.01474 55242	5193	9.72	—.01172 91824	3289
8.74	—.01467 42906	5143	9.74	—.01167 86087	3261
8.76	—.01460 35714	5094	9.76	—.01162 83610	3233
8.78	—.01453 33615	5045	9.78	—.01157 84366	3205
8.80	—.01446 36562	4997	9.80	—.01152 88327	3178
8.82	—.01439 44505	4949	9.82	—.01147 95466	3151
8.84	—.01432 57398	4902	9.84	—.01143 05756	3124
8.86	—.01425 75194	4856	9.86	—.01138 19170	3098
8.88	—.01418 97845	4810	9.88	—.01133 35681	3071
8.90	—.01412 25306	4765	9.90	—.01128 55264	3045
8.92	—.01405 57532	4720	9.92	—.01123 77892	3020
8.94	—.01398 94477	4675	9.94	—.01119 03540	2994
8.96	—.01392 36098	4632	9.96	—.01114 32182	2969
8.98	—.01385 82351	4589	9.98	—.01109 63794	2944
9.00	—.01379 33192	4546	10.00	—.01104 98350	2920

x	$\Psi''(x)$	$-\delta^2$	x	$\Psi''(x)$	$-\delta^2$
10.00	—.01104 98350	2920	11.00	—.00904 98350	1960
10.02	—.01100 35825	2895	11.02	—.00901 55347	1945
10.04	—.01095 76196	2871	11.04	—.00898 14289	1930
10.06	—.01091 19438	2847	11.06	—.00894 75161	1916
10.08	—.01086 65528	2824	11.08	—.00891 37949	1901
10.10	—.01082 14442	2801	11.10	—.00888 02639	1887
10.12	—.01077 66156	2777	11.12	—.00884 69215	1873
10.14	—.01073 20647	2754	11.14	—.00881 37665	1859
10.16	—.01068 77893	2732	11.16	—.00878 07973	1845
10.18	—.01064 37871	2709	11.18	—.00874 80127	1831
10.20	—.01060 00558	2687	11.20	—.00871 54112	1818
10.22	—.01055 65933	2665	11.22	—.00868 29914	1804
10.24	—.01051 33973	2644	11.24	—.00865 07521	1791
10.26	—.01047 04656	2622	11.26	—.00861 86919	1778
10.28	—.01042 77962	2601	11.28	—.00858 68095	1765
10.30	—.01038 53868	2580	11.30	—.00855 51035	1752
10.32	—.01034 32354	2559	11.32	—.00852 35727	1739
10.34	—.01030 13399	2538	11.34	—.00849 22157	1726
10.36	—.01025 96982	2518	11.36	—.00846 10314	1713
10.38	—.01021 83083	2498	11.38	—.00843 00184	1701
10.40	—.01017 71682	2477	11.40	—.00839 91755	1688
10.42	—.01013 62758	2458	11.42	—.00836 85014	1676
10.44	—.01009 56291	2438	11.44	—.00833 79949	1664
10.46	—.01005 52263	2419	11.46	—.00830 76548	1652
10.48	—.01001 50653	2399	11.48	—.00827 74799	1640
10.50	—.00997 51442	2380	11.50	—.00824 74691	1628
10.52	—.00993 54612	2361	11.52	—.00821 76210	1616
10.54	—.00989 60143	2343	11.54	—.00818 79345	1605
10.56	—.00985 68017	2324	11.56	—.00815 84086	1593
10.58	—.00981 78215	2306	11.58	—.00812 90418	1582
10.60	—.00977 90719	2288	11.60	—.00809 98333	1570
10.62	—.00974 05511	2270	11.62	—.00807 07819	1559
10.64	—.00970 22572	2252	11.64	—.00804 18863	1548
10.66	—.00966 41885	2234	11.66	—.00801 31456	1537
10.68	—.00962 63433	2217	11.68	—.00798 45585	1526
10.70	—.00958 87198	2200	11.70	—.00795 61241	1515
10.72	—.00955 13163	2183	11.72	—.00792 78411	1504
10.74	—.00951 41310	2166	11.74	—.00789 97087	1494
10.76	—.00947 71622	2149	11.76	—.00787 17256	1483
10.78	—.00944 04084	2132	11.78	—.00784 38908	1473
10.80	—.00940 38678	2116	11.80	—.00781 62033	1462
10.82	—.00936 75388	2100	11.82	—.00778 86621	1452
10.84	—.00933 14197	2083	11.84	—.00776 12660	1442
10.86	—.00929 55089	2067	11.86	—.00773 40142	1432
10.88	—.00925 98049	2052	11.88	—.00770 69056	1422
10.90	—.00922 43061	2036	11.90	—.00767 99392	1412
10.92	—.00918 90108	2020	11.92	—.00765 31139	1402
10.94	—.00915 39177	2005	11.94	—.00762 64289	1392
10.96	—.00911 90250	1990	11.96	—.00759 98831	1383
10.98	—.00908 43312	1974	11.98	—.00757 34756	1373
11.00	—.00904 98350	1960	12.00	—.00754 72054	1364

x	$\Psi''(x)$	$-\delta^2$	x	$\Psi''(x)$	$-\delta^2$
12.00	—.00754 72054	1364	13.00	—.00638 97980	978
12.02	—.00752 10715	1354	13.02	—.00636 94319	972
12.04	—.00749 50731	1345	13.04	—.00634 91630	965
12.06	—.00746 92092	1336	13.06	—.00632 89907	959
12.08	—.00744 34788	1326	13.08	—.00630 89143	953
12.10	—.00741 78811	1317	13.10	—.00628 89332	947
12.12	—.00739 24151	1308	13.12	—.00626 90469	941
12.14	—.00736 70800	1299	13.14	—.00624 92547	935
12.16	—.00734 18748	1291	13.16	—.00622 95560	929
12.18	—.00731 67986	1282	13.18	—.00620 99503	923
12.20	—.00729 18507	1273	13.20	—.00619 04369	918
12.22	—.00726 70300	1264	13.22	—.00617 10153	912
12.24	—.00724 23358	1256	13.24	—.00615 16849	906
12.26	—.00721 77671	1247	13.26	—.00613 24452	901
12.28	—.00719 33232	1239	13.28	—.00611 32955	895
12.30	—.00716 90032	1231	13.30	—.00609 42353	890
12.32	—.00714 48062	1222	13.32	—.00607 52641	884
12.34	—.00712 07315	1214	13.34	—.00605 63813	879
12.36	—.00709 67781	1206	13.36	—.00603 75864	873
12.38	—.00707 29454	1198	13.38	—.00601 88788	868
12.40	—.00704 92324	1190	13.40	—.00600 02579	862
12.42	—.00702 56384	1182	13.42	—.00598 17233	857
12.44	—.00700 21627	1174	13.44	—.00596 32744	852
12.46	—.00697 88042	1166	13.46	—.00594 49106	847
12.48	—.00695 55625	1158	13.48	—.00592 66315	841
12.50	—.00693 24366	1151	13.50	—.00590 84366	836
12.52	—.00690 94257	1143	13.52	—.00589 03253	831
12.54	—.00688 65292	1136	13.54	—.00587 22970	826
12.56	—.00686 37462	1128	13.56	—.00585 43514	821
12.58	—.00684 10761	1121	13.58	—.00583 64879	816
12.60	—.00681 85180	1113	13.60	—.00581 87060	811
12.62	—.00679 60712	1106	13.62	—.00580 10051	806
12.64	—.00677 37351	1099	13.64	—.00578 33849	801
12.66	—.00675 15088	1092	13.66	—.00576 58448	796
12.68	—.00672 93916	1084	13.68	—.00574 83843	792
12.70	—.00670 73830	1077	13.70	—.00573 10030	787
12.72	—.00668 54820	1070	13.72	—.00571 37004	782
12.74	—.00666 36881	1063	13.74	—.00569 64760	777
12.76	—.00664 20005	1056	13.76	—.00567 93293	773
12.78	—.00662 04185	1050	13.78	—.00566 22599	768
12.80	—.00659 89416	1043	13.80	—.00564 52673	763
12.82	—.00657 75689	1036	13.82	—.00562 83510	759
12.84	—.00655 62998	1029	13.84	—.00561 15106	754
12.86	—.00653 51336	1023	13.86	—.00559 47456	750
12.88	—.00651 40698	1016	13.88	—.00557 80556	745
12.90	—.00649 31075	1010	13.90	—.00556 14402	741
12.92	—.00647 22462	1003	13.92	—.00554 48989	737
12.94	—.00645 14853	997	13.94	—.00552 84312	732
12.96	—.00643 08240	990	13.96	—.00551 20367	728
12.98	—.00641 02618	984	13.98	—.00549 57150	724
13.00	—.00638 97980	978	14.00	—.00547 94657	719

x	$\Psi''(x)$	$-\delta^2$	x	$\Psi''(x)$	$-\delta^2$
14.00	—.00547 94657	719	15.00	—.00475 06027	541
14.02	—.00546 32883	715	15.02	—.00473 75401	538
14.04	—.00544 71824	711	15.04	—.00472 45313	535
14.06	—.00543 11476	707	15.06	—.00471 15759	532
14.08	—.00541 51834	703	15.08	—.00469 86738	530
14.10	—.00539 92895	698	15.10	—.00468 58246	526
14.12	—.00538 34655	694	15.12	—.00467 30279	523
14.14	—.00536 77109	690	15.14	—.00466 02836	520
14.16	—.00535 20253	686	15.16	—.00464 75914	518
14.18	—.00533 64083	682	15.18	—.00463 49509	515
14.20	—.00532 08596	678	15.20	—.00462 23619	512
14.22	—.00530 53786	674	15.22	—.00460 98241	509
14.24	—.00528 99652	670	15.24	—.00459 73372	506
14.26	—.00527 46187	667	15.26	—.00458 49010	504
14.28	—.00525 93389	663	15.28	—.00457 25151	501
14.30	—.00524 41254	659	15.30	—.00456 01793	498
14.32	—.00522 89778	655	15.32	—.00454 78934	495
14.34	—.00521 38957	651	15.34	—.00453 56570	492
14.36	—.00519 88787	648	15.36	—.00452 34699	490
14.38	—.00518 39265	644	15.38	—.00451 13319	488
14.40	—.00516 90386	640	15.40	—.00449 92427	485
14.42	—.00515 42148	637	15.42	—.00448 72019	482
14.44	—.00513 94546	633	15.44	—.00447 52094	480
14.46	—.00512 47577	629	15.46	—.00446 32649	477
14.48	—.00511 01238	626	15.48	—.00445 13682	475
14.50	—.00509 55524	622	15.50	—.00443 95189	472
14.52	—.00508 10432	619	15.52	—.00442 77169	470
14.54	—.00506 65958	615	15.54	—.00441 59618	467
14.56	—.00505 22100	612	15.56	—.00440 42535	465
14.58	—.00503 78853	608	15.58	—.00439 25916	462
14.60	—.00502 36215	605	15.60	—.00438 09760	460
14.62	—.00500 94181	601	15.62	—.00436 94064	458
14.64	—.00499 52748	598	15.64	—.00435 78826	455
14.66	—.00498 11913	595	15.66	—.00434 64043	453
14.68	—.00496 71673	591	15.68	—.00433 49712	450
14.70	—.00495 32024	588	15.70	—.00432 35832	448
14.72	—.00493 92962	585	15.72	—.00431 22400	446
14.74	—.00492 54486	581	15.74	—.00430 09413	443
14.76	—.00491 16591	578	15.76	—.00428 96870	440
14.78	—.00489 79273	575	15.78	—.00427 84767	439
14.80	—.00488 42531	572	15.80	—.00426 73104	437
14.82	—.00487 06360	568	15.82	—.00425 61876	434
14.84	—.00485 70758	565	15.84	—.00424 51083	432
14.86	—.00484 35720	562	15.86	—.00423 40722	430
14.88	—.00483 01245	559	15.88	—.00422 30791	427
14.90	—.00481 67329	556	15.90	—.00421 21287	425
14.92	—.00480 33969	553	15.92	—.00420 12208	423
14.94	—.00479 01162	550	15.94	—.00419 03552	421
14.96	—.00477 68905	547	15.96	—.00417 95317	419
14.98	—.00476 37194	544	15.98	—.00416 87501	416
15.00	—.00475 06027	541	16.00	—.00415 80101	414

x	$\Psi''(x)$	$-\delta^2$	x	$\Psi''(x)$	$-\delta^2$
16.00	—.00415 80101	414	17.00	—.00366 97289	323
16.02	—.00414 73116	412	17.02	—.00366 08568	321
16.04	—.00413 66543	410	17.04	—.00365 20169	320
16.06	—.00412 60380	408	17.06	—.00364 32089	318
16.08	—.00411 54624	406	17.08	—.00363 44328	317
16.10	—.00410 49275	404	17.10	—.00362 56883	315
16.12	—.00409 44330	402	17.12	—.00361 69753	314
16.14	—.00408 39787	400	17.14	—.00360 82937	312
16.16	—.00407 35643	398	17.16	—.00359 96433	311
16.18	—.00406 31897	396	17.18	—.00359 10240	309
16.20	—.00405 28547	394	17.20	—.00358 24356	308
16.22	—.00404 25590	392	17.22	—.00357 38779	306
16.24	—.00403 23025	390	17.24	—.00356 53509	305
16.26	—.00402 20850	388	17.26	—.00355 68543	303
16.28	—.00401 19063	386	17.28	—.00354 83881	302
16.30	—.00400 17661	384	17.30	—.00353 99520	300
16.32	—.00399 16643	382	17.32	—.00353 15460	299
16.34	—.00398 16007	380	17.34	—.00352 31699	298
16.36	—.00397 15751	378	17.36	—.00351 48235	296
16.38	—.00396 15873	376	17.38	—.00350 65067	295
16.40	—.00395 16371	374	17.40	—.00349 82195	293
16.42	—.00394 17244	372	17.42	—.00348 99615	292
16.44	—.00393 18489	371	17.44	—.00348 17328	291
16.46	—.00392 20104	369	17.46	—.00347 35331	289
16.48	—.00391 22089	367	17.48	—.00346 53623	288
16.50	—.00390 24440	365	17.50	—.00345 72204	287
16.52	—.00389 27156	363	17.52	—.00344 91071	285
16.54	—.00388 30235	361	17.54	—.00344 10223	284
16.56	—.00387 33676	359	17.56	—.00343 29658	283
16.58	—.00386 37476	358	17.58	—.00342 49377	281
16.60	—.00385 41634	356	17.60	—.00341 69376	280
16.62	—.00384 46148	354	17.62	—.00340 89656	279
16.64	—.00383 51016	353	17.64	—.00340 10214	277
16.66	—.00382 56237	351	17.66	—.00339 31049	276
16.68	—.00381 61809	349	17.68	—.00338 52161	275
16.70	—.00380 67730	347	17.70	—.00337 73547	273
16.72	—.00379 73998	346	17.72	—.00336 95206	272
16.74	—.00378 80612	344	17.74	—.00336 17138	271
16.76	—.00377 87570	342	17.76	—.00335 39341	270
16.78	—.00376 94870	340	17.78	—.00334 61813	268
16.80	—.00376 02511	339	17.80	—.00333 84554	267
16.82	—.00375 10490	337	17.82	—.00333 07562	266
16.84	—.00374 18807	336	17.84	—.00332 30836	265
16.86	—.00373 27460	334	17.86	—.00331 54374	263
16.88	—.00372 36446	332	17.88	—.00330 78176	262
16.90	—.00371 45765	331	17.90	—.00330 02241	261
16.92	—.00370 55415	329	17.92	—.00329 26566	260
16.94	—.00369 65394	328	17.94	—.00328 51152	259
16.96	—.00368 75700	326	17.96	—.00327 75996	257
16.98	—.00367 86332	324	17.98	—.00327 01098	256
17.00	—.00366 97289	323	18.00	—.00326 26456	255

x	$\Psi''(x)$	$-\delta^2$	x	$\Psi''(x)$	$-\delta^2$
18.00	—.00326 26456	255	19.00	—.00291 97101	205
18.02	—.00325 52070	254	19.02	—.00291 34120	203
18.04	—.00324 77937	253	19.04	—.00290 71343	203
18.06	—.00324 04057	252	19.06	—.00290 08768	202
18.08	—.00323 30429	251	19.08	—.00289 46395	201
18.10	—.00322 57052	249	19.10	—.00288 84223	200
18.12	—.00321 83924	248	19.12	—.00288 22251	199
18.14	—.00321 11044	247	19.14	—.00287 60479	198
18.16	—.00320 38412	246	19.16	—.00286 98904	198
18.18	—.00319 66026	245	19.18	—.00286 37527	197
18.20	—.00318 93884	244	19.20	—.00285 76347	196
18.22	—.00318 21987	243	19.22	—.00285 15362	195
18.24	—.00317 50332	242	19.24	—.00284 54573	194
18.26	—.00316 78919	241	19.26	—.00283 93977	193
18.28	—.00316 07747	240	19.28	—.00283 33575	193
18.30	—.00315 36814	238	19.30	—.00282 73366	192
18.32	—.00314 66119	237	19.32	—.00282 13348	191
18.34	—.00313 95662	236	19.34	—.00281 53520	190
18.36	—.00313 25441	235	19.36	—.00280 93883	189
18.38	—.00312 55456	234	19.38	—.00280 34436	188
18.40	—.00311 85705	233	19.40	—.00279 75176	187
18.42	—.00311 16187	232	19.42	—.00279 16105	187
18.44	—.00310 46901	231	19.44	—.00278 57220	186
18.46	—.00309 77846	230	19.46	—.00277 98521	185
18.48	—.00309 09021	229	19.48	—.00277 40007	185
18.50	—.00308 40425	228	19.50	—.00276 81679	184
18.52	—.00307 72058	227	19.52	—.00276 23533	183
18.54	—.00307 03917	226	19.54	—.00275 65571	182
18.56	—.00306 36002	225	19.56	—.00275 07791	181
18.58	—.00305 68313	224	19.58	—.00274 50192	181
18.60	—.00305 00847	223	19.60	—.00273 92774	180
18.62	—.00304 33605	222	19.62	—.00273 35536	179
18.64	—.00303 66584	221	19.64	—.00272 78478	178
18.66	—.00302 99785	220	19.66	—.00272 21597	178
18.68	—.00302 33205	219	19.68	—.00271 64894	177
18.70	—.00301 66845	218	19.70	—.00271 08369	176
18.72	—.00301 00703	217	19.72	—.00270 52019	175
18.74	—.00300 34779	216	19.74	—.00269 95845	175
18.76	—.00299 69070	216	19.76	—.00269 39846	174
18.78	—.00299 03577	214	19.78	—.00268 84020	173
18.80	—.00298 38299	213	19.80	—.00268 28368	173
18.82	—.00297 73233	213	19.82	—.00267 72889	172
18.84	—.00297 08381	212	19.84	—.00267 17581	171
18.86	—.00296 43740	211	19.86	—.00266 62445	170
18.88	—.00295 79309	210	19.88	—.00266 07479	170
18.90	—.00295 15089	209	19.90	—.00265 52683	169
18.92	—.00294 51077	208	19.92	—.00264 98056	168
18.94	—.00293 87273	207	19.94	—.00264 43597	168
18.96	—.00293 23677	206	19.96	—.00263 89306	167
18.98	—.00292 60286	205	19.98	—.00263 35182	166
19.00	—.00291 97101	205	20.00	—.00262 81224	166

TABLE 20

THE TETRAGAMMA FUNCTION

Description: $\Psi''(x)$ to 17 decimal places with central differences from $x = 20.0$ to $x = 100.0$ at intervals of .1.

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
20.0	−.(2)26281 22402 31465	414077 88299	217 30061	23930
20.1	26013 90605 02219	405700 59815	210 74224	22972
20.2	25750 64508 32788	397534 05555	204 41359	22058
20.3	25491 35945 68913	389571 92654	198 30552	21184
20.4	25235 96954 97691	381808 10305	192 40929	20348
20.5	24984 39772 36773	374236 68884	186 71654	19550
20.6	24736 56826 44741	366851 99118	181 21928	18786
20.7	24492 40732 51826	359648 51279	175 90989	18056
20.8	24251 84287 10190	352620 94430	170 78107	17358
20.9	24014 80462 62984	345764 15687	165 82582	16690
21.0	23781 22402 31465	339073 19527	161 03748	16051
21.1	23551 03415 19474	332543 27115	156 40964	15439
21.2	23324 16971 34597	326169 75666	151 93620	14853
21.3	23100 56697 25387	319948 17838	147 61128	14292
21.4	22880 16371 34014	313874 21138	143 42929	13755
21.5	22662 89919 63780	307943 67367	139 38486	13241
21.6	22448 71411 60912	302152 52081	135 47282	12748
21.7	22237 55056 10125	296496 84078	131 68827	12275
21.8	22029 35197 43417	290972 84902	128 02647	11822
21.9	21824 06311 61611	285576 88373	124 48289	11388
22.0	21621 63002 68179	280305 40133	121 05319	10972
22.1	21421 99999 14878	275154 97211	117 73321	10573
22.2	21225 12150 58790	270122 27611	114 51896	10190
22.3	21030 94424 30312	265204 09908	111 40661	9822
22.4	20839 41902 11743	260397 32866	108 39248	9470
22.5	20650 49777 26040	255698 95071	105 47305	9131
22.6	20464 13351 35407	251106 04582	102 64492	8806
22.7	20280 28031 49357	246615 78584	99 90486	8494
22.8	20098 89327 41890	242225 43073	97 24975	8195
22.9	19919 92848 77497	237932 32536	94 67658	7907
23.0	19743 34302 45639	233733 89658	92 18248	7631
23.1	19569 09490 03439	229627 65027	89 76469	7365
23.2	19397 14305 26266	225611 16865	87 42055	7110
23.3	19227 44731 65958	221682 10758	85 14750	6864
23.4	19059 96840 16408	217838 19401	82 94310	6629
23.5	18894 66786 86259	214077 22355	80 80499	6402
23.6	18731 50810 78465	210397 05807	78 73089	6184
23.7	18570 45231 76478	206795 62349	76 71863	5974
23.8	18441 46448 36840	203270 90753	74 76611	5772
23.9	18254 50935 87955	199820 95769	72 87131	5578
24.0	18099 55244 34839	196443 87916	71 03229	5391
24.1	17946 55996 69640	193137 83292	69 24718	5211
24.2	17795 49886 87732	189901 03386	67 51418	5038
24.3	17646 33678 09211	186731 74899	65 83157	4871
24.4	17499 04201 05588	183628 29568	64 19767	4711
24.5	17353 58352 31533	180589 04004	62 61088	4556
24.6	17209 93092 61483	177612 39528	61 06965	4407
24.7	17068 05445 30960	174696 82017	59 57250	4264
24.8	16927 92494 82455	171840 81755	58 11798	4126
24.9	16789 51385 15705	169042 93292	56 70473	3993
25.0	−.(2)16652 79318 42247	166301 75302	55 33140	3864

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
25.0	−.(2) 16652 79318 42247	166301 75302	55 33140	3864
25.1	16517 73553 44091	163615 90453	53 99672	3741
25.2	16384 31404 36388	160984 05275	52 69944	3621
25.3	16252 50239 33959	158404 90041	51 43837	3506
25.4	16122 27479 21573	155877 18645	50 21237	3395
25.5	15993 60596 27831	153399 68486	49 02031	3288
25.6	15866 47113 02574	150971 20357	47 86114	3185
25.7	15740 84600 97676	148590 58343	46 73382	3085
25.8	15616 70679 51120	146256 69712	45 63735	2989
25.9	15494 03014 74276	143968 44815	44 57077	2896
26.0	15372 79318 42247	141724 76995	43 53316	2807
26.1	15252 97346 87213	139524 62492	42 52362	2720
26.2	15134 54899 94671	137367 00351	41 54128	2637
26.3	15017 49820 02480	135250 92338	40 58531	2556
26.4	14901 79991 02626	133175 42856	39 65491	2479
26.5	14787 43337 45629	131139 58865	38 74929	2403
26.6	14674 37823 47496	129142 49804	37 86771	2331
26.7	14562 61451 99168	127183 27514	37 00944	2261
26.8	14452 12263 78354	125261 06168	36 17377	2193
26.9	14342 88336 63707	123375 02199	35 36003	2127
27.0	14234 87784 51259	121524 34233	34 56757	2064
27.1	14128 08756 73044	119708 23024	33 79576	2003
27.2	14022 49437 17852	117925 91389	33 04395	1944
27.3	13918 08043 54050	116176 64150	32 31159	1886
27.4	13814 82826 54397	114459 68070	31 59810	1831
27.5	13712 72069 22815	112774 31800	30 90292	1778
27.6	13611 74086 23033	111119 85822	30 22551	1726
27.7	13511 87223 09073	109495 62395	29 56537	1676
27.8	13413 09855 57508	107900 95505	28 92198	1627
27.9	13315 40389 01448	106335 20813	28 29486	1581
28.0	13218 77257 66201	104797 75607	27 68356	1535
28.1	13123 18924 06560	103287 98756	27 08760	1491
28.2	13028 63878 45676	101805 30666	26 50655	1449
28.3	12935 10638 15457	100349 13230	25 94000	1408
28.4	12842 57746 98469	98918 89794	25 38751	1368
28.5	12751 03774 71275	97514 05110	24 84871	1329
28.6	12660 47316 49191	96134 05297	24 32320	1292
28.7	12570 86992 32403	94778 37803	23 81061	1256
28.8	12482 21446 53419	93446 51371	23 31058	1221
28.9	12394 49347 25806	92137 95997	22 82275	1187
29.0	12307 69385 94189	90852 22897	22 34679	1154
29.1	12221 80276 85470	89588 84477	21 88237	1122
29.2	12136 80756 61228	88347 34294	21 42917	1091
29.3	12052 69583 71280	87127 27028	20 98688	1061
29.4	11969 45538 08359	85928 18450	20 55521	1032
29.5	11887 07420 63889	84749 65393	20 13386	1004
29.6	11805 54052 84812	83591 25722	19 72254	977
29.7	11724 84276 31458	82452 58306	19 32100	950
29.8	11644 96952 36409	81333 22989	18 92896	925
29.9	11565 90961 64350	80232 80569	18 54617	900
30.0	−.(2) 11487 65203 72860	79150 92767	18 17238	876

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
30.0	−.(2)11487 65203 72860	79150 92767	18 17238	876
30.1	11410 18596 74137	78087 22203	17 80735	853
30.2	11333 50076 97617	77041 32374	17 45085	830
30.3	11257 58598 53472	76012 87631	17 10264	808
30.4	11182 43132 96958	75001 53151	16 76251	786
30.5	11108 02668 93594	74006 94923	16 43025	766
30.6	11034 36211 85154	73028 79719	16 10564	746
30.7	10961 42783 56433	72066 75080	15 78849	726
30.8	10889 21422 02792	71120 49290	15 47861	707
30.9	10817 71180 98442	70189 71361	15 17579	689
31.0	10746 91129 65453	69274 11011	14 87986	671
31.1	10676 80352 43474	68373 38646	14 59065	654
31.2	10607 37948 60142	67487 25347	14 30796	637
31.3	10538 63032 02156	66615 42844	14 03165	620
31.4	10470 54730 87014	65757 63506	13 76155	605
31.5	10403 12187 35378	64913 60322	13 49748	589
31.6	10336 34557 44065	64083 06887	13 23932	574
31.7	10270 21010 59638	63265 77384	12 98689	560
31.8	10204 70729 52596	62461 46569	12 74006	546
31.9	10139 82909 92122	61669 89760	12 49868	532
32.0	10075 56760 21409	60890 82820	12 26264	518
32.1	10011 91501 33516	60124 02142	12 03175	505
32.2	−.(3)9948 86366 47765	59369 24640	11 80594	493
32.3	9886 40600 86653	58626 27731	11 58505	481
32.4	9824 53461 53272	57894 89327	11 36897	469
32.5	9763 24217 09219	57174 87821	11 15757	457
32.6	9702 52147 52987	56466 02071	10 95075	446
32.7	9642 36543 98825	55768 11396	10 74838	435
32.8	9582 76708 56060	55080 95560	10 55037	424
32.9	9523 71954 08854	54404 34760	10 35659	414
33.0	9465 21603 96409	53738 09620	10 16696	404
33.1	9407 24991 93584	53082 01176	9 98136	394
33.2	9349 81461 91934	52435 90868	9 79971	385
33.3	9292 90367 81152	51799 60531	9 62190	375
33.4	9236 51073 30901	51172 95384	9 44784	366
33.5	9180 62951 73033	50555 69021	9 27744	357
33.6	9125 25385 84187	49947 73402	9 11062	349
33.7	9070 37767 68743	49348 88846	8 94729	341
33.8	9015 99498 42145	48758 99019	8 78736	332
33.9	8962 09988 14567	48177 87928	8 63076	325
34.0	8908 68655 74916	47605 39914	8 47741	317
34.1	8855 74928 75179	47041 39639	8 32722	309
34.2	8803 28243 15081	46485 72087	8 18014	302
34.3	8751 28043 27071	45938 22548	8 03606	295
34.4	8699 73781 61608	45398 76615	7 89494	288
34.5	8648 64918 72761	44867 20175	7 75670	282
34.6	8598 00923 04088	44343 39406	7 62128	275
34.7	8547 81270 74821	43827 20766	7 48861	269
34.8	8498 05445 66320	43318 50986	7 35863	262
34.9	8448 72939 08806	42817 17070	7 23127	256
35.0	−.(3)8399 83249 68362	42323 06282	7 10648	251

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
35.0	−.(3) 8399 83249 68362	42323 06282	7 10648	251
35.1	8351 35883 34199	41836 06140	6 98419	245
35.2	8303 30353 06177	41356 04418	6 86434	239
35.3	8255 66178 82573	40882 89129	6 74689	234
35.4	8208 42887 48098	40416 48531	6 63178	228
35.5	8161 60012 62153	39956 71110	6 51895	223
35.6	8115 17094 47319	39503 45585	6 40836	218
35.7	8069 13679 78070	39056 60896	6 29995	213
35.8	8023 49321 69718	38616 06202	6 19367	209
35.9	7978 23579 67567	38181 70875	6 08948	204
36.0	7933 36019 36292	37753 44496	5 98732	199
36.1	7888 86212 49512	37331 16849	5 88716	195
36.2	7844 73736 79581	36914 77918	5 78895	191
36.3	7800 98175 87567	36504 17883	5 69265	186
36.4	7757 59119 13436	36099 27112	5 59821	182
36.5	7714 56161 66417	35699 96162	5 50559	178
36.6	7671 88904 15560	35306 15771	5 41475	174
36.7	7629 56952 80475	34917 76856	5 32566	171
36.8	7587 59919 22245	34534 70506	5 23828	167
36.9	7545 97420 34522	34156 87985	5 15256	163
37.0	7504 69078 34783	33784 20718	5 06847	160
37.1	7463 74520 55763	33416 60299	4 98598	156
37.2	7423 13379 37041	33053 98478	4 90505	153
37.3	7382 85292 16798	32696 27162	4 82565	150
37.4	7342 89901 23718	32343 38412	4 74775	146
37.5	7303 26853 69048	31995 24436	4 67131	143
37.6	7263 95801 38815	31651 77591	4 59630	140
37.7	7224 96400 86172	31312 90375	4 52269	137
37.8	7186 28313 23905	30978 55429	4 45045	134
37.9	7147 91204 17067	30648 65528	4 37956	131
38.0	7109 84743 75758	30323 13583	4 30998	129
38.1	7072 08606 48032	30001 92637	4 24169	126
38.2	7034 62471 12942	29684 95858	4 17465	123
38.3	6997 46020 73711	29372 16545	4 10885	121
38.4	6960 58942 51025	29063 48117	4 04425	118
38.5	6924 00927 76456	28758 84114	3 98084	116
38.6	6887 71671 86001	28458 18196	3 91858	113
38.7	6851 70874 13742	28161 44136	3 85746	111
38.8	6815 98237 85619	27868 55821	3 79745	109
38.9	6780 53470 13317	27579 47252	3 73852	106
39.0	6745 36281 88267	27294 12534	3 68065	104
39.1	6710 46387 75751	27012 45882	3 62383	102
39.2	6675 83506 09118	26734 41613	3 56803	100
39.3	6641 47358 84097	26459 94147	3 51323	98
39.4	6607 37671 53224	26188 98004	3 45941	96
39.5	6573 54173 20355	25921 47803	3 40655	94
39.6	6539 96596 35289	25657 38255	3 35462	92
39.7	6506 64676 88478	25396 64170	3 30362	90
39.8	6473 58154 05838	25139 20448	3 25352	88
39.9	6440 76770 43645	24885 02077	3 20431	87
40.0	−.(3) 6408 20271 83530	24634 04138	3 15596	85

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
40.0	—.(3)6408 20271 83530	24634 04138	3 15596	85
40.1	6375 88407 27553	24386 21794	3 10846	83
40.2	6343 80928 93370	24141 50297	3 06179	82
40.3	6311 97592 09484	23899 84978	3 01594	80
40.4	6280 38155 10577	23661 21254	2 97089	78
40.5	6249 02379 32923	23425 54618	2 92662	77
40.6	6217 90029 09888	23192 80644	2 88311	75
40.7	6187 00871 67496	22962 94980	2 84036	74
40.8	6156 34677 20084	22735 93354	2 79835	72
40.9	6125 91218 66026	22511 71562	2 75706	71
41.0	6095 70271 83530	22290 25476	2 71648	70
41.1	6065 71615 26509	22071 51038	2 67660	68
41.2	6035 95030 20527	21855 44260	2 63739	67
41.3	6006 40300 58804	21642 01221	2 59886	66
41.4	5977 07212 98303	21431 18068	2 56098	64
41.5	5947 95556 55869	21222 91012	2 52374	63
41.6	5919 05123 04447	21017 16330	2 48713	62
41.7	5890 35706 69355	20813 90361	2 45114	61
41.8	5861 87104 24625	20613 09507	2 41576	59
41.9	5833 59114 89401	20414 70228	2 38097	58
42.0	5805 51540 24406	20218 69046	2 34676	57
42.1	5777 64184 28456	20025 02541	2 31313	56
42.2	5749 96853 35048	19833 67348	2 28006	55
42.3	5722 49356 08988	19644 60162	2 24753	54
42.4	5695 21503 43090	19457 77728	2 21555	53
42.5	5668 13108 54920	19273 16850	2 18410	52
42.6	5641 23986 83600	19090 74382	2 15316	51
42.7	5614 53955 86662	18910 47230	2 12274	50
42.8	5588 02835 36953	18732 32352	2 09282	49
42.9	5561 70447 19596	18556 26756	2 06339	48
43.0	5535 56615 28995	18382 27498	2 03444	47
43.1	5509 61165 65892	18210 31685	2 00596	46
43.2	5483 83926 34474	18040 36467	1 97794	46
43.3	5458 24727 39522	17872 39043	1 95039	45
43.4	5432 83400 83614	17706 36658	1 92327	44
43.5	5407 59780 64365	17542 26601	1 89660	43
43.6	5382 53702 71716	17380 06204	1 87036	42
43.7	5357 65004 85271	17219 72843	1 84454	41
43.8	5332 93526 71669	17061 23935	1 81914	41
43.9	5308 39109 82002	16904 56942	1 79414	40
44.0	5284 01597 49277	16749 69362	1 76954	39
44.1	5259 80834 85914	16596 58736	1 74533	39
44.2	5235 76668 81286	16445 22643	1 72151	38
44.3	5211 88947 99301	16295 58702	1 69807	37
44.4	5188 17522 76018	16147 64569	1 67500	36
44.5	5164 62245 17304	16001 37935	1 65230	36
44.6	5141 22968 96525	15856 76532	1 62995	35
44.7	5117 99549 52278	15713 78124	1 60796	35
44.8	5094 91843 86155	15572 40511	1 58631	34
44.9	5071 99710 60543	15432 61529	1 56500	33
45.0	—.(3)5049 23009 96460	15294 39047	1 54402	33

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
45.0	−.(3)5049 23009 96460	15294 39047	1 54402	33
45.1	5026 61603 71424	15157 70967	1 52337	32
45.2	5004 15355 17356	15022 55224	1 50304	32
45.3	4981 84129 18512	14888 89785	1 48303	31
45.4	4959 67792 09453	14756 72650	1 46333	30
45.5	4937 66211 73044	14626 01847	1 44393	30
45.6	4915 79257 38481	14496 75436	1 42483	29
45.7	4894 06799 79355	14368 91509	1 40603	29
45.8	4872 48711 11738	14242 48185	1 38751	28
45.9	4851 04864 92307	14117 43612	1 36928	28
46.0	4829 75136 16487	13993 75967	1 35133	27
46.1	4808 59401 16635	13871 43455	1 33365	27
46.2	4787 57537 60238	13750 44308	1 31624	26
46.3	4766 69424 48148	13630 76784	1 29909	26
46.4	4745 94942 12843	13512 39170	1 28221	26
46.5	4725 33972 16708	13395 29777	1 26558	25
46.6	4704 86397 50350	13279 46942	1 24920	25
46.7	4684 52102 30934	13164 89027	1 23307	24
46.8	4664 30972 00545	13051 54420	1 21718	24
46.9	4644 22893 24576	12939 41530	1 20153	23
47.0	4624 27753 90137	12828 48793	1 18611	23
47.1	4604 45443 04491	12718 74668	1 17093	23
47.2	4584 75850 93513	12610 17635	1 15597	22
47.3	4565 18869 00170	12502 76199	1 14123	22
47.4	4545 74389 83026	12396 48886	1 12671	22
47.5	4526 42307 14769	12291 34245	1 11241	21
47.6	4507 22515 80756	12187 30844	1 09832	21
47.7	4488 14911 77587	12084 37274	1 08443	20
47.8	4469 19392 11693	11982 52148	1 07075	20
47.9	4450 35854 97946	11881 74097	1 05727	20
48.0	4431 64199 58297	11782 01773	1 04399	19
48.1	4413 04326 20420	11683 33848	1 03090	19
48.2	4394 56136 16390	11585 69013	1 01801	19
48.3	4376 19531 81374	11489 05979	1 00530	18
48.4	4357 94416 52337	11393 43474	99277	18
48.5	4339 80694 66774	11298 80248	98043	18
48.6	4321 78271 61458	11205 15064	96827	18
48.7	4303 87053 71207	11112 46707	95628	17
48.8	4286 06948 27662	11020 73978	94447	17
48.9	4268 37863 58094	10929 95695	93282	17
49.0	4250 79708 84223	10840 10695	92134	16
49.1	4233 32394 21045	10751 17828	91003	16
49.2	4215 95830 75697	10663 15965	89888	16
49.3	4198 69930 46313	10576 03989	88788	16
49.4	4181 54606 20919	10489 80802	87705	15
49.5	4164 49771 76326	10404 45319	86637	15
49.6	4147 55341 77052	10319 96473	85583	15
49.7	4130 71231 74251	10236 33210	84545	15
49.8	4113 97358 04660	10153 54493	83522	14
49.9	4097 33637 89561	10071 59297	82513	14
50.0	−.(3)4080 79989 33760	9990 46614	81518	14

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
50.0	—.(3)4080 79989 33760	9990 46614	81518	14
50.1	4064 36331 24572	9910 15449	80537	14
50.2	4048 02583 30833	9830 64820	79570	14
50.3	4031 78666 01915	9751 93762	78616	13
50.4	4015 64500 66758	9674 01319	77676	13
50.5	3999 60009 32920	9596 86552	76748	13
50.6	3983 65114 85635	9520 48534	75834	13
50.7	3697 79740 86883	9444 86349	74932	12
50.8	3952 03811 74481	9369 99097	74043	12
50.9	3936 37252 61176	9295 85888	73166	12
51.0	3920 79989 33760	9222 45846	72301	12
51.1	3905 31948 52189	9149 78105	71449	12
51.2	3889 93057 48723	9077 81812	70607	12
51.3	3874 63244 27070	9006 56127	69778	11
51.4	3859 42437 61543	8936 00219	68959	11
51.5	3844 30566 96236	8866 13271	68152	11
51.6	3829 27562 44200	8796 94475	67356	11
51.7	3814 33354 86639	8728 43035	66571	11
51.8	3799 47875 72113	8660 58166	65796	10
51.9	3784 71057 15752	8593 39092	65032	10
52.0	3770 02831 98484	8526 85051	64278	10
52.1	3755 43133 66268	8460 95288	63534	10
52.2	3740 91896 29339	8395 69059	62801	10
52.3	3726 49054 61469	8331 05631	62077	10
52.4	3712 14543 99230	8267 04280	61363	10
52.5	3697 88300 41271	8203 64292	60658	9
52.6	3683 70260 47604	8140 84962	59963	9
52.7	3669 60361 38900	8078 65596	59278	9
52.8	3655 58540 95792	8017 05508	58601	9
52.9	3641 64737 58191	7956 04020	57933	9
53.0	3627 78890 24611	7895 60466	57274	9
53.1	3614 00938 51496	7835 74186	56624	9
53.2	3600 30822 52567	7776 44530	55983	8
53.3	3586 68482 98169	7717 70857	55350	8
53.4	3573 13861 14628	7659 52534	54725	8
53.5	3559 66898 83621	7601 88937	54109	8
53.6	3546 27538 41550	7544 79447	53500	8
53.7	3532 95722 78928	7488 23458	52900	8
53.8	3519 71395 39763	7432 20369	52307	8
53.9	3506 54500 20968	7376 69587	51722	8
54.0	3493 44981 71759	7321 70527	51145	8
54.1	3480 42784 93078	7267 22613	50575	7
54.2	3467 47855 37010	7213 25273	50013	7
54.3	3454 60139 06214	7159 77947	49458	7
54.4	3441 79582 53366	7106 80078	48910	7
54.5	3429 06132 80595	7054 31120	48369	7
54.6	3416 39737 38945	7002 30530	47835	7
54.7	3403 80344 27825	6950 77776	47308	7
54.8	3391 27901 94481	6899 72331	46788	7
54.9	3378 82359 33467	6849 13673	46274	7
55.0	—.(3)3366 43665 86127	6799 01290	45767	6

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
55.0	—.(3) 3366 43665 86127	6799 01290	45767	6
55.1	3354 11771 40076	6749 34674	45267	6
55.2	3341 86626 28699	6700 13325	44773	6
55.3	3329 68181 30647	6651 36749	44285	6
55.4	3317 56387 69344	6603 04457	43803	6
55.5	3305 51197 12498	6555 15969	43328	6
55.6	3293 52561 71621	6507 70808	42858	6
55.7	3281 60434 01552	6460 68506	42394	6
55.8	3269 74766 99990	6414 08597	41936	6
55.9	3257 95514 07024	6367 90625	41484	6
56.0	3246 22629 04684	6322 14138	41038	6
56.1	3234 56066 16482	6276 78688	40597	6
56.2	3222 95780 06969	6231 83836	40162	5
56.3	3211 41725 81291	6187 29146	39732	5
56.4	3199 93858 84760	6143 14188	39308	5
56.5	3188 52135 02416	6099 38537	38888	5
56.6	3177 16510 58610	6056 01775	38474	5
56.7	3165 86942 16578	6013 03488	38066	5
56.8	3154 63386 78034	5970 43266	37662	5
56.9	3143 45801 82756	5928 20705	37263	5
57.0	3132 34145 08183	5886 35408	36869	5
57.1	3121 28374 69018	5844 86980	36480	5
57.2	3110 28449 16833	5803 75031	36096	5
57.3	3099 34327 39679	5762 99179	35716	5
57.4	3088 45968 61704	5722 59043	35341	5
57.5	3077 63332 42772	5682 54248	34971	5
57.6	3066 86378 78087	5642 84424	34605	4
57.7	3056 15067 97827	5603 49205	34244	4
57.8	3045 49360 66771	5564 48230	33887	4
57.9	3034 89217 83944	5525 81142	33534	4
58.0	3024 34600 82260	5487 47588	33186	4
58.1	3013 85471 28163	5449 47220	32842	4
58.2	3003 41791 21285	5411 79693	32502	4
58.3	2993 03522 94101	5374 44669	32166	4
58.4	2982 70629 11586	5337 41810	31834	4
58.5	2972 43072 70881	5300 70786	31506	4
58.6	2962 20817 00961	5264 31267	31182	4
58.7	2952 03825 62309	5228 22932	30862	4
58.8	2941 92062 46589	5192 45458	30546	4
58.9	2931 85491 76326	5156 98530	30234	4
59.0	2921 84078 04594	5121 81836	29925	4
59.1	2911 87786 14698	5086 95067	29620	4
59.2	2901 96581 19869	5052 37918	29318	4
59.3	2892 10428 62958	5018 10087	29021	4
59.4	2882 29294 16134	4984 11277	28726	3
59.5	2872 53143 80588	4950 41193	28435	3
59.6	2862 81943 86235	4916 99545	28148	3
59.7	2853 15660 91427	4883 86044	27864	3
59.8	2843 54261 82663	4851 00408	27583	3
59.9	2833 97713 74308	4818 42355	27306	3
60.0	—.(3) 2824 45984 08307	4786 11607	27032	3

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
60.0	—.(3)2824 45984 08307	4786 11607	27032	3
60.1	2814 99040 53913	4754 07892	26761	3
60.2	2805 56851 07411	4722 30937	26493	3
60.3	2796 19383 91846	4690 80475	26228	3
60.4	2786 86607 56756	4659 56241	25967	3
60.5	2777 58490 77907	4628 57974	25708	3
60.6	2768 35002 57033	4597 85416	25453	3
60.7	2759 16112 21574	4567 38310	25200	3
60.8	2750 01789 24425	4537 16403	24950	3
60.9	2740 92003 43679	4507 19448	24704	3
61.0	2731 86724 82381	4477 47196	24460	3
61.1	2722 85923 68278	4447 99403	24218	3
61.2	2713 89570 53578	4418 75829	23980	3
61.3	2704 97636 14708	4389 76235	23744	3
61.4	2696 10091 52072	4361 00385	23512	3
61.5	2687 26907 89821	4332 48048	23281	3
61.6	2678 48056 75619	4304 18991	23054	3
61.7	2669 73509 80406	4276 12988	22829	3
61.8	2661 03238 98182	4248 29813	22606	3
61.9	2652 37216 45771	4220 69245	22386	2
62.0	2643 75414 62604	4193 31062	22169	2
62.1	2635 17806 10499	4166 15048	21954	2
62.2	2626 64363 73442	4139 20988	21741	2
62.3	2618 15060 57373	4112 48668	21531	2
62.4	2609 69869 89972	4085 97880	21323	2
62.5	2601 28765 20451	4059 68414	21117	2
62.6	2592 91720 19344	4033 60066	20914	2
62.7	2584 58708 78302	4007 72632	20713	2
62.8	2576 29705 09892	3982 05911	20515	2
62.9	2568 04683 47393	3956 59705	20318	2
63.0	2559 83618 44598	3931 33817	20124	2
63.1	2551 66484 75621	3906 28053	19932	2
63.2	2543 53257 34698	3881 42222	19742	2
63.3	2535 43911 35996	3856 76132	19554	2
63.4	2527 38422 13425	3832 29596	19368	2
63.5	2519 36765 20451	3808 02429	19185	2
63.6	2511 38916 29906	3783 94446	19003	2
63.7	2503 44851 33807	3760 05466	18823	2
63.8	2495 54546 43174	3736 35310	18646	2
63.9	2487 67977 87852	3712 83799	18470	2
64.0	2479 85122 16329	3689 50758	18296	2
64.1	2472 05955 95564	3666 36014	18124	2
64.2	2464 30456 10813	3643 39393	17954	2
64.3	2456 58599 65455	3620 60727	17786	2
64.4	2448 90363 80824	3597 99846	17620	2
64.5	2441 25725 96039	3575 56585	17455	2
64.6	2433 64663 67839	3553 30780	17292	2
64.7	2426 07154 70419	3531 22266	17131	2
64.8	2418 53176 95266	3509 30885	16972	2
64.9	2411 02708 50997	3487 56475	16815	2
65.0	—.(3)2403 55727 63204	3465 98880	16659	2

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
65.0	−.(3)2403 55727 63204	3465 98880	16659	2
65.1	2396 12212 74290	3444 57944	16505	2
65.2	2388 72142 43321	3423 33513	16352	2
65.3	2381 35495 45866	3402 25435	16202	2
65.4	2374 02250 73845	3381 33558	16052	2
65.5	2366 72387 35382	3360 57733	15905	2
65.6	2359 45884 54651	3339 97813	15759	2
65.7	2352 22721 71734	3319 53652	15614	2
65.8	2345 02878 42469	3299 25105	15471	2
65.9	2337 86334 38310	3279 12030	15330	2
66.0	2330 73069 46180	3259 14285	15190	1
66.1	2323 63063 68336	3239 31730	15052	1
66.2	2316 56297 22221	3219 64227	14915	1
66.3	2309 52750 40334	3200 11639	14779	1
66.4	2302 52403 70085	3180 73830	14645	1
66.5	2295 55237 73665	3161 50666	14513	1
66.6	2288 61233 27912	3142 42016	14382	1
66.7	2281 70371 24175	3123 47746	14252	1
66.8	2274 82632 68183	3104 67729	14123	1
66.9	2267 97998 79921	3086 01835	13996	1
67.0	2261 16450 93494	3067 49937	13870	1
67.1	2254 37970 57003	3049 11909	13746	1
67.2	2247 62539 32421	3030 87627	13623	1
67.3	2240 90138 95466	3012 76968	13501	1
67.4	2234 20751 35478	2994 79809	13380	1
67.5	2227 54358 55300	2976 96031	13261	1
67.6	2220 90942 71152	2959 25513	13143	1
67.7	2214 30486 12518	2941 68138	13026	1
67.8	2207 72971 22022	2924 23789	12910	1
67.9	2201 18380 55315	2906 92351	12796	1
68.0	2194 66696 80960	2889 73707	12682	1
68.1	2188 17902 80311	2872 67746	12570	1
68.2	2181 71981 47408	2855 74355	12459	1
68.3	2175 28915 88861	2838 93424	12349	1
68.4	2168 88689 23738	2822 24841	12241	1
68.5	2162 51284 83456	2805 68500	12133	1
68.6	2156 16686 11674	2789 24291	12027	1
68.7	2149 84876 64183	2772 92109	11921	1
68.8	2143 55840 08802	2756 71848	11817	1
68.9	2137 29560 25269	2740 63404	11714	1
69.0	2131 06021 05140	2724 66674	11611	1
69.1	2124 85206 51686	2708 81555	11510	1
69.2	2118 67100 79786	2693 07946	11410	1
69.3	2112 51688 15832	2677 45747	11311	1
69.4	2106 38952 97625	2661 94859	11213	1
69.5	2100 28879 74276	2646 55183	11116	1
69.6	2094 21453 06112	2631 26624	11019	1
69.7	2088 16657 64571	2616 09084	10924	1
69.8	2082 14478 32113	2601 02468	10830	1
69.9	2076 14900 02123	2586 06682	10737	1
70.0	−.(3)2070 17907 78814	2571 21632	10644	1

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
70.0	—.(3)2070 17907 78814	2571 21632	10644
70.1	2064 23486 77138	2556 47228	10553
70.2	2058 31622 22690	2541 83376	10462
70.3	2052 42299 51617	2527 29986	10373
70.4	2046 55504 10531	2512 86970	10284
70.5	2040 71221 56415	2498 54238	10196
70.6	2034 89437 56536	2484 31702	10109
70.7	2029 10137 88360	2470 19275	10023
70.8	2023 33308 39458	2456 16872	9938
70.9	2017 58935 07429	2442 24407	9854
71.0	2011 87003 99806	2428 41795	9770
71.1	2006 17501 33978	2414 68954	9687
71.2	2000 50413 37104	2401 05800	9605
71.3	1994 85726 46030	2387 52251	9524
71.4	1989 23427 07206	2374 08227	9444
71.5	1983 63501 76610	2360 73647	9365
71.6	1978 05937 19661	2347 48431	9286
71.7	1972 50720 11142	2334 32501	9208
71.8	1966 97837 35125	2321 25779	9131
71.9	1961 47275 84888	2308 28188	9054
72.0	1955 99022 62839	2295 39651	8978
72.1	1950 53064 80440	2282 60092	8904
72.2	1945 09389 58134	2269 89437	8829
72.3	1939 67984 25265	2257 27611	8756
72.4	1934 28836 20007	2244 74541	8683
72.5	1928 91932 89290	2232 30154	8611
72.6	1923 57261 88727	2219 94377	8539
72.7	1918 24810 82540	2207 67140	8469
72.8	1912 94567 43495	2195 48372	8399
72.9	1907 66519 52821	2183 38003	8329
73.0	1902 40655 00150	2171 35962	8261
73.1	1897 16961 83441	2159 42183	8193
73.2	1891 95428 08915	2147 56596	8125
73.3	1886 76041 90985	2135 79134	8059
73.4	1881 58791 52190	2124 09731	7992
73.5	1876 43665 23125	2112 48320	7927
73.6	1871 30651 42380	2100 94837	7862
73.7	1866 19738 56473	2089 49215	7798
73.8	1861 10915 19780	2078 11391	7734
73.9	1856 04169 94478	2066 81302	7671
74.0	1850 99491 50479	2055 58884	7609
74.1	1845 96868 65363	2044 44075	7547
74.2	1840 96290 24322	2033 36813	7486
74.3	1835 97745 20094	2022 37036	7425
74.4	1831 01222 52902	2011 44685	7365
74.5	1826 06711 30395	2000 59700	7306
74.6	1821 14200 67588	1989 82019	7247
74.7	1816 23679 86800	1979 11586	7188
74.8	1811 35138 17598	1968 48341	7130
74.9	1806 48564 96737	1957 92226	7073
75.0	—.(3)1801 63949 68101	1947 43184	7016

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
75.0	—.(3)1801 63949 68101	1947 43184	7016
75.1	1796 81281 82649	1937 01159	6960
75.2	1792 00550 98356	1926 66093	6904
75.3	1787 21746 80156	1916 37933	6849
75.4	1782 44858 99889	1906 16621	6795
75.5	1777 69877 36244	1896 02104	6740
75.6	1772 96791 74702	1885 94328	6687
75.7	1768 25592 07488	1875 93238	6634
75.8	1763 56268 33512	1865 98781	6581
75.9	1758 88810 58316	1856 10906	6529
76.0	1754 23208 94027	1846 29559	6477
76.1	1749 59453 59296	1836 54689	6426
76.2	1744 97534 79254	1826 86245	6375
76.3	1740 37442 85457	1817 24176	6325
76.4	1735 79168 15836	1807 68431	6275
76.5	1731 22701 14645	1798 18961	6225
76.6	1726 68032 32416	1788 75717	6177
76.7	1722 15152 25905	1779 38650	6128
76.8	1717 64051 58043	1770 07710	6080
76.9	1713 14720 97891	1760 82851	6032
77.0	1708 67151 20590	1751 64024	5985
77.1	1704 21333 07314	1742 51182	5939
77.2	1699 77257 45219	1733 44279	5892
77.3	1695 34915 27404	1724 43269	5846
77.4	1690 94297 52857	1715 48104	5801
77.5	1686 55395 26415	1706 58741	5756
77.6	1682 18199 58714	1697 75134	5711
77.7	1677 82701 66147	1688 97238	5667
77.8	1673 48892 70818	1680 25008	5623
77.9	1669 16764 00496	1671 58403	5580
78.0	1664 86306 88578	1662 97376	5537
78.1	1660 57512 74035	1654 41887	5494
78.2	1656 30373 01380	1645 91891	5452
78.3	1652 04879 20616	1637 47348	5410
78.4	1647 81022 87199	1629 08214	5368
78.5	1643 58795 61997	1620 74448	5327
78.6	1639 38189 11242	1612 46010	5286
78.7	1635 19195 06497	1604 22857	5246
78.8	1631 01805 24609	1596 04951	5206
78.9	1626 86011 47672	1587 92251	5166
79.0	1622 71805 62986	1579 84716	5127
79.1	1618 59179 63016	1571 82309	5088
79.2	1614 48125 45355	1563 84989	5049
79.3	1610 38635 12684	1555 92719	5011
79.4	1606 30700 72731	1548 05459	4973
79.5	1602 24314 38237	1540 23172	4935
79.6	1598 19468 26915	1532 45820	4898
79.7	1594 16154 61413	1524 73366	4861
79.8	1590 14365 69278	1517 05773	4824
79.9	1586 14093 82915	1509 43004	4788
80.0	—.(3)1582 15331 39557	1501 85023	4752

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
80.0	—.(3)1582 15331 39557	1501 85023	4752
80.1	1578 18070 81221	1494 31794	4716
80.2	1574 22304 54680	1486 83281	4681
80.3	1570 28025 11420	1479 39449	4646
80.4	1566 35225 07608	1472 00262	4611
80.5	1562 43897 04059	1464 65687	4576
80.6	1558 54033 66196	1457 35687	4542
80.7	1554 65627 64021	1450 10230	4508
80.8	1550 78671 72075	1442 89282	4475
80.9	1546 93158 69412	1435 72808	4442
81.0	1543 09081 39557	1428 60776	4409
81.1	1539 26432 70477	1421 53152	4376
81.2	1535 45205 54550	1414 49904	4343
81.3	1531 65392 88526	1407 51000	4311
81.4	1527 86987 73503	1400 56406	4279
81.5	1524 09983 14886	1393 66093	4248
81.6	1520 34372 22361	1386 80026	4216
81.7	1516 60148 09863	1379 98177	4185
81.8	1512 87303 95541	1373 20512	4155
81.9	1509 15833 01732	1366 47003	4124
82.0	1505 45728 54925	1359 77617	4094
82.1	1501 76983 85735	1353 12325	4064
82.2	1498 09592 28870	1346 51097	4034
82.3	1494 43547 23101	1339 93903	4005
82.4	1490 78842 11235	1333 40713	3975
82.5	1487 15470 40083	1326 91499	3946
82.6	1483 53425 60429	1320 46231	3918
82.7	1479 92701 27006	1314 04880	3889
82.8	1476 33290 98463	1307 67419	3861
82.9	1472 75188 37339	1301 33818	3833
83.0	1469 18387 10034	1295 04051	3805
83.1	1465 62880 86780	1288 78088	3777
83.2	1462 08663 41613	1282 55902	3750
83.3	1458 55728 52348	1276 37467	3723
83.4	1455 04070 00550	1270 22754	3696
83.5	1451 53681 71507	1264 11738	3669
83.6	1448 04557 54202	1258 04391	3643
83.7	1444 56691 41288	1252 00688	3617
83.8	1441 10077 29061	1246 00601	3591
83.9	1437 64709 17436	1240 04105	3565
84.0	1434 20581 09916	1234 11174	3540
84.1	1430 77687 13570	1228 21783	3514
84.2	1427 36021 39007	1222 35907	3489
84.3	1423 95578 00351	1216 53519	3464
84.4	1420 56351 15214	1210 74596	3440
84.5	1417 18335 04672	1204 99112	3415
84.6	1413 81523 93243	1199 27043	3391
84.7	1410 45912 08856	1193 58366	3367
84.8	1407 11493 82836	1187 93054	3343
84.9	1403 78263 49870	1182 31086	3319
85.0	—.(3)1400 46215 47989	1176 72437	3296

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
85.0	−.(3) 1400 46215 47989	1176 72437	3296
85.1	1397 15344 18546	1171 17083	3272
85.2	1393 85644 06187	1165 65002	3249
85.3	1390 57109 58829	1160 16170	3226
85.4	1387 29735 27642	1154 70565	3204
85.5	1384 03515 67019	1149 28163	3181
85.6	1380 78445 34559	1143 88941	3159
85.7	1377 54518 91041	1138 52879	3136
85.8	1374 31731 00401	1133 19953	3114
85.9	1371 10076 29715	1127 90142	3093
86.0	1367 89549 49170	1122 63423	3071
86.1	1364 70145 32048	1117 39775	3050
86.2	1361 51858 54701	1112 19177	3028
86.3	1358 34683 96531	1107 01607	3007
86.4	1355 18616 39968	1101 87044	2986
86.5	1352 03650 70450	1096 75468	2965
86.6	1348 89781 76399	1091 66857	2945
86.7	1345 77004 49205	1086 61191	2924
86.8	1342 65313 83202	1081 58449	2904
86.9	1339 54704 75648	1076 58611	2884
87.0	1336 45172 26705	1071 61658	2864
87.1	1333 36711 39420	1066 67568	2844
87.2	1330 29317 19704	1061 76323	2825
87.3	1327 22984 76310	1056 87903	2805
87.4	1324 17709 20820	1052 02287	2786
87.5	1321 13485 67616	1047 19458	2767
87.6	1318 10309 33871	1042 39395	2748
87.7	1315 08175 39522	1037 62081	2729
87.8	1312 07079 07253	1032 87495	2710
87.9	1309 07015 62478	1028 15619	2692
88.0	1306 07980 33323	1023 46435	2673
88.1	1303 09968 50602	1018 79924	2655
88.2	1300 12975 47805	1014 16068	2637
88.3	1297 16996 61076	1009 54849	2619
88.4	1294 22027 29196	1004 96249	2601
88.5	1291 28062 93564	1000 40250	2583
88.6	1288 35098 98182	995 86834	2566
88.7	1285 43130 89634	991 35984	2548
88.8	1282 52154 17070	986 87683	2531
88.9	1279 62164 32189	982 41913	2514
89.0	1276 73156 89221	977 98656	2497
89.1	1273 85127 44908	973 57897	2480
89.2	1270 98071 58494	969 19618	2463
89.3	1268 11984 91697	964 83803	2447
89.4	1265 26863 08704	960 50435	2430
89.5	1262 42701 76146	956 19496	2414
89.6	1259 59496 63083	951 90972	2398
89.7	1256 77243 40993	947 64846	2382
89.8	1253 95937 83749	943 41102	2366
89.9	1251 15575 67607	939 19723	2350
90.0	−.(3) 1248 36152 71188	935 00694	2334

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
90.0	—.(3)1248 36152 71188	935 00694	2334
90.1	1245 57664 75463	930 84000	2319
90.2	1242 80107 63738	926 69624	2303
90.3	1240 03477 21638	922 57552	2288
90.4	1237 27769 37089	918 47767	2273
90.5	1234 52980 00307	914 40255	2258
90.6	1231 79105 03781	910 35001	2243
90.7	1229 06140 42256	906 31989	2228
90.8	1226 34082 12720	902 31205	2213
90.9	1223 62926 14389	898 32634	2198
91.0	1220 92668 48691	894 36261	2184
91.1	1218 23305 19255	890 42072	2169
91.2	1215 54832 31890	886 50052	2155
91.3	1212 87245 94576	882 60187	2141
91.4	1210 20542 17449	878 72463	2127
91.5	1207 54717 12785	874 86865	2113
91.6	1204 89766 94986	871 03381	2099
91.7	1202 25687 80567	867 21995	2085
91.8	1199 62475 88144	863 42694	2071
91.9	1197 00127 38414	859 65465	2058
92.0	1194 38638 54149	855 90293	2044
92.1	1191 78005 60178	852 17166	2031
92.2	1189 18224 83373	848 46070	2018
92.3	1186 59292 52639	844 76992	2005
92.4	1184 01204 98896	841 09919	1992
92.5	1181 43958 55073	837 44837	1979
92.6	1178 87549 56087	833 81734	1966
92.7	1176 31974 38834	830 20596	1953
92.8	1173 77229 42177	826 61412	1940
92.9	1171 23311 06933	823 04168	1928
93.0	1168 70215 75856	819 48851	1915
93.1	1166 17939 93630	815 95451	1903
93.2	1163 66480 06855	812 43953	1891
93.3	1161 15832 64033	808 94345	1878
93.4	1158 65994 15556	805 46617	1866
93.5	1156 16961 13696	802 00754	1854
93.6	1153 68730 12589	798 56746	1842
93.7	1151 21297 68230	795 14581	1831
93.8	1148 74660 38450	791 74246	1819
93.9	1146 28814 82917	788 35730	1807
94.0	1143 83757 63113	784 99021	1796
94.1	1141 39485 42331	781 64108	1784
94.2	1138 95994 85656	778 30979	1773
94.3	1136 53282 59960	774 99623	1761
94.4	1134 11345 33888	771 70028	1750
94.5	1131 70179 77843	768 42184	1739
94.6	1129 29782 63983	765 16079	1728
94.7	1126 90150 66201	761 91702	1717
94.8	1124 51280 60122	758 69042	1706
94.9	1122 13169 23083	755 48088	1695
95.0	—.(3)1119 75813 34133	752 28829	1685

x	$\Psi''(x)$	$-\delta^2$	$-\delta^4$
95.0	−.(3)1119 75813 34133	752 28829	1685
95.1	1117 39209 74012	749 11256	1674
95.2	1115 03355 25147	745 95356	1663
95.3	1112 68246 71637	742 81119	1653
95.4	1110 33880 99247	739 68536	1642
95.5	1108 00254 95393	736 57595	1632
95.6	1105 67365 49133	733 48286	1622
95.7	1103 35209 51160	730 40599	1612
95.8	1101 03783 93785	727 34524	1602
95.9	1098 73085 70935	724 30050	1592
96.0	1096 43111 78134	721 27168	1582
96.1	1094 13859 12501	718 25867	1572
96.2	1091 85324 72735	715 26138	1562
96.3	1089 57505 59107	712 27971	1552
96.4	1087 30398 73450	709 31356	1542
96.5	1085 04001 19148	706 36283	1533
96.6	1082 78310 01129	703 42743	1523
96.7	1080 53322 25854	700 50726	1514
96.8	1078 29035 01304	697 60223	1504
96.9	1076 05445 36977	694 71224	1495
97.0	1073 82550 43875	691 83721	1486
97.1	1071 60347 34492	688 97703	1477
97.2	1069 38833 22813	686 13161	1467
97.3	1067 18005 24294	683 30087	1458
97.4	1064 97860 55863	680 48471	1449
97.5	1062 78396 35903	677 68305	1440
97.6	1060 59609 84247	674 89577	1431
97.7	1058 41498 22170	672 12284	1423
97.8	1056 24058 72377	669 36412	1414
97.9	1054 07288 58996	666 61954	1405
98.0	1051 91185 07569	663 88901	1397
98.1	1049 75745 45043	661 17245	1388
98.2	1047 60966 99762	658 46977	1380
98.3	1045 46847 01459	655 78088	1371
98.4	1043 33382 81243	653 10571	1363
98.5	1041 20571 71599	650 44416	1354
98.6	1039 08411 06372	647 79616	1346
98.7	1036 96898 20760	645 16162	1338
98.8	1034 86030 51310	642 54046	1330
98.9	1032 75805 35906	639 93259	1322
99.0	1030 66220 13761	637 33794	1314
99.1	1028 57272 25410	634 75643	1306
99.2	1026 48959 12703	632 18798	1297
99.3	1024 41278 18794	629 63251	1290
99.4	1022 34226 88135	627 08993	1282
99.5	1020 27802 66470	624 56017	1274
99.6	1018 22003 00822	622 04316	1267
99.7	1016 16825 39490	619 53882	1259
99.8	1014 12267 32040	617 04706	1251
99.9	1012 08326 29296	614 56782	1244
100.0	1010 04999 83335	612 10102	1236

TABLE 21

THE PENTAGAMMA FUNCTION

Description: $\Psi^{(3)}(x)$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1.

$\text{Log}_{10}\Psi^{(3)}(x)$ to 10 decimal places from $x = 0.0$ to $x = -10.00$ by increments of .1.

$\Psi^{(3)}(x)$ to 12 decimal places from $x = 0.00$ to x 1.00 by increments of .01.

$\text{Log}_{10}\Psi^{(3)}(x)$ to 10 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

x	$\Psi^{(3)}(-x)$	$\mathrm{Log}_{10}\Psi^{(3)}(-x)$	x	$\Psi^{(3)}(-x)$	$\mathrm{Log}_{10}\Psi^{(3)}(-x)$
0.0	∞	∞	5.0	∞	∞
0.1	60009.73901 40938	4.77822 17380	5.1	60014.24067 91330	4.77825 43156
0.2	3765.37089 20705	3.57580 77610	5.2	3768.60524 93032	3.57618 06487
0.3	766.61989 04192	2.88458 00831	5.3	769.01081 23670	2.88593 24461
0.4	281.78204 10453	2.44991 33105	5.4	283.59269 84844	2.45269 50450
0.5	193.40909 10340	2.28647 68838	5.5	194.80904 84045	2.28960 91248
0.6	282.49155 53302	2.45100 54697	5.6	283.59360 44888	2.45269 64325
0.7	768.13135 23269	2.88543 54918	5.7	769.01262 92973	2.88593 34722
0.8	3767.89343 23647	3.57609 86111	5.8	3768.60798 70578	3.57618 09642
0.9	60013.65782 42068	4.77825 00977	5.9	60014.24435 27002	4.77825 43422
1.0	∞	∞	6.0	∞	∞
1.1	60013.83709 48260	4.77825 13950	6.1	60014.24501 25642	4.77825 43469
1.2	3768.26441 05890	3.57614 13686	6.2	3768.60930 98498	3.57618 11167
1.3	768.72065 71990	2.88576 85517	6.3	769.01462 11747	2.88593 45971
1.4	283.34389 02748	2.45231 38524	6.4	283.59627 47631	2.45270 05217
1.5	194.59427 62192	2.28913 00617	6.5	194.81240 96314	2.28961 66180
1.6	283.40708 26740	2.45241 06995	6.6	283.59676 65882	2.45270 12749
1.7	768.84973 45301	2.88584 14687	6.7	769.01560 67991	2.88593 51537
1.8	3768.46499 15783	3.57616 44851	6.8	3768.61079 32383	3.57618 12876
1.9	60014.11822 58305	4.77825 34294	6.9	60014.24699 97060	4.77825 43613
2.0	∞	∞	7.0	∞	∞
2.1	60014.14560 82540	4.77825 36275	7.1	60014.24737 36831	4.77825 43640
2.2	3768.52054 06348	3.57617 08867	7.2	3768.61154 25029	3.57618 13739
2.3	768.93506 46661	2.88588 96658	7.3	769.01673 39816	2.88593 57902
2.4	283.52473 51822	2.45259 09534	7.4	283.59827 56584	2.45270 35859
2.5	194.74787 62192	2.28947 27301	7.5	194.81430 59277	2.28962 08454
2.6	283.53838 05977	2.45261 18545	7.6	283.59856 50320	2.45270 40290
2.7	768.96263 51155	2.88590 52374	7.7	769.01731 36216	2.88593 61176
2.8	3768.56260 71551	3.57617 57346	7.8	3768.61241 42003	3.57618 14744
2.9	60014.20305 77431	4.77825 40433	7.9	60014.24854 01392	4.77825 43725
3.0	∞	∞	8.0	∞	∞
3.1	60014.21057 69986	4.77825 40977	8.1	60014.24876 75175	4.77825 43741
3.2	3768.57776 10937	3.57617 74809	8.2	3768.61286 95791	3.57618 15269
3.3	768.98565 82554	2.88591 82402	8.3	769.01799 82489	2.88593 65042
3.4	283.56963 40699	2.45265 97228	8.4	283.59948 07890	2.45270 54314
3.5	194.78785 95595	2.28956 18853	8.5	194.81545 53392	2.28962 34077
3.6	283.57410 30486	2.45266 65670	8.6	283.59966 19078	2.45270 57087
3.7	768.99464 94409	2.88592 33180	8.7	769.01836 09291	2.88593 67091
3.8	3768.59138 22566	3.57617 90507	8.8	3768.61341 47083	3.57618 15897
3.9	60014.22899 31355	4.77825 42310	8.9	60014.24949 64328	4.77825 43794
4.0	∞	∞	9.0	∞	∞
4.1	60014.23181 02169	4.77825 42514	9.1	60014.24964 24724	4.77825 43804
4.2	3768.59704 31830	3.57617 97031	9.2	3768.61370 71083	3.57618 16234
4.3	769.00320 82678	2.88592 81517	9.3	769.01880 03321	2.88593 69572
4.4	283.58564 21978	2.45268 42389	9.4	283.60024 92819	2.45270 66082
4.5	194.80249 14753	2.28959 45069	9.5	194.81619 19818	2.28962 50499
4.6	283.58750 35153	2.45268 70894	9.6	283.60036 83332	2.45270 67905
4.7	769.00694 53266	2.88593 02622	9.7	769.01903 86710	2.88593 70918
4.8	3768.60268 50633	3.57618 03532	9.8	3768.61406 52078	3.57618 16647
4.9	60014.23940 11270	4.77825 43063	9.9	60014.25012 10450	4.77825 43839
5.0	∞	∞	10.0	∞	∞

x	$\Psi^{(3)}(x)$	$\log_{10}\Psi^{(3)}(x)$	x	$\Psi^{(3)}(x)$	$\mathrm{Log}_{10}\Psi^{(3)}(x)$
.00	∞	∞	.50	97.409091 034002	1.98859 94908
.01	600000006.251061 872883	8.77815 12549	.51	90.064057 951274	1.95455 15107
.02	37500006.019694 989010	7.57403 13374	.52	83.402974 449219	1.92118 15394
.03	7407413.206591 264672	6.86966 65715	.53	77.350667 295222	1.88846 40647
.04	2343755.588916 839889	6.36991 23206	.54	71.841434 683900	1.85637 49967
.05	960005.388322 313195	5.98227 36706	.55	66.817706 331072	1.82489 15634
.06	462968.159828 659993	5.66555 11239	.56	62.228913 110931	1.79399 22157
.07	249900.890764 764475	5.39776 78041	.57	58.030530 449132	1.76365 65404
.08	146489.214396 970237	5.16580 56499	.58	54.183266 294418	1.73386 51817
.09	91454.146643 018277	4.96120 34017	.59	50.652369 791560	1.70459 97688
.10	60004.512876 790267	4.77818 39144	.60	47.407041 045346	1.67584 28493
.11	40985.167530 712526	4.61262 67146	.61	44.419925 813845	1.64757 78289
.12	28939.399296 360683	4.46148 95120	.62	41.666681 766520	1.61978 89782
.13	21011.742042 291220	4.32246 20603	.63	39.125605 220147	1.59246 10685
.14	15622.432580 950850	4.19374 86589	.64	36.777309 126023	1.56557 99502
.15	11855.663789 873839	4.07392 58744	.65	34.604444 606955	1.53913 18833
.16	9158.962353 954041	3.96184 62739	.66	32.591459 596683	1.51310 38106
.17	7187.392987 823735	3.85657 13916	.67	30.724389 168991	1.48748 32574
.18	5719.049938 300615	3.75732 38888	.68	28.990672 999863	1.46225 82973
.19	4607.365460 670764	3.66345 26623	.69	27.378996 116505	1.43741 75202
.20	3753.244994 864726	3.57440 69141	.70	25.879149 678428	1.41295 00024
.21	3088.279186 311721	3.48971 65544	.71	24.481909 029247	1.38884 52798
.22	2564.349216 032898	3.40897 71675	.72	23.178926 670914	1.36509 33215
.23	2147.031036 266365	3.33183 83223	.73	21.962638 158558	1.34168 45064
.24	1811.316623 632950	3.25799 43728	.74	20.826179 205636	1.31860 96011
.25	1538.782144 009188	3.18717 71380	.75	19.763312 534851	1.29585 97384
.26	1315.679227 969460	3.11915 00179	.76	18.768363 218051	1.27342 63995
.27	1131.626793 518048	3.05370 32214	.77	17.836161 424344	1.25130 13942
.28	978.700618 129805	2.99064 98626	.78	16.961991 645094	1.22947 68450
.29	850.790713 118995	2.92982 27407	.79	16.141547 591700	1.20794 51707
.30	743.141764 655050	2.87107 16694	.80	15.370892 070483	1.18669 90730
.31	652.020487 032575	2.81426 12418	.81	14.646421 231746	1.16573 15201
.32	574.472113 346920	2.75926 89515	.82	13.964832 669453	1.14503 57360
.33	508.140256 630845	2.70598 36026	.83	13.323096 916078	1.12460 51870
.34	451.132336 062045	2.65430 39574	.84	12.718431 935761	1.10443 35698
.35	401.918114 585505	2.60413 75804	.85	12.148280 269339	1.08451 48025
.36	359.252538 206280	2.55539 98452	.86	11.610288 528348	1.06484 30122
.37	322.116578 317432	2.50801 30771	.87	11.102288 972741	1.04541 25267
.38	289.671528 549204	2.46190 58110	.88	10.622282 939618	1.02621 78649
.39	261.223440 402937	2.41701 21449	.89	10.168425 918569	1.00725 37284
.40	236.195259 033947	2.37327 11760	.90	9.739014 093777	0.98851 49943
.41	214.104850 516858	2.33062 65062	.91	9.332472 194394	0.96999 67046
.42	194.547568 458811	2.28902 58070	.92	8.947342 513373	0.95169 40629
.43	177.182341 482412	2.24842 04365	.93	8.582274 971155	0.93360 24249
.44	161.720508 918988	2.20876 50993	.94	8.236018 114901	0.91571 72931
.45	147.916814 544070	2.17001 75454	.95	7.907410 956358	0.89803 43101
.46	135.562104 651558	2.13213 83026	.96	7.595375 562408	0.88054 92529
.47	124.477379 516472	2.09509 04367	.97	7.298910 321945	0.86325 80278
.48	114.508925 177412	2.05883 93381	.98	7.017083 821124	0.84615 66644
.49	105.524311 871461	2.02335 25283	.99	6.749029 266528	0.82924 13114
.50	97.409091 034002	1.98859 94908	1.00	6.493939 402267	0.81250 82317

TABLE 22

THE PENTAGAMMA FUNCTION

Description: $\Psi^{(3)}(x)$ to 15 decimal places with central differences from $x = 1.00$ to $x = 4.00$ by increments of .01.

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6	δ^8	δ^{10}
1.00	6.49393 94022 66829	1221 23348 77910	506660 88847	3642 40897	4013486	62834
1.01	6.25106 18728 83403	1151 06455 09837	467959 86153	3297 41407	3561458	54637
1.02	6.01969 49890 09813	1085 57521 27917	432556 24866	2988 03375	3164067	47597
1.03	5.79918 38572 64140	1024 41143 70864	400140 66954	2710 29409	2814273	41512
1.04	5.58891 68398 89332	967 24906 80764	370435 38451	2460 69716	2505991	36254
1.05	5.38832 23131 95287	913 79105 29116	343190 79664	2236 16015	2233964	31705
1.06	5.19686 56970 30358	863 76494 57131	318182 36892	2033 96277	1993641	27759
1.07	5.01404 67303 22561	816 92066 22039	295207 90397	1851 70181	1781078	24339
1.08	4.83939 69702 36803	773 02845 77344	274085 14083	1687 25162	1592854	21360
1.09	4.67247 74947 28389	731 87710 46732	254649 62931	1538 72998	1425990	18776
1.10	4.51287 67902 66707	693 27224 79052	236752 84777	1404 46823	1277902	16518
1.11	4.36020 88082 84077	657 03491 96148	220260 53446	1282 98551	1146332	14551
1.12	4.21411 11754 97596	623 00019 66691	205051 20666	1172 96610	1029313	12833
1.13	4.07424 35446 77805	591 01598 57900	191014 84495	1073 23982	925127	11330
1.14	3.94028 60737 15914	560 94192 33603	178051 72306	982 76481	832271	10004
1.15	3.81193 80219 87627	532 64837 81613	166071 36599	900 61252	749418	8921
1.16	3.68891 64540 40951	506 01554 66222	154991 62144	825 95440	675487	7698
1.17	3.57095 50415 60498	480 93263 12975	144737 83128	758 05115	609252	7160
1.18	3.45780 29553 93020	457 29709 42866	135242 09228	696 24043	550178	6026
1.19	3.34922 38401 68408	435 01397 81985	126442 59370	639 93148	497129	5548
1.20	3.24499 48647 25780	413 99528 80474	118283 02661	588 59383	449628	4871
1.21	3.14490 58421 63626	394 15942 81624	110712 05335	541 75245	406998	4355
1.22	3.04875 84138 83097	375 43068 88109	103682 83253	498 98106	368723	3812
1.23	2.95636 52924 90677	357 73877 77848	97152 59279	459 89690	334260	3602
1.24	2.86754 95588 76104	341 01839 26865	91082 24994	424 15533	303398	2884
1.25	2.78214 40091 88396	325 20883 00865	85436 06243	391 44775	275422	2904
1.26	2.69999 05478 01553	310 25362 81109	80181 32267	361 49439	250349	2404
1.27	2.62093 96226 95819	296 10023 93619	75288 07729	334 04450	227680	2215
1.28	2.54484 96999 83704	282 69973 13858	70728 87642	308 87142	207226	1977
1.29	2.47158 67745 85446	270 00651 21739	66478 54697	285 77060	188749	1771
1.30	2.40102 39143 08927	257 97807 84317	62513 98813	264 55728	172044	1591
1.31	2.33304 08348 16724	246 57478 45707	58813 98656	245 06439	156929	1428
1.32	2.26752 35031 70229	235 75963 05755	55359 04939	227 14079	143242	1285
1.33	2.20436 37678 29488	225 49806 70742	52131 25302	210 64962	130840	1155
1.34	2.14345 90131 59489	215 75781 61030	49114 10626	195 46683	119592	1040
1.35	2.08471 18366 50521	206 50870 61945	46292 42633	181 47997	109385	938
1.36	2.02802 97472 03497	197 72252 05493	43652 22638	168 58696	100115	845
1.37	1.97332 48829 61966	189 37285 71678	41180 61339	156 69511	91691	763
1.38	1.92051 37472 92113	181 43499 99204	38865 69552	145 72016	84029	689
1.39	1.86951 69616 21463	173 88579 96280	36696 49779	135 58549	77056	623
1.40	1.82025 90339 47094	166 70356 43137	34662 88557	126 22139	70706	563
1.41	1.77266 81419 15862	159 86795 78550	32755 49473	117 56435	64919	510
1.42	1.72667 59294 63180	153 35990 63436	30965 66824	109 55649	59642	462
1.43	1.68221 73160 73934	147 16151 15146	29285 39825	102 14505	54826	418
1.44	1.63923 03177 99833	141 25597 06681	27707 27330	95 28187	50429	380
1.45	1.59765 58792 32414	135 62750 25546	26224 43023	88 92299	46412	344
1.46	1.55743 77156 90541	130 26127 87433	24830 51014	83 02822	42739	313
1.47	1.51852 21649 36101	125 14336 00335	23519 61828	77 56084	39379	284
1.48	1.48085 80477 81996	120 26063 75065	22286 28725	72 48725	36303	259
1.49	1.44439 65370 02957	115 60077 78520	21125 44348	67 77669	33486	235
1.50	1.40909 10340 02437	111 15217 26323	20032 37639	63 40100	30905	214

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6	δ^8	δ^{10}
1.50	1.40909 10340 02437	111 15217 26323	20032 37639	63 40100	30905	214
1.51	1.37489 70527 28241	106 90389 11765	19002 71030	59 33435	28537	195
1.52	1.34177 21103 65810	102 84563 68237	18032 37856	55 55308	26365	178
1.53	1.30967 56243 71615	98 96770 62565	17117 59990	52 03546	24371	162
1.54	1.27856 88154 39985	95 26095 16883	16254 85670	48 76156	22540	149
1.55	1.24841 46160 25239	91 71674 56872	15440 87505	45 71305	20856	135
1.56	1.21917 75840 67365	88 32694 84366	14672 60646	42 87310	19308	124
1.57	1.19082 38215 93856	85 08387 72505	13947 21096	40 22625	17884	114
1.58	1.16332 08978 92853	81 98027 81741	13262 04172	37 75823	16573	103
1.59	1.13663 77769 73592	79 00929 95149	12614 63070	35 45595	15365	96
1.60	1.11074 47490 49479	76 16446 71626	12002 67563	33 30732	14253	86
1.61	1.08561 33657 96992	73 43966 15666	11424 02787	31 30122	13227	80
1.62	1.06121 63791 60172	70 82909 62493	10876 68134	29 42739	12281	72
1.63	1.03752 76834 85844	68 32729 77454	10358 76219	27 67637	11407	69
1.64	1.01452 22607 88971	65 92908 68634	9868 51941	26 03941	10601	59
1.65	.99217 61289 60731	63 62956 11755	9404 31604	24 50847	9855	58
1.66	.97046 62927 44247	61 42407 86480	8964 62114	23 07608	9167	51
1.67	.94937 06973 14244	59 30824 23320	8548 00231	21 73535	8530	48
1.68	.92886 81843 07560	57 27788 60390	8153 11884	20 47994	7941	44
1.69	.90893 84501 61265	55 32906 09345	7778 71531	19 30393	7396	40
1.70	.88956 20066 24316	53 45802 29830	7423 61570	18 20187	6891	38
1.71	.87072 01433 17197	51 66122 11885	7086 71796	17 16873	6423	34
1.72	.85239 48922 21964	49 93528 65736	6766 98895	16 19982	5989	32
1.73	.83456 89939 92466	48 27702 18482	6463 45976	15 29080	5597	29
1.74	.81722 58659 81449	46 68339 17204	6175 22137	14 43766	5215	26
1.75	.80034 95718 87637	45 15151 38063	5901 42064	13 63666	4868	25
1.76	.78392 47929 31887	43 67865 00986	5641 25656	12 88435	4547	22
1.77	.76793 68004 77123	42 26219 89565	5393 97684	12 17751	4249	22
1.78	.75237 14300 11925	40 89968 75829	5158 87462	11 51315	3971	19
1.79	.73721 50564 22555	39 58876 49554	4935 28556	10 88852	3714	18
1.80	.72245 45704 82739	38 32719 51835	4722 58501	10 30101	3474	17
1.81	.70807 73564 94758	37 11285 12617	4520 18547	9 74824	3251	16
1.82	.69407 12710 19394	35 94370 91947	4327 53418	9 22798	3043	14
1.83	.68042 46226 35977	34 81784 24695	4144 11087	8 73815	2850	13
1.84	.66712 61526 77254	33 73341 68530	3969 42572	8 27682	2670	12
1.85	.65416 50168 87061	32 68868 54937	3803 01738	7 84218	2502	12
1.86	.64153 07679 51805	31 68198 43082	3644 45123	7 43256	2345	10
1.87	.62921 33388 59630	30 71172 76350	3493 31764	7 04640	2199	10
1.88	.61720 30270 43805	29 77640 41381	3349 23044	6 68222	2063	9
1.89	.60549 04792 69361	28 87457 29458	3211 82547	6 33868	1936	9
1.90	.59406 66772 24375	28 00486 00082	3080 75919	6 01451	1818	8
1.91	.58292 29237 79470	27 16595 46624	2955 70741	5 70851	1707	7
1.92	.57205 08298 81190	26 35660 63908	2836 36414	5 41958	1603	7
1.93	.56144 23020 46817	25 57562 17605	2722 44045	5 14669	1507	6
1.94	.55108 95304 30049	24 82186 15348	2613 66345	4 88886	1416	6
1.95	.54098 49774 28630	24 09423 79436	2509 77531	4 64520	1332	6
1.96	.53112 13668 06646	23 39171 21054	2410 53236	4 41485	1253	5
1.97	.52149 16733 05716	22 71329 15909	2315 70427	4 19703	1179	5
1.98	.51208 91127 20695	22 05802 81191	2225 07321	3 99100	1109	4
1.99	.50290 71324 16865	21 42501 53794	2138 43316	3 79607	1044	4
2.00	.49393 94022 66829	20 81338 69712	2055 58917	3 61157	983	4

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
2.00	.49393 94022 66829	20 81338 69712	2055 58917	361157
2.01	.48517 98059 86505	20 22231 44547	1976 35675	343691
2.02	.47662 24328 50728	19 65100 55057	1900 56124	327152
2.03	.46826 15697 70007	19 09870 21690	1828 03725	311486
2.04	.46009 16937 10976	18 56467 92049	1758 62813	296643
2.05	.45210 74644 43995	18 04824 25221	1692 18543	282576
2.06	.44430 37176 02235	17 54872 76935	1628 56848	269240
2.07	.43667 54580 37410	17 06549 85498	1567 64394	256596
2.08	.42921 78534 58082	16 59794 58454	1509 28536	244602
2.09	.42192 62283 37209	16 14548 59947	1453 37280	233225
2.10	.41479 60580 76283	15 70755 98719	1399 79249	222427
2.11	.40782 29634 14075	15 28363 16740	1348 43645	212179
2.12	.40100 27050 68608	14 87318 78406	1299 20220	202448
2.13	.39433 11786 01546	14 47573 60292	1251 99243	193208
2.14	.38780 44094 94776	14 09080 41421	1206 71474	184430
2.15	.38141 85484 29427	13 71793 94025	1163 28136	176090
2.16	.37516 98667 58104	13 35670 74764	1121 60887	168164
2.17	.36905 47521 61544	13 00669 16390	1081 61802	160630
2.18	.36306 97044 81374	12 66749 19818	1043 23347	153466
2.19	.35721 13317 21022	12 33872 46592	1006 38357	146653
2.20	.35147 63462 07262	12 02002 11724	971 00021	140172
2.21	.34586 15609 05226	11 71102 76877	937 01856	134005
2.22	.34036 38858 80067	11 41140 43886	904 37696	128136
2.23	.33498 03248 98793	11 12082 48590	873 01673	122550
2.24	.32970 79721 66110	10 83897 54969	842 88200	117231
2.25	.32454 40091 88396	10 56555 49546	813 91958	112165
2.26	.31948 57017 60228	10 30027 36082	786 07881	107340
2.27	.31453 03970 68142	10 04285 30499	759 31144	102743
2.28	.30967 55209 06555	9 79302 56060	733 57151	98362
2.29	.30491 85750 01028	9 55053 38772	708 81519	94186
2.30	.30025 71344 34272	9 31513 03003	685 00074	90205
2.31	.29568 88451 70520	9 08657 67308	662 08834	86409
2.32	.29121 14216 74075	8 86464 40448	640 04003	82788
2.33	.28682 26446 18078	8 64911 17590	618 81960	79334
2.34	.28252 03586 79671	8 43976 76692	598 39250	76038
2.35	.27830 24704 17957	8 23640 75044	578 72579	72893
2.36	.27416 69462 31286	8 03883 45976	559 78801	69890
2.37	.27011 18103 90591	7 84685 95708	541 54913	67024
2.38	.26613 51431 45605	7 66030 00353	523 98048	64286
2.39	.26223 50789 00970	7 47898 03045	507 05470	61672
2.40	.25840 98044 59381	7 30273 11208	490 74563	59174
2.41	.25465 75573 29000	7 13138 93933	475 02831	56787
2.42	.25097 66240 92551	6 96479 79490	459 87885	54507
2.43	.24736 53388 35593	6 80280 52932	445 27447	52326
2.44	.24382 20816 31566	6 64526 53820	431 19335	50242
2.45	.24034 52770 81360	6 49203 74044	417 61465	48249
2.46	.23693 33929 05196	6 34298 55732	404 51844	46343
2.47	.23358 49385 84765	6 19797 89264	391 88566	44520
2.48	.23029 84640 53597	6 05689 11362	379 69808	42775
2.49	.22707 25584 33791	5 91960 03267	367 93824	41106
2.50	.22390 58488 17252	5 78598 88997	356 58946	39502

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
2.50	.22390 58488 17252	5 78598 88997	356 58946	39502
2.51	.22079 69990 89710	5 65594 33672	345 63576	37978
2.52	.21774 47087 95840	5 52935 41924	335 06184	36514
2.53	.21474 77120 43894	5 40611 56360	324 85306	35111
2.54	.21180 47764 48309	5 28612 56102	314 99540	33768
2.55	.20891 47021 08826	5 16928 55384	305 47542	32482
2.56	.20607 63206 24727	5 05550 02208	296 28025	31249
2.57	.20328 84941 42835	4 94467 77056	287 39758	30068
2.58	.20055 01144 38000	4 83672 91662	278 81558	28936
2.59	.19786 01020 24827	4 73156 87826	270 52293	27850
2.60	.19521 74052 99479	4 62911 36283	262 50879	26810
2.61	.19262 09997 10414	4 52928 35618	254 76274	25812
2.62	.19006 98869 56968	4 43200 11229	247 27481	24855
2.63	.18756 30942 14750	4 33719 14320	240 03543	23937
2.64	.18509 96733 86851	4 24478 20954	233 03541	23056
2.65	.18267 87003 79907	4 15470 31129	226 26596	22211
2.66	.18029 92744 04092	4 06688 67901	219 71862	21400
2.67	.17796 05172 96177	3 98126 76534	213 38528	20622
2.68	.17566 15728 64796	3 89778 23695	207 25816	19874
2.69	.17340 16062 57111	3 81636 96672	201 32978	19157
2.70	.17117 98033 46099	3 73697 02627	195 59297	18468
2.71	.16899 53701 37713	3 65952 67879	190 04084	17806
2.72	.16684 75321 97206	3 58398 37215	184 66678	17171
2.73	.16473 55340 93914	3 51028 73228	179 46442	16560
2.74	.16265 86388 63850	3 43838 55684	174 42766	15973
2.75	.16061 61274 89469	3 36822 80906	169 55064	15409
2.76	.15860 72983 95995	3 29976 61192	164 82770	14867
2.77	.15663 14669 63712	3 23295 24248	160 25345	14346
2.78	.15468 79650 55677	3 16774 12648	155 82265	13845
2.79	.15277 61405 60290	3 10408 83314	151 53031	13364
2.80	.15089 53569 48217	3 04195 07010	147 37160	12900
2.81	.14904 49928 43154	2 98128 67866	143 34190	12455
2.82	.14722 44416 05958	2 92205 62912	139 43674	12026
2.83	.14543 31109 31675	2 86422 01632	135 65185	11614
2.84	.14367 04224 59023	2 80774 05537	131 98309	11217
2.85	.14193 58113 91909	2 75258 07752	128 42651	10835
2.86	.14022 87261 32547	2 69870 52617	124 97827	10467
2.87	.13854 86279 25802	2 64607 95310	121 63471	10114
2.88	.13689 49905 14368	2 59467 01475	118 39229	9773
2.89	.13526 72998 04408	2 54444 46867	115 24759	9445
2.90	.13366 50535 41315	2 49537 17019	112 19734	9129
2.91	.13208 77609 95242	2 44742 06905	109 23838	8825
2.92	.13053 49426 56074	2 40056 20629	106 36766	8531
2.93	.12900 61299 37535	2 35476 71118	103 58225	8249
2.94	.12750 08648 90114	2 31000 79833	100 87934	7977
2.95	.12601 86999 22525	2 26625 76482	98 25619	7715
2.96	.12455 91975 31418	2 22348 98749	95 71019	7462
2.97	.12312 19300 39061	2 18167 92036	93 23880	7218
2.98	.12170 64793 38739	2 14080 09202	90 83960	6983
2.99	.12031 24366 47620	2 10083 10329	88 51022	6757
3.00	.11893 94022 66829	2 06174 62478	86 24842	6538

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
3.00	.11893 94022 66829	2 06174 62479	86 24842	6538
3.01	.11758 69853 48516	2 02352 39468	84 05200	6328
3.02	.11625 48036 69672	1 98614 21659	81 91885	6125
3.03	.11494 24834 12486	1 94957 95734	79 84696	5929
3.04	.11364 96589 51034	1 91381 54505	77 83434	5740
3.05	.11237 59726 44087	1 87882 96711	75 87913	5557
3.06	.11112 10746 33851	1 84460 26829	73 97949	5381
3.07	.10988 46226 50445	1 81111 54896	72 13365	5211
3.08	.10866 62818 21935	1 77834 96329	70 33994	5047
3.09	.10746 57244 89754	1 74628 71755	68 59669	4889
3.10	.10628 26300 29327	1 71491 06849	66 90233	4736
3.11	.10511 66846 75750	1 68420 32177	65 25534	4589
3.12	.10396 75813 54350	1 65414 83040	63 65423	4446
3.13	.10283 50195 15990	1 62472 99325	62 09758	4308
3.14	.10171 87049 76954	1 59593 25368	60 58402	4176
3.15	.10061 83497 63287	1 56774 09814	59 11221	4047
3.16	.09953 36719 59433	1 54014 05481	57 68088	3923
3.17	.09846 43955 61061	1 51311 69236	56 28877	3803
3.18	.09741 02503 31925	1 48665 61868	54 93470	3687
3.19	.09637 09716 64657	1 46074 47971	53 61750	3575
3.20	.09534 63004 45360	1 43536 95823	52 33605	3467
3.21	.09433 59829 21886	1 41051 77280	51 08928	3363
3.22	.09333 97705 75692	1 38617 67666	49 87613	3261
3.23	.09235 74199 97163	1 36233 45664	48 69560	3164
3.24	.09138 86927 64299	1 33897 93222	47 54670	3069
3.25	.09043 33553 24656	1 31609 95449	46 42849	2978
3.26	.08949 11788 80462	1 29368 40526	45 34006	2889
3.27	.08856 19392 76794	1 27172 19609	44 28053	2804
3.28	.08764 54168 92734	1 25020 26744	43 24903	2721
3.29	.08674 13965 35419	1 22911 58782	42 24474	2641
3.30	.08584 96673 36885	1 20845 15294	41 26686	2563
3.31	.08497 00226 53645	1 18819 98492	40 31461	2489
3.32	.08410 22599 68897	1 16835 13151	39 38725	2416
3.33	.08324 61807 97299	1 14889 66535	38 48406	2346
3.34	.08240 15905 92237	1 12982 68325	37 60431	2278
3.35	.08156 82986 55499	1 11113 30547	36 74735	2212
3.36	.08074 61180 49308	1 09280 67503	35 91251	2148
3.37	.07993 48655 10620	1 07483 95710	35 09915	2087
3.38	.07913 43613 67643	1 05722 33833	34 30666	2027
3.39	.07834 44294 58498	1 03995 02620	33 53443	1969
3.40	.07756 48970 51974	1 02301 24852	32 78190	1913
3.41	.07679 55947 70301	1 00640 25273	32 04850	1859
3.42	.07603 63565 13901	99011 30545	31 33369	1806
3.43	.07528 70193 88047	97413 69185	30 63694	1755
3.44	.07454 74236 31377	95846 71519	29 95774	1706
3.45	.07381 74125 46226	94309 69628	29 29561	1658
3.46	.07309 68324 30704	92801 97297	28 65005	1612
3.47	.07238 55325 12478	91322 89971	28 02062	1567
3.48	.07168 33648 84222	89871 84706	27 40685	1523
3.49	.07099 01844 40674	88448 20127	26 80832	1481
3.50	.07030 58488 17252	87051 36379	26 22460	1440

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
3.50	.07030 58488 17252	87051 36379	26 22460	1440
3.51	.06963 02183 30209	85680 75091	25 65528	1401
3.52	.06896 31559 18257	84335 79330	25 09997	1362
3.53	.06830 45270 85635	83015 93566	24 55828	1325
3.54	.06765 41998 46579	81720 63630	24 02984	1289
3.55	.06701 20446 71153	80449 36678	23 51429	1254
3.56	.06637 79344 32405	79201 61154	23 01127	1220
3.57	.06575 17443 54811	77976 86757	22 52045	1187
3.58	.06513 33519 63974	76774 64405	22 04150	1155
3.59	.06452 26370 37542	75594 46203	21 57409	1124
3.60	.06391 94815 57313	74435 85409	21 11791	1093
3.61	.06332 37696 62493	73298 36407	20 67268	1064
3.62	.06273 53876 04080	72181 54672	20 23808	1036
3.63	.06215 42237 00338	71084 96745	19 81384	1008
3.64	.06158 01682 93342	70008 20202	19 39968	981
3.65	.06101 31137 06547	68950 83626	18 99534	955
3.66	.06045 29542 03378	67912 46585	18 60055	930
3.67	.05989 95859 46794	66892 69597	18 21506	906
3.68	.05935 29069 59808	65891 14116	17 83863	882
3.69	.05881 28170 86937	64907 42497	17 47102	859
3.70	.05827 92179 56564	63941 17981	17 11200	837
3.71	.05775 20129 44171	62992 04665	16 76135	815
3.72	.05723 11071 36443	62059 67483	16 41884	794
3.73	.05671 64072 96198	61143 72185	16 08427	773
3.74	.05620 78218 28139	60243 85315	15 75743	753
3.75	.05570 52607 45395	59359 74188	15 43812	734
3.76	.05520 86356 36838	58491 06873	15 12616	715
3.77	.05471 78596 35155	57637 52174	14 82133	697
3.78	.05423 28473 85646	56798 79608	14 52348	679
3.79	.05375 35150 15745	55974 59391	14 23242	662
3.80	.05327 97801 05235	55164 62415	13 94797	645
3.81	.05281 15616 57139	54368 60235	13 66996	628
3.82	.05234 87800 69279	53586 25053	13 39825	613
3.83	.05189 13571 06472	52817 29694	13 13266	597
3.84	.05143 92158 73359	52061 47602	12 87304	582
3.85	.05099 22807 87848	51318 52814	12 61924	568
3.86	.05055 04775 55151	50588 19949	12 37112	553
3.87	.05011 37331 42403	49870 24197	12 12853	540
3.88	.04968 19757 53851	49164 41297	11 89134	526
3.89	.04925 51348 06597	48470 47532	11 65941	513
3.90	.04883 31409 06874	47788 19708	11 43261	500
3.91	.04841 59258 26860	47117 35145	11 21082	488
3.92	.04800 34224 81990	46457 71665	10 99391	476
3.93	.04759 55649 08785	45809 07575	10 78176	464
3.94	.04719 22882 43155	45171 21662	10 57426	453
3.95	.04679 35286 99187	44543 93174	10 37128	442
3.96	.04639 92235 48393	43927 01815	10 17273	431
3.97	.04600 93110 99415	43320 27729	9 97849	421
3.98	.04562 37306 78166	42723 51493	9 78846	411
3.99	.04524 24226 08410	42136 54102	9 60253	401
4.00	.04486 53281 92755	41559 16964	9 42061	391

TABLE 23

THE PENTAGAMMA FUNCTION

Description: $\Psi^{(3)}(x)$ to 15 decimal places with central differences from $x = 4.00$ to $x = 20.00$ by increments of .02.

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
4.00	.(1) 4486 53281 92755	166246 09919	150 76116	25042
4.02	4412 35503 17263	161739 11139	145 12453	23854
4.04	4339 79463 52910	157377 24811	139 72644	22729
4.06	4268 80801 13369	153155 11128	134 55564	21662
4.08	4199 35293 84955	149067 53009	129 60147	20651
4.10	4131 38854 09551	145109 55038	124 85380	19691
4.12	4064 87523 89185	141276 42446	120 30305	18781
4.14	3999 77470 11265	137563 60160	115 94009	17916
4.16	3936 04979 93505	133966 71882	111 75631	17096
4.18	3873 66456 47627	130481 59236	107 74348	16317
4.20	3812 58414 60985	127104 20938	103 89383	15577
4.22	3752 77476 95281	123830 72022	100 19995	14874
4.24	3694 20370 01598	120657 43102	96 65481	14206
4.26	3636 83920 51018	117580 79663	93 25174	13572
4.28	3580 65051 80102	114597 41398	89 98438	12968
4.30	3525 60780 50583	111704 01572	86 84671	12394
4.32	3471 68213 22637	108897 46417	83 83297	11848
4.34	3418 84543 41108	106174 74558	80 93771	11328
4.36	3367 07048 34136	103532 96470	78 15573	10834
4.38	3316 33086 23633	100969 33954	75 48208	10363
4.40	3266 60093 47085	98481 19647	72 91207	9915
4.42	3217 85581 90184	96065 96547	70 44121	9488
4.44	3170 07136 29830	93721 17569	68 06523	9082
4.46	3123 22411 87044	91444 45114	65 78007	8694
4.48	3077 29131 89373	89233 50666	63 58185	8325
4.50	3032 25085 42367	87086 14403	61 46689	7973
4.52	2988 08125 09764	85000 24830	59 43166	7638
4.54	2944 76165 01991	82973 78422	57 47281	7318
4.56	2902 27178 72639	81004 79294	55 58713	7013
4.58	2860 59197 22582	79091 38880	53 77159	6722
4.60	2819 70307 11406	77231 75626	52 02327	6444
4.62	2779 58648 75855	75424 14698	50 33939	6179
4.64	2740 22414 55002	73666 87710	48 71730	5926
4.66	2701 59847 21860	71958 32451	47 15447	5684
4.68	2663 69238 21168	70296 92639	45 64847	5453
4.70	2626 48926 13117	68681 17675	44 19702	5233
4.72	2589 97295 22740	67109 62412	42 79789	5022
4.74	2554 12773 94775	65580 86939	41 44899	4821
4.76	2518 93833 53748	64093 56364	40 14830	4629
4.78	2484 38986 69086	62646 40620	38 89389	4445
4.80	2450 46786 25044	61238 14264	37 68394	4269
4.82	2417 15823 95265	59867 56302	36 51667	4101
4.84	2384 44729 21789	58533 50007	35 39041	3940
4.86	2352 32167 98320	57234 82753	34 30355	3786
4.88	2320 76841 57605	55970 45853	33 25454	3639
4.90	2289 77485 62743	54739 34408	32 24193	3498
4.92	2259 32869 02288	53540 47155	31 26429	3363
4.94	2229 41792 88989	52372 86331	30 32028	3234
4.96	2200 03089 62021	51235 57535	29 40860	3110
4.98	2171 15621 92588	50127 69600	28 52803	2991
5.00	.(1) 2142 78281 92755	49048 34467	27 67737	2878

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
5.00	.(1) 2142 78281 92755	49048 34467	27 67737	2878
5.02	2114 89990 27389	47996 67071	26 85548	2769
5.04	2087 49695 29094	46971 85224	26 06129	2665
5.06	2060 56372 16024	45973 09506	25 29375	2565
5.08	2034 09022 12459	44999 63163	24 55186	2469
5.10	2008 06671 72057	44050 72006	23 83466	2377
5.12	1982 48372 03661	43125 64315	23 14123	2289
5.14	1957 33197 99580	42223 70747	22 47070	2205
5.16	1932 60247 66246	41344 24249	21 82222	2124
5.18	1908 28641 57161	40486 59974	21 19498	2046
5.20	1884 37522 08050	39650 15196	20 58820	1972
5.22	1860 86052 74135	38834 29237	20 00113	1900
5.24	1837 73417 69457	38038 43392	19 43307	1831
5.26	1814 98821 08171	37262 00853	18 88331	1765
5.28	1792 61486 47738	36504 46645	18 35121	1702
5.30	1770 60656 33950	35765 27559	17 83614	1641
5.32	1748 95591 47720	35043 92086	17 33747	1583
5.34	1727 65570 53577	34339 90361	16 85463	1527
5.36	1706 69889 49795	33652 74098	16 38705	1473
5.38	1686 07861 20111	32981 96540	15 93420	1421
5.40	1665 78814 86966	32327 12403	15 49556	1371
5.42	1645 82095 66225	31687 77822	15 07063	1323
5.44	1626 17064 23306	31063 50305	14 65893	1277
5.46	1606 83096 30692	30453 88681	14 26000	1233
5.48	1587 79582 26759	29858 53057	13 87340	1190
5.50	1569 05926 75883	29277 04772	13 49870	1149
5.52	1550 61548 29779	28709 06357	13 13549	1110
5.54	1532 45878 90032	28154 21491	12 78337	1072
5.56	1514 58363 71776	27612 14962	12 44198	1035
5.58	1496 98460 68482	27082 52631	12 11094	1000
5.60	1479 65640 17819	26565 01394	11 78990	966
5.62	1462 59384 68550	26059 29147	11 47853	934
5.64	1445 79188 48428	25565 04754	11 17650	903
5.66	1429 24557 33061	25081 98010	10 88349	872
5.68	1412 95008 15704	24609 79616	10 59921	843
5.70	1396 90068 77963	24148 21144	10 32337	815
5.72	1381 09277 61366	23696 95008	10 05567	788
5.74	1365 52183 39776	23255 74439	9 79587	763
5.76	1350 18344 92626	22824 33457	9 54368	737
5.78	1335 07330 78932	22402 46843	9 29888	713
5.80	1320 18719 12081	21989 90117	9 06120	690
5.82	1305 52097 35346	21586 39511	8 83043	668
5.84	1291 07061 98123	21191 71948	8 60633	646
5.86	1276 83218 32849	20805 65019	8 38870	625
5.88	1262 80180 32593	20427 96960	8 17732	605
5.90	1248 97570 29297	20058 46632	7 97199	586
5.92	1235 35018 72633	19696 93503	7 77251	567
5.94	1221 92164 09471	19343 17625	7 57871	549
5.96	1208 68652 63935	18996 99619	7 39040	532
5.98	1195 64138 18018	18658 20653	7 20741	515
6.00	.(1) 1182 78281 92755	18326 62429	7 02956	499

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
6.00	.(1)1182 78281 92756	18326 62429	7 02956	499
6.02	1170 10752 29920	18002 07160	6 85670	483
6.04	1157 61224 74246	17684 37562	6 68868	468
6.06	1145 29381 56133	17373 36831	6 52533	453
6.08	1133 14911 74851	17068 88633	6 36651	439
6.10	1121 17510 82203	16770 77087	6 21209	426
6.12	1109 36880 66641	16478 86749	6 06192	413
6.14	1097 72729 37828	16193 02603	5 91588	400
6.16	1086 24771 11619	15913 10046	5 77384	388
6.18	1074 92725 95456	15638 94872	5 63567	376
6.20	1063 76319 74165	15370 43266	5 50127	364
6.22	1052 75283 96140	15107 41787	5 37050	353
6.24	1041 89355 59902	14849 77357	5 24327	343
6.26	1031 18277 01021	14597 37254	5 11946	332
6.28	1020 61795 79393	14350 09098	4 99898	322
6.30	.(1)1010 19664 66865	14107 80840	4 88172	313
6.32	.(2) 999 91641 35176	13870 40754	4 76759	303
6.34	989 77488 44241	13637 77427	4 65649	294
6.36	979 76973 30733	13409 79749	4 54833	286
6.38	969 89867 96975	13186 36904	4 44303	277
6.40	960 15949 00120	12967 38363	4 34050	269
6.42	950 54997 41629	12752 73872	4 24066	261
6.44	941 06798 57008	12542 33447	4 14344	253
6.46	931 71142 05835	12336 07366	4 04874	246
6.48	922 47821 62027	12133 86159	3 95651	239
6.50	913 36635 04378	11935 60602	3 86666	232
6.52	904 37384 07331	11741 21712	3 77914	225
6.54	895 49874 31997	11550 60736	3 69387	219
6.56	886 73915 17399	11363 69147	3 61078	212
6.58	878 09319 71948	11180 38636	3 52982	206
6.60	869 55904 65132	11000 61107	3 45093	201
6.62	861 13490 19424	10824 28670	3 37404	195
6.64	852 81900 02386	10651 33638	3 29909	189
6.66	844 60961 18986	10481 68514	3 22604	184
6.68	836 50504 04100	10315 25995	3 15483	179
6.70	828 50362 15209	10151 98960	3 08541	174
6.72	820 60372 25279	9991 80466	3 01773	169
6.74	812 80374 15814	9834 63744	2 95173	164
6.76	805 10210 70093	9680 42196	2 88737	160
6.78	797 49727 66568	9529 09385	2 82462	155
6.80	789 98773 72428	9380 59035	2 76341	151
6.82	782 57200 37323	9234 85026	2 70371	147
6.84	775 24861 87243	9091 81388	2 64547	142
6.86	768 01615 18552	8951 42298	2 58867	139
6.88	760 87319 92158	8813 62074	2 53325	135
6.90	753 81838 27838	8678 35175	2 47918	131
6.92	746 85034 98694	8545 56195	2 42642	128
6.94	739 96777 25743	8415 19856	2 37494	124
6.96	733 16934 72649	8287 21012	2 32471	121
6.98	726 45379 40568	8161 54639	2 27568	118
7.00	.(2)719 81985 63125	8038 15834	2 22783	114

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
7.00	.(2)719 81985 63125	8038 15834	2 22783	114
7.02	713 26630 01517	7916 99811	2 18112	111
7.04	706 79191 39720	7798 01900	2 13552	108
7.06	700 39550 79823	7681 17542	2 09101	106
7.08	694 07591 37468	7566 42285	2 04756	103
7.10	687 83198 37398	7453 71784	2 00513	100
7.12	681 66259 09111	7343 01796	1 96371	97
7.14	675 56662 82620	7234 28179	1 92325	95
7.16	669 54300 84308	7127 46887	1 88375	92
7.18	663 59066 32883	7022 53970	1 84517	90
7.20	657 70854 35429	6919 45570	1 80748	88
7.22	651 89561 83545	6818 17918	1 77068	85
7.24	646 15087 49579	6718 67334	1 73472	83
7.26	640 47331 82947	6620 90222	1 69960	81
7.28	634 86197 06538	6524 83070	1 66528	79
7.30	629 31587 13198	6430 42446	1 63176	77
7.32	623 83407 62305	6337 64998	1 59900	75
7.34	618 41565 76410	6246 47450	1 56699	73
7.36	613 05970 37965	6156 86600	1 53571	71
7.38	607 76531 86120	6068 79322	1 50514	69
7.40	602 53162 13597	5982 22557	1 47526	67
7.42	597 35774 63631	5897 13318	1 44606	66
7.44	592 24284 26983	5813 48685	1 41751	64
7.46	587 18607 39019	5731 25802	1 38961	63
7.48	582 18661 76859	5650 41881	1 36233	61
7.50	577 24366 56579	5570 94192	1 33566	59
7.52	572 35642 30491	5492 80069	1 30958	58
7.54	567 52410 84473	5415 96905	1 28409	56
7.56	562 74595 35359	5340 42149	1 25916	55
7.58	558 02120 28395	5266 13309	1 23478	54
7.60	553 34911 34739	5193 07946	1 21093	52
7.62	548 72895 49029	5121 23676	1 18761	51
7.64	544 16000 86995	5050 58168	1 16481	50
7.66	539 64156 83129	4981 09141	1 14250	49
7.68	535 17293 88404	4912 74362	1 12067	47
7.70	530 75343 68041	4845 51652	1 09932	46
7.72	526 38238 99330	4779 38873	1 07844	45
7.74	522 05913 69492	4714 33938	1 05800	44
7.76	517 78302 73593	4650 34804	1 03801	43
7.78	513 55342 12497	4587 39471	1 01845	42
7.80	509 36968 90872	4525 45982	99930	41
7.82	505 23121 15230	4464 52424	98057	40
7.84	501 13737 92011	4404 56922	96223	39
7.86	497 08759 25714	4345 57644	94429	38
7.88	493 08126 17062	4287 52795	92673	37
7.90	489 11780 61204	4230 40619	90954	36
7.92	485 19665 45965	4174 19396	89271	35
7.94	481 31724 50121	4118 87443	87623	35
7.96	477 47902 41721	4064 43114	86010	34
7.98	473 68144 76435	4010 84796	84431	33
8.00	.(2)469 92397 95945	3958 10909	82885	32

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
8.00	.(2) 469 92397 95945	3958 10909	82885	32
8.02	466 20609 26364	3906 19907	81372	31
8.04	462 52726 76689	3855 10277	79889	31
8.06	458 88699 37292	3804 80537	78438	30
8.08	455 28476 78431	3755 29234	77016	29
8.10	451 72009 48805	3706 54948	75624	29
8.12	448 19248 74126	3658 56285	74261	28
8.14	444 70146 55732	3611 31883	72925	27
8.16	441 24655 69221	3564 80407	71617	27
8.18	437 82729 63117	3519 00547	70335	26
8.20	434 44322 57560	3473 91022	69080	26
8.22	431 09389 43024	3429 50577	67850	25
8.24	427 77885 79065	3385 77982	66645	24
8.26	424 49767 93088	3342 72032	65464	24
8.28	421 24992 79142	3300 31546	64308	23
8.30	418 03517 96742	3258 55368	63174	23
8.32	414 85301 69710	3217 42364	62063	22
8.34	411 70302 85043	3176 91424	60975	22
8.36	408 58480 91800	3137 01459	59908	21
8.38	405 49796 00015	3097 71401	58863	21
8.40	402 44208 79631	3059 00206	57838	20
8.42	399 41680 59454	3020 86849	56833	20
8.44	396 42173 26126	2983 30326	55849	19
8.46	393 45649 23124	2946 29651	54884	19
8.48	390 52071 49773	2909 83860	53938	19
8.50	387 61403 60282	2873 92006	53010	18
8.52	384 73609 62798	2838 53163	52101	18
8.54	381 88654 18476	2803 66420	51209	17
8.56	379 06502 40574	2769 30885	50335	17
8.58	376 27119 93558	2735 45686	49477	17
8.60	373 50472 92227	2702 09963	48637	16
8.62	370 76528 00859	2669 22878	47812	16
8.64	368 05252 32369	2636 83604	47004	16
8.66	365 36613 47483	2604 91334	46211	15
8.68	362 70579 53931	2573 45276	45433	15
8.70	360 07119 05655	2542 44650	44670	15
8.72	357 46201 02029	2511 88695	43922	14
8.74	354 87794 87097	2481 76662	43188	14
8.76	352 31870 48827	2452 07817	42468	14
8.78	349 78398 18375	2422 81441	41762	13
8.80	347 27348 69364	2393 96827	41069	13
8.82	344 78693 17181	2365 53283	40389	13
8.84	342 32403 18280	2337 50127	39722	13
8.86	339 88450 69506	2309 86694	39068	12
8.88	337 46808 07426	2282 62329	38426	12
8.90	335 07448 07676	2255 76390	37796	12
8.92	332 70343 84317	2229 28247	37177	12
8.94	330 35468 89204	2203 17281	36570	11
8.96	328 02797 11373	2177 42885	35975	11
8.98	325 72302 76426	2152 04465	35390	11
9.00	.(2) 323 43960 45945	2127 01434	34817	11

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
9.00	.(2)323 43960 45945	2127 01434	34817	11
9.02	321 17745 16898	2102 33221	34254	10
9.04	318 93632 21072	2077 99261	33701	10
9.06	316 71597 24507	2053 99002	33158	10
9.08	314 51616 26943	2030 31901	32626	10
9.10	312 33665 61280	2006 97425	32103	10
9.12	310 17721 93041	1983 95053	31589	9
9.14	308 03762 19856	1961 24269	31085	9
9.16	305 91763 70940	1938 84572	30591	9
9.18	303 81704 06596	1916 75465	30105	9
9.20	301 73561 17716	1894 96462	29628	9
9.22	299 67313 25299	1873 47088	29159	8
9.24	297 62938 79970	1852 26872	28699	8
9.26	295 60416 61513	1831 35356	28247	8
9.28	293 59725 78411	1810 72087	27804	8
9.30	291 60845 67398	1790 36623	27368	8
9.32	289 63755 93007	1770 28526	26940	8
9.34	287 68436 47141	1750 47369	26520	7
9.36	285 47867 48646	1730 92733	26107	7
9.38	283 83029 42883	1711 64203	25702	7
9.40	281 92903 01323	1692 61375	25303	7
9.42	280 04469 21139	1673 83851	24912	7
9.44	278 17709 24805	1655 31238	24527	7
9.46	276 32604 59708	1637 03152	24150	7
9.48	274 49136 97764	1618 99217	23779	6
9.50	272 67288 35037	1601 19060	23414	6
9.52	270 87040 91370	1583 62317	23056	6
9.54	269 08377 10020	1566 28631	22704	6
9.56	267 31279 57301	1549 17649	22358	6
9.58	265 55731 22232	1532 29025	22019	6
9.60	263 81715 16187	1515 62420	21685	6
9.62	262 09214 72562	1499 17499	21356	6
9.64	260 38213 46437	1482 93935	21034	6
9.66	258 68695 14246	1466 91405	20717	5
9.68	257 00643 73460	1451 09592	20406	5
9.70	255 34043 42267	1435 48184	20099	5
9.72	253 68878 59257	1420 06876	19798	5
9.74	252 05133 83124	1404 85366	19503	5
9.76	250 42793 92357	1389 83359	19212	5
9.78	248 81843 84950	1375 00564	18926	5
9.80	247 22268 78107	1360 36695	18645	5
9.82	245 64054 07959	1345 91471	18369	5
9.84	244 07185 29282	1331 64616	18097	5
9.86	242 51648 15221	1317 55858	17830	4
9.88	240 97428 57018	1303 64930	17568	4
9.90	239 44512 63746	1289 91571	17310	4
9.92	237 92886 62045	1276 35521	17056	4
9.94	236 42536 95864	1262 96527	16806	4
9.96	234 93450 26209	1249 74339	16561	4
9.98	233 45613 30894	1236 68712	16320	4
10.00	.(2)231 99013 04290	1223 79405	16082	4

x	$\Psi^{(3)}(x)$	δ^2	δ^4	x	$\Psi^{(3)}(x)$	δ^2	δ^4
10.00	.(2)231 99013 04290	1223 79405	16082	11.00	.(2)171 99013 04290	743 78733	8018
10.02	230 53636 57091	1211 06180	15849	11.02	171 01397 52756	736 77508	7913
10.04	229 09471 16073	1198 48805	15620	11.04	170 04518 78731	729 84197	7809
10.06	227 66504 23860	1186 07049	15394	11.06	169 08369 88903	722 98693	7706
10.08	226 24723 38695	1173 80687	15172	11.08	168 12943 97767	716 20896	7606
10.10	224 84116 34218	1161 69497	14954	11.10	167 18234 27528	709 50705	7506
10.12	223 44670 99237	1149 73260	14739	11.12	166 24234 07994	702 88020	7408
10.14	222 06375 37517	1137 91762	14528	11.14	165 30936 76480	696 32743	7312
10.16	220 69217 67559	1126 24792	14320	11.16	164 38335 77708	689 84778	7217
10.18	219 33186 22392	1114 72142	14115	11.18	163 46424 63715	683 44030	7124
10.20	217 98269 49367	1103 33606	13914	11.20	162 55196 93751	677 10406	7031
10.22	216 64456 09948	1092 08985	13716	11.22	161 64646 34194	670 83813	6941
10.24	215 31734 79513	1080 98080	13522	11.24	160 74766 58449	664 64161	6851
10.26	214 00094 47158	1070 00697	13330	11.26	159 85551 46866	658 51359	6763
10.28	212 69524 15501	1059 16644	13142	11.28	158 96994 86641	652 45321	6676
10.30	211 40013 00487	1048 45733	12957	11.30	158 09090 71737	646 45959	6590
10.32	210 11550 31207	1037 87779	12774	11.32	157 21833 02793	640 53187	6506
10.34	208 84125 49706	1027 42599	12595	11.34	156 35215 87034	634 66921	6423
10.36	207 57728 10804	1017 10014	12418	11.36	155 49233 38197	628 87079	6341
10.38	206 32347 81916	1006 89847	12244	11.38	154 63879 76439	623 13577	6260
10.40	205 07974 42876	996 81924	12073	11.40	153 79149 28258	617 46336	6181
10.42	203 84597 85760	986 86075	11905	11.42	152 95036 26413	611 85276	6103
10.44	202 62208 14719	977 02131	11740	11.44	152 11535 09844	606 30319	6025
10.46	201 40795 45809	967 29927	11577	11.46	151 28640 23594	600 81386	5949
10.48	200 20350 06827	957 69300	11416	11.48	150 46346 18730	595 38403	5874
10.50	199 00862 37144	948 20088	11258	11.50	149 64647 52269	590 01294	5800
10.52	197 82322 87550	938 82135	11103	11.52	148 83538 87103	584 69986	5727
10.54	196 64722 20091	929 55286	10950	11.54	148 03014 91922	579 44404	5655
10.56	195 48051 07917	920 39386	10800	11.56	147 23070 41145	574 24478	5585
10.58	194 32300 35129	911 34286	10651	11.58	146 43700 14846	569 10136	5515
10.60	193 17460 96627	902 39837	10506	11.60	145 64898 98684	564 01310	5446
10.62	192 03523 97962	893 55894	10362	11.62	144 86661 83831	558 97929	5378
10.64	190 90480 55192	884 82314	10221	11.64	144 08983 66908	553 99926	5311
10.66	189 78321 94735	876 18953	10082	11.66	143 31859 49910	549 07235	5245
10.68	188 67039 53231	867 65675	9945	11.68	142 55284 40148	544 19789	5180
10.70	187 56624 77403	859 22341	9810	11.70	141 79253 50174	539 37523	5116
10.72	186 47069 23915	850 88817	9677	11.72	141 03761 97724	534 60374	5053
10.74	185 38364 59245	842 64970	9546	11.74	140 28805 05647	529 88277	4991
10.76	184 30502 59544	834 50669	9418	11.76	139 54378 01848	525 21172	4929
10.78	183 23475 10512	826 45786	9291	11.78	138 80476 19220	520 58996	4869
10.80	182 17274 07266	818 50193	9166	11.80	138 07094 95589	516 01688	4809
10.82	181 11891 54214	810 63767	9043	11.82	137 34229 73645	511 49190	4750
10.84	180 07319 64929	802 86384	8922	11.84	136 61876 00891	507 01442	4692
10.86	179 03550 62027	795 17923	8803	11.86	135 90029 29580	502 58387	4635
10.88	178 00576 77049	787 58265	8686	11.88	135 18685 16655	498 19966	4579
10.90	176 98390 50335	780 07293	8570	11.90	134 47839 23696	493 86124	4523
10.92	175 96984 30914	772 64891	8456	11.92	133 77487 16861	489 56805	4468
10.94	174 96350 76384	765 30945	8344	11.94	133 07624 66830	485 31954	4414
10.96	173 96482 52799	758 05343	8234	11.96	132 38247 48753	481 11516	4361
10.98	172 97372 34556	750 87976	8125	11.98	131 69351 42192	476 95440	4308
11.00	.(2)171 99013 04290	743 78733	8018	12.00	(2)131 00932 31071	472 83671	4256

x	$\Psi^{(3)}(x)$	δ^2	δ^4	x	$\Psi^{(3)}(x)$	δ^2	δ^4
12.00	.(2) 131 00932 31071	472 83671	4256	13.00	.(2) 102 07413 79219	312 08412	2380
12.02	130 32986 03620	468 76158	4205	13.02	101 58677 53277	309 60718	2354
12.04	129 65508 52328	464 72850	4154	13.04	101 10250 88053	307 15379	2328
12.06	128 98495 73886	460 73696	4104	13.06	100 62131 38209	304 72368	2302
12.08	128 31943 69140	456 78647	4055	13.08	.(2) 100 14316 60732	302 31659	2277
12.10	127 65848 43041	452 87653	4007	13.10	.(3) 99 66804 14916	299 93228	2252
12.12	127 00206 04596	449 00666	3959	13.12	99 19591 62326	297 57048	2227
12.14	126 35012 66816	445 17638	3912	13.14	98 72676 66785	295 23095	2203
12.16	125 70264 46675	441 38522	3865	13.16	98 26056 94338	292 91345	2179
12.18	125 05957 65056	437 63272	3820	13.18	97 79730 13237	290 61773	2155
12.20	124 42088 46708	433 91841	3774	13.20	97 33693 93908	288 34356	2131
12.22	123 78653 20202	430 24184	3730	13.22	96 87946 08936	286 09070	2108
12.24	123 15648 17879	426 60257	3686	13.24	96 42484 33033	283 85892	2085
12.26	122 53069 75813	423 00016	3642	13.26	95 97306 43024	281 64799	2062
12.28	121 90914 33763	419 43416	3599	13.28	95 52410 17813	279 45768	2040
12.30	121 29178 35130	415 90417	3557	13.30	95 07793 38371	277 28777	2018
12.32	120 67858 26913	412 40974	3515	13.32	94 63453 87706	275 13804	1996
12.34	120 06950 59669	408 95046	3474	13.34	94 19389 50845	273 00827	1974
12.36	119 46451 87472	405 52593	3434	13.36	93 75598 14811	270 89824	1953
12.38	118 86358 67868	402 13573	3393	13.38	93 32077 68601	268 80774	1932
12.40	118 26667 61836	398 77947	3354	13.40	92 88826 03165	266 73655	1911
12.42	117 67375 33750	395 45674	3315	13.42	92 45841 11384	264 68449	1891
12.44	117 08478 51339	392 16716	3276	13.44	92 03120 88051	262 65132	1870
12.46	116 49973 85643	388 91035	3238	13.46	91 60663 29851	260 63686	1850
12.48	115 91858 10982	385 68592	3201	13.48	91 18466 35337	258 64090	1831
12.50	115 34128 04913	382 49350	3164	13.50	90 76528 04913	256 66325	1811
12.52	114 76780 48195	379 33272	3127	13.52	90 34846 40815	254 70371	1792
12.54	114 19812 24748	376 20321	3091	13.54	89 93419 47088	252 76209	1773
12.56	113 63220 21623	373 10462	3056	13.56	89 52245 29569	250 83819	1754
12.58	113 07001 28960	370 03659	3021	13.58	89 11321 95869	248 93182	1735
12.60	112 51152 39956	366 99877	2986	13.60	88 70647 55352	247 04281	1717
12.62	111 95670 50828	363 99080	2952	13.62	88 30220 19116	245 17097	1699
12.64	111 40552 60781	361 01236	2918	13.64	87 90037 99977	243 31611	1681
12.66	110 85795 71970	358 06310	2885	13.66	87 50099 12449	241 47806	1663
12.68	110 31396 89468	355 14268	2852	13.68	87 10401 72728	239 65664	1645
12.70	109 77353 21234	352 25079	2820	13.70	86 70943 98672	237 85167	1628
12.72	109 23661 78080	349 38710	2788	13.72	86 31724 09782	236 06299	1611
12.74	108 70319 73636	346 55128	2756	13.74	85 92740 27192	234 29041	1594
12.76	108 17324 24321	343 74303	2725	13.76	85 53990 73642	232 53377	1577
12.78	107 64672 49308	340 96202	2694	13.78	85 15473 73470	230 79291	1561
12.80	107 12361 70497	338 20796	2664	13.80	84 77187 52590	229 06766	1545
12.82	106 60389 12482	335 48053	2634	13.82	84 39130 38475	227 35785	1529
12.84	106 08752 02520	332 77944	2604	13.84	84 01300 60145	225 66333	1513
12.86	105 57447 70502	330 10439	2575	13.86	83 63696 48149	223 98393	1497
12.88	105 06473 48922	327 45509	2546	13.88	83 26316 34545	222 31951	1481
12.90	104 55826 72851	324 83125	2518	13.90	82 89158 52893	220 66989	1466
12.92	104 05504 79905	322 23259	2489	13.92	82 52221 38229	219 03494	1451
12.94	103 55505 10218	319 65882	2462	13.94	82 15503 27060	217 41449	1436
12.96	103 05825 06412	317 10966	2434	13.96	81 79002 57339	215 80840	1421
12.98	102 56462 13573	314 58485	2407	13.98	81 42717 68459	214 21653	1406
13.00	.(2) 102 07413 79219	312 08412	2380	14.00	.(3) 81 06647 01232	212 63871	1392

x	$\Psi^{(3)}(x)$	δ^2	δ^4	x	$\Psi^{(3)}(x)$	δ^2	δ^4
14.00	.(3) 81 06647 01232	212 63871	1392	15.00	.(3) 65 44797 78283	148 88931	845
14.02	80 70788 97875	211 07482	1378	15.02	65 17832 81837	147 86912	837
14.04	80 35142 02001	209 52470	1363	15.04	64 91015 72303	146 85730	829
14.06	79 99704 58596	207 98821	1350	15.06	64 64345 48499	145 85377	821
14.08	79 64475 14012	206 46522	1336	15.08	64 37821 10073	144 85846	814
14.10	79 29452 15951	204 95559	1322	15.10	64 11441 57492	143 87129	806
14.12	78 94634 13449	203 45918	1309	15.12	63 85205 92041	142 89217	798
14.14	78 60019 56865	201 97585	1295	15.14	63 59113 15807	141 92104	791
14.16	78 25606 97866	200 50548	1282	15.16	63 33162 31677	140 95781	783
14.18	77 91394 89416	199 04793	1269	15.18	63 07352 43328	140 00242	776
14.20	77 57381 85759	197 60307	1256	15.20	62 81682 55221	139 05478	768
14.22	77 23566 42408	196 17077	1243	15.22	62 56151 72593	138 11483	761
14.24	76 89947 16134	194 75090	1231	15.24	62 30759 01447	137 18248	754
14.26	76 56522 64950	193 34335	1218	15.26	62 05503 48550	136 25768	747
14.28	76 23291 48101	191 94798	1206	15.28	61 80384 21420	135 34035	740
14.30	75 90252 26050	190 56467	1194	15.30	61 55400 28326	134 43041	733
14.32	75 57403 60465	189 19330	1182	15.32	61 30550 78272	133 52780	726
14.34	75 24744 14210	187 83375	1170	15.34	61 05834 80998	132 63245	719
14.36	74 92272 51330	186 48591	1158	15.36	60 81251 46970	131 74430	712
14.38	74 59987 37041	185 14964	1147	15.38	60 56799 87372	130 86327	706
14.40	74 27887 37717	183 82485	1135	15.40	60 32479 14100	129 98929	699
14.42	73 95971 20878	182 51141	1124	15.42	60 08288 39758	129 12231	693
14.44	73 64237 55180	181 20922	1113	15.44	59 84226 77647	128 26226	686
14.46	73 32685 10403	179 91815	1102	15.46	59 60293 41762	127 40907	680
14.48	73 01312 57442	178 63810	1091	15.48	59 36487 46785	126 56268	674
14.50	72 70118 68290	177 36895	1080	15.50	59 12808 08075	125 72302	667
14.52	72 39102 16034	176 11061	1069	15.52	58 89254 41668	124 89004	661
14.54	72 08261 74839	174 86296	1059	15.54	58 65825 64264	124 06367	655
14.56	71 77596 19940	173 62590	1048	15.56	58 42520 93227	123 24385	649
14.58	71 47104 27631	172 39932	1038	15.58	58 19339 46576	122 43052	643
14.60	71 16784 75254	171 18312	1028	15.60	57 96280 42976	121 62362	637
14.62	70 86636 41190	169 97720	1018	15.62	57 73343 01738	120 82309	631
14.64	70 56658 04846	168 78145	1008	15.64	57 50526 42809	120 02887	625
14.66	70 26848 46647	167 59578	998	15.66	57 27829 86767	119 24091	620
14.68	69 97206 48026	166 42009	988	15.68	57 05252 54816	118 45914	614
14.70	69 67730 91414	165 25427	978	15.70	56 82793 68779	117 68352	608
14.72	69 38420 60230	164 09824	969	15.72	56 60452 51095	116 91398	603
14.74	69 09274 38870	162 95190	959	15.74	56 38228 24808	116 15047	597
14.76	68 80291 12699	161 81514	950	15.76	56 16120 13567	115 39293	592
14.78	68 51469 68042	160 68789	941	15.78	55 94127 41619	114 64131	587
14.80	68 22808 92175	159 57004	932	15.80	55 72249 33802	113 89555	581
14.82	67 94307 73312	158 46151	922	15.82	55 50485 15540	113 15561	576
14.84	67 65965 00600	157 36220	914	15.84	55 28834 12839	112 42142	571
14.86	67 37779 64108	156 27203	905	15.86	55 07295 52280	111 69295	566
14.88	67 09750 54819	155 19091	896	15.88	54 85868 61015	110 97013	560
14.90	66 81876 64621	154 11874	887	15.90	54 64552 66764	110 25291	555
14.92	66 54156 86296	153 05545	879	15.92	54 43346 97803	109 54124	550
14.94	66 26590 13517	152 00094	870	15.94	54 22250 82966	108 83508	545
14.96	65 99175 40831	150 95514	862	15.96	54 01263 51638	108 13438	540
14.98	65 71911 63659	149 91796	854	15.98	53 80384 33747	107 43907	536
15.00	.(3) 65 44797 78283	148 88931	845	16.00	.(3) 53 59612 59764	106 74913	531

	$\Psi^{(3)}(x)$	δ^2	δ^4		$\Psi^{(3)}(x)$	δ^2	δ^4
16.00	(3) 53 59612 59764	106 74913	531	17.00	.(3)44 44085 25389	78 13874	343
16.02	53 38947 60694	106 06449	526	17.02	44 27983 63048	77 66775	340
16.04	53 18388 68073	105 38511	521	17.04	44 11959 67481	77 20015	337
16.06	52 97935 13963	104 71095	517	17.06	43 96012 91930	76 73593	334
16.08	52 77586 30948	104 04195	512	17.08	43 80142 89972	76 27506	332
16.10	52 57341 52128	103 37808	508	17.10	43 64349 15520	75 81750	329
16.12	52 37200 11117	102 71928	503	17.12	43 48631 22818	75 36323	326
16.14	52 17161 42033	102 06551	499	17.14	43 32988 66438	74 91223	323
16.16	51 97224 79499	101 41672	494	17.16	43 17421 01281	74 46445	321
16.18	51 77389 58638	100 77288	490	17.18	43 01927 82569	74 01988	318
16.20	51 57655 15064	100 13393	485	17.20	42 86508 65844	73 57849	315
16.22	51 38020 84884	99 49984	481	17.22	42 71163 06969	73 14026	313
16.24	51 18486 04687	98 87055	477	17.24	42 55890 62119	72 70515	310
16.26	50 99050 11545	98 24604	473	17.26	42 40690 87784	72 27315	308
16.28	50 79712 43006	97 62625	468	17.28	42 25563 40764	71 84422	305
16.30	50 60472 37093	97 01114	464	17.30	42 10507 78166	71 41834	302
16.32	50 41329 32294	96 40068	460	17.32	41 95523 57401	70 99549	300
16.34	50 22282 67563	95 79483	456	17.34	41 80610 36186	70 57563	298
16.36	50 03331 82316	95 19353	452	17.36	41 65767 72533	70 15875	295
16.38	49 84476 16420	94 59675	448	17.38	41 50995 24756	69 74482	293
16.40	49 65715 10201	94 00446	444	17.40	41 36292 51461	69 33382	290
16.42	49 47048 04428	93 41661	440	17.42	41 21659 11548	68 92572	288
16.44	49 28474 40316	92 83317	437	17.44	41 07094 64208	68 52050	285
16.46	49 09993 59521	92 25409	433	17.46	40 92598 68918	68 11814	283
16.48	48 91605 04135	91 67934	429	17.48	40 78170 85442	67 71860	281
16.50	48 73308 16683	91 10888	425	17.50	40 63810 73825	67 32187	279
16.52	48 55102 40120	90 54267	422	17.52	40 49517 94396	66 92793	276
16.54	48 36987 17823	89 98068	418	17.54	40 35292 07760	66 53675	274
16.56	48 18961 93595	89 42287	414	17.56	40 21132 74799	66 14831	272
16.58	48 01026 11653	88 86920	411	17.58	40 07039 56669	65 76259	270
16.60	47 83179 16632	88 31964	407	17.60	39 93012 14798	65 37956	267
16.62	47 65420 53574	87 77415	404	17.62	39 79050 10882	64 99920	265
16.64	47 47749 67931	87 23269	400	17.64	39 65153 06887	64 62150	263
16.66	47 30166 05557	86 69524	397	17.66	39 51320 65041	64 24643	261
16.68	47 12669 12708	86 16176	393	17.68	39 37552 47839	63 87396	259
16.70	46 95258 36034	85 63221	390	17.70	39 23848 18032	63 50408	257
16.72	46 77933 22582	85 10656	387	17.72	39 10207 38634	63 13677	255
16.74	46 60693 19785	84 58477	383	17.74	38 96629 72914	62 77201	253
16.76	46 43537 75465	84 06682	380	17.76	38 83114 84394	62 40977	250
16.78	46 26466 37827	83 55267	377	17.78	38 69662 36852	62 05003	248
16.80	46 09478 55457	83 04228	374	17.80	38 56271 94312	61 69278	246
16.82	45 92573 77314	82 53564	370	17.82	38 42943 21052	61 33800	244
16.84	45 75751 52736	82 03269	367	17.84	38 29675 81591	60 98566	243
16.86	45 59011 31426	81 53342	364	17.86	38 16469 40696	60 63574	241
16.88	45 42352 63458	81 03779	361	17.88	38 03323 63376	60 28823	239
16.90	45 25774 99269	80 54576	358	17.90	37 90238 14878	59 94311	237
16.92	45 09277 89656	80 05732	355	17.92	37 77212 60692	59 60036	235
16.94	44 92860 85775	79 57243	352	17.94	37 64246 66542	59 25995	233
16.96	44 76523 39137	79 09105	349	17.96	37 51339 98387	58 92187	231
16.98	44 60265 01605	78 61317	346	17.98	37 38492 22419	58 58611	229
17.00	(3) 44 44085 25389	78 13874	343	18.00	.(3)37 25703 05061	58 25263	227

x	$\Psi^{(3)}(x)$	δ^2	δ^4	x	$\Psi^{(3)}(x)$	δ^2	δ^4
	.(3)				.(3)		
18.00	37 25703 05061	58 25263	227	19.00	31 54143 83708	44 14000	154
18.02	37 12972 12967	57 92144	226	19.02	31 43946 13867	43 90252	153
18.04	37 00299 13017	57 59249	224	19.04	31 33792 34279	43 66657	152
18.06	36 87683 72316	57 26579	222	19.06	31 23682 21348	43 43214	151
18.08	36 75125 58193	56 94131	220	19.08	31 13615 51631	43 19923	150
18.10	36 62624 38202	56 61903	219	19.10	31 03592 01837	42 96780	149
18.12	36 50179 80113	56 29893	217	19.12	30 93611 48823	42 73787	147
18.14	36 37791 51918	55 98101	215	19.14	30 83673 69595	42 50940	146
18.16	36 25459 21823	55 66524	213	19.16	30 73778 41308	42 28240	145
18.18	36 13182 58253	55 35160	212	19.18	30 63925 41261	42 05686	144
18.20	36 00961 29842	55 04008	210	19.20	30 54114 46900	41 83275	143
18.22	35 88795 05438	54 73066	208	19.22	30 44345 35815	41 61008	142
18.24	35 76683 54100	54 42332	207	19.24	30 34617 85737	41 38883	141
18.26	35 64626 45094	54 11805	205	19.26	30 24931 74543	41 16899	140
18.28	35 52623 47893	53 81484	204	19.28	30 15286 80247	40 95054	139
18.30	35 40674 32176	53 51366	202	19.30	30 05682 81006	40 73349	138
18.32	35 28778 67824	53 21450	200	19.32	29 96119 55113	40 51781	137
18.34	35 16936 24922	52 91734	199	19.34	29 86596 81001	40 30351	136
18.36	35 05146 73754	52 62217	197	19.36	29 77114 37241	40 09056	135
18.38	34 93409 84802	52 32898	196	19.38	29 67672 02536	39 87896	134
18.40	34 81725 28749	52 03774	194	19.40	29 58269 55727	39 66870	133
18.42	34 70092 76470	51 74845	193	19.42	29 48906 75788	39 45976	132
18.44	34 58511 99035	51 46108	191	19.44	29 39583 41825	39 25215	131
18.46	34 46982 67703	51 17562	190	19.46	29 30299 33078	39 04585	130
18.48	34 35504 53943	50 89207	188	19.48	29 21054 28915	38 84084	129
18.50	34 24077 29386	50 61039	187	19.50	29 11848 08836	38 63713	128
18.52	34 12700 65867	50 33059	185	19.52	29 02680 52470	38 43469	127
18.54	34 01374 35407	50 05263	184	19.54	28 93551 39573	38 23353	126
18.56	33 90098 10210	49 77652	183	19.56	28 84460 50028	38 03363	125
18.58	33 78871 62665	49 50223	181	19.58	28 75407 63846	37 83498	124
18.60	33 67694 65344	49 22976	180	19.60	28 66392 61162	37 63757	123
18.62	33 56566 90998	48 95908	178	19.62	28 57415 22235	37 44140	123
18.64	33 45488 12560	48 69018	177	19.64	28 48475 27448	37 24645	122
18.66	33 34458 03141	48 42306	176	19.66	28 39572 57306	37 05272	121
18.68	33 23476 36026	48 15769	174	19.68	28 30706 92437	36 86020	120
18.70	33 12542 84681	47 89406	173	19.70	28 21878 13588	36 66888	119
18.72	33 01657 22742	47 63216	172	19.72	28 13086 01627	36 47875	118
18.74	32 90819 24019	47 37198	170	19.74	28 04330 37540	36 28980	117
18.76	32 80028 62494	47 11350	169	19.76	27 95611 02433	36 10202	116
18.78	32 69285 12319	46 85672	168	19.78	27 86927 77528	35 91540	116
18.80	32 58588 47817	46 60161	166	19.80	27 78280 44164	35 72995	114
18.82	32 47938 43475	46 34816	165	19.82	27 69668 83794	35 54564	114
18.84	32 37334 73949	46 09637	164	19.84	27 61092 77988	35 36247	113
18.86	32 26777 14060	45 84621	163	19.86	27 52552 08428	35 18043	112
18.88	32 16265 38792	45 59768	161	19.88	27 44046 56911	34 99951	112
18.90	32 05799 23292	45 35077	160	19.90	27 35576 05346	34 81971	111
18.92	31 95378 42870	45 10546	159	19.92	27 27140 35751	34 64102	110
18.94	31 85002 72994	44 86174	158	19.94	27 18739 30258	34 46342	109
18.96	31 74671 89291	44 61960	157	19.96	27 10372 71108	34 28692	108
18.98	31 64385 67548	44 37902	155	19.98	27 02040 40649	34 11150	108
19.00	31 54143 83708	44 14000	154	20.00	26 93742 21340	33 93715	107
	.(3)				.(3)		

TABLE 24

THE PENTAGAMMA FUNCTION

Description: $\Psi^{(3)}(x)$ to 17 and 18 decimal places with central differences from $x = 20.0$ to $x = 100.0$ by increments of .1.

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
20.0	.(3)2693 74221 33964	84848 22463	66 76388	9799
20.1	2652 75685 55816	82708 40114	64 41948	9360
20.2	2612 59858 17781	80632 99712	62 16867	8941
20.3	2573 24663 79458	78619 76178	60 00728	8544
20.4	2534 68089 17312	76666 53371	57 93132	8166
20.5	2496 88181 08538	74771 23695	55 93702	7806
20.6	2459 83044 23458	72931 87722	54 02078	7464
20.7	2423 50839 26100	71146 53825	52 17918	7139
20.8	2387 89780 82568	69413 37847	50 40897	6829
20.9	2352 98135 76883	67730 62766	48 70705	6534
21.0	2318 74221 33964	66096 58391	47 07048	6253
21.1	2285 16403 49436	64509 61063	45 49643	5986
21.2	2252 23095 25970	62968 13378	43 98225	5731
21.3	2219 92755 15883	61470 63918	42 52537	5488
21.4	2188 23885 69713	60015 66995	41 12338	5257
21.5	2157 15031 90539	58601 82410	39 77395	5036
21.6	2126 64779 93774	57227 75220	38 47489	4826
21.7	2096 71755 72231	55892 15519	37 22408	4625
21.8	2067 34623 66206	54593 78226	36 01952	4433
21.9	2038 52085 38408	53331 42884	34 85929	4251
22.0	2010 22878 53494	52103 93472	33 74158	4076
22.1	1982 45775 62052	50910 18217	32 66462	3910
22.2	1955 19582 88828	49749 09424	31 62677	3751
22.3	1928 43139 25027	48619 63308	30 62642	3599
22.4	1902 15315 24535	47520 79835	29 66207	3454
22.5	1876 35012 03877	46451 62568	28 73226	3316
22.6	1851 01160 45788	45411 18528	27 83561	3183
22.7	1826 12720 06226	44398 58048	26 47078	3057
22.8	1801 68678 24712	43412 94647	26 13653	2936
22.9	1777 68049 37845	42453 44898	25 33163	2820
23.0	1754 09873 95875	41519 28313	24 55493	2709
23.1	1730 93217 82219	40609 67221	23 80533	2603
23.2	1708 17171 35784	39723 86662	23 08176	2502
23.3	1685 80848 76011	38861 14279	22 38321	2406
23.4	1663 83387 30517	38020 80217	21 70872	2312
23.5	1642 23946 65240	37202 17027	21 05735	2224
23.6	1621 01708 16990	36404 59572	20 42822	2139
23.7	1600 15874 28312	35627 44939	19 82048	2057
23.8	1579 65667 84574	34870 12353	19 23331	1979
23.9	1559 50331 53188	34132 03098	18 66593	1905
24.0	1539 69127 24901	33412 60436	18 11760	1833
24.1	1520 21335 57050	32711 29533	17 58759	1764
24.2	1501 06255 18732	32027 57389	17 07523	1699
24.3	1482 23202 37803	31360 92768	16 57986	1635
24.4	1463 71510 49642	30710 86134	16 10084	1575
24.5	1445 50529 47615	30076 89582	15 63757	1517
24.6	1427 59625 35170	29458 56787	15 18946	1461
24.7	1409 98179 79513	28855 42939	14 75598	1408
24.8	1392 65589 66794	28267 04688	14 33657	1357
24.9	1375 61266 58764	27693 00094	13 93072	1308
25.0	.(3)1358 84636 50827	27132 88572	13 53796	1260

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
25.0	.(3)1358 84636 50827	27132 88572	13 53796	1260
25.1	1342 35139 31463	26586 30846	13 15779	1215
25.2	1326 12228 42946	26052 88900	12 78978	1171
25.3	1310 15370 43328	25532 25931	12 43348	1130
25.4	1294 44044 69640	25024 06311	12 08848	1090
25.5	1278 97743 02264	24527 95538	11 75438	1051
25.6	1263 75969 30425	24043 60203	11 43078	1014
25.7	1248 78239 18790	23570 67946	11 11733	978
25.8	1234 04079 75100	23108 87422	10 81366	944
25.9	1219 53029 18833	22657 88263	10 51942	912
26.0	1205 24636 50827	22217 41046	10 23431	879
26.1	1191 18461 23868	21787 17261	9 95799	849
26.2	1177 34073 14170	21366 89274	9 69016	820
26.3	1163 71051 93745	20956 30303	9 43053	792
26.4	1150 28987 03623	20555 14386	9 17882	765
26.5	1137 07477 27887	20163 16351	8 93476	739
26.6	1124 06130 68502	19780 11793	8 69809	714
26.7	1111 24564 20910	19405 77043	8 46855	690
26.8	1098 62403 50360	19039 89149	8 24591	666
26.9	1086 19282 68960	18682 25846	8 02994	644
27.0	1073 94844 13406	18332 65537	7 82040	622
27.1	1061 88738 23389	17990 87268	7 61709	602
27.2	1050 00623 20641	17656 70708	7 41979	582
27.3	1038 30164 88601	17329 96127	7 22832	562
27.4	1026 77036 52688	17010 44378	7 04247	544
27.5	1015 40918 61154	16697 96876	6 86206	526
27.6	1004 21498 66495	16392 35579	6 68691	509
27.7	.(4)993 18471 07416	16093 42974	6 51685	492
27.8	982 31536 91310	15801 02053	6 35172	476
27.9	971 60403 77258	15514 96305	6 19135	461
28.0	961 04785 59510	15235 09691	6 03559	446
28.1	950 64402 51454	14961 26636	5 88429	432
28.2	940 38980 70033	14693 32011	5 73731	418
28.3	930 28252 20624	14431 11117	5 59452	405
28.4	920 31954 82331	14174 49675	5 45577	392
28.5	910 49831 93713	13923 33809	5 32093	379
28.6	900 81632 38903	13677 50036	5 18989	367
28.7	891 27110 34130	13436 85253	5 06253	356
28.8	881 86025 14610	13201 26723	4 93873	345
28.9	872 58141 21813	12970 62065	4 81837	334
29.0	863 43227 91081	12744 79245	4 70135	324
29.1	854 41059 39594	12523 66560	4 58757	313
29.2	845 51414 54666	12307 12632	4 47693	304
29.3	836 74076 82372	12095 06398	4 36932	294
29.4	828 08834 16474	11887 37095	4 26466	286
29.5	819 55478 87672	11683 94259	4 16286	277
29.6	811 13807 53129	11484 67709	4 06382	269
29.7	802 83620 86296	11287 47542	3 96747	260
29.8	794 64723 67005	11098 24121	3 87372	252
29.9	786 56924 71834	10910 88072	3 78249	245
30.0	.(4)778 60036 64736	10727 30272	3 69371	237

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
30.0	.(4) 778 60036 64736	10727 30272	3 69371	237
30.1	770 73875 87910	10547 41842	3 60730	230
30.2	762 98262 52926	10371 14143	3 52319	223
30.3	755 33020 32084	10198 38762	3 44132	217
30.4	747 77976 50005	10029 07514	3 36161	210
30.5	740 32961 75439	9863 12426	3 28401	204
30.6	732 97810 13299	9700 45740	3 20845	198
30.7	725 72358 96900	9540 99899	3 13487	192
30.8	718 56448 80399	9384 67544	3 06321	186
30.9	711 49923 31442	9231 41510	2 99341	182
31.0	704 52629 23995	9081 14817	2 92543	175
31.1	697 64416 31365	8933 80667	2 85920	171
31.2	690 85137 19402	8789 32437	2 79469	166
31.3	684 14647 39875	8647 63676	2 73183	161
31.4	677 52805 24024	8508 68098	2 67058	156
31.5	670 99471 76270	8372 39578	2 61090	152
31.6	664 54510 68094	8238 72148	2 55273	148
31.7	658 17788 32066	8107 59991	2 49605	143
31.8	651 89173 56029	7978 97439	2 44079	139
31.9	645 68537 77431	7852 78965	2 38693	135
32.0	639 55754 77798	7728 99185	2 33442	132
32.1	633 50700 77350	7607 52848	2 28323	128
32.2	627 53254 29750	7488 34833	2 23332	124
32.3	621 63296 16983	7371 40151	2 18465	121
32.4	615 80709 44367	7256 63933	2 13719	117
32.5	610 05379 35683	7144 01435	2 09090	114
32.6	604 37193 28435	7033 48027	2 04576	111
32.7	598 76040 69213	6924 99195	2 00173	108
32.8	593 21813 09186	6818 50535	1 95877	105
32.9	587 74403 99694	6713 97752	1 91687	102
33.0	582 33708 87954	6611 36657	1 87598	99
33.1	576 99625 12870	6510 63159	1 83610	97
33.2	571 72052 00946	6411 73271	1 79717	94
33.3	566 50890 62292	6314 63101	1 75919	92
33.4	561 36043 86740	6219 28849	1 72212	89
33.5	556 27416 40036	6125 66809	1 68594	87
33.6	551 24914 60141	6033 73364	1 65063	84
33.7	546 28446 53610	5943 44981	1 61616	82
33.8	541 37921 92060	5854 78216	1 58251	80
33.9	536 53252 08725	5767 69700	1 54966	78
34.0	531 74349 95091	5682 16151	1 51759	76
34.1	527 01129 97607	5598 14360	1 48627	73
34.2	522 33508 14483	5515 61196	1 45569	72
34.3	517 71401 92555	5434 53601	1 42582	70
34.4	513 14730 24228	5354 88588	1 39665	68
34.5	508 63413 44489	5276 63240	1 36817	66
34.6	504 17373 27989	5199 74709	1 34034	65
34.7	499 76532 86198	5124 20213	1 31316	63
34.8	495 40816 64620	5049 97032	1 28661	61
34.9	491 10150 40074	4977 02514	1 26067	60
35.0	.(4) 486 84461 18042	4905 34062	1 23533	58

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
35.0	.(4)486 84461 18042	4905 34062	1 23533	58
35.1	482 63677 30072	4834 89144	1 21057	57
35.2	478 47728 31246	4765 65282	1 18637	56
35.3	474 36544 97701	4697 60056	1 16272	54
35.4	470 30059 24213	4630 71103	1 13961	52
35.5	466 28204 21827	4564 96111	1 11703	51
35.6	462 30914 15552	4500 32821	1 09495	50
35.7	458 38124 42099	4436 79026	1 07337	49
35.8	454 49771 47671	4374 32569	1 05228	47
35.9	450 65792 85813	4312 91339	1 03165	46
36.0	446 86127 15293	4252 53274	1 01149	45
36.1	443 10713 98048	4193 16359	99178	44
36.2	439 39493 97161	4134 78622	97251	43
36.3	435 72408 74896	4077 38135	95366	42
36.4	432 09400 90767	4020 93015	93523	41
36.5	428 50413 99652	3965 41417	91720	40
36.6	424 95392 49955	3910 81539	89957	39
36.7	421 44281 81797	3857 11619	88233	38
36.8	417 97028 25257	3804 29932	86546	37
36.9	414 53578 98650	3752 34791	84896	36
37.0	411 13882 06834	3701 24546	83282	35
37.1	407 77886 39564	3650 97584	81703	34
37.2	404 45541 69879	3601 52325	80158	33
37.3	401 16798 52519	3552 87225	78647	32
37.4	397 91608 22384	3505 00771	77167	32
37.5	394 69922 93019	3457 91484	75720	31
37.6	391 51695 55140	3411 57918	74303	30
37.7	388 36879 75178	3365 98655	72917	30
37.8	385 25429 93871	3321 12309	71560	29
37.9	382 17301 24873	3276 97524	70232	28
38.0	379 12449 53400	3233 52970	68932	27
38.1	376 10831 34896	3190 77349	67660	27
38.2	373 12403 93742	3148 69388	66414	26
38.3	370 17125 21976	3107 27841	65194	25
38.4	367 24953 78050	3066 51488	64000	25
38.5	364 35848 85612	3026 39135	62831	24
38.6	361 49770 32309	2986 89613	61686	24
38.7	358 66678 68618	2948 01777	60565	23
38.8	355 86535 06705	2909 74507	59467	23
38.9	353 09301 19300	2872 06704	58392	22
39.0	350 34939 38598	2834 97293	57339	22
39.1	347 63412 55189	2798 45222	56308	21
39.2	344 94684 17002	2762 49458	55298	21
39.3	342 28718 28272	2727 08992	54308	20
39.4	339 65479 48534	2692 22834	53339	20
39.5	337 04932 91630	2657 90014	52389	19
39.6	334 47044 24740	2624 09584	51459	19
39.7	331 91779 67435	2590 80613	50547	18
39.8	329 39105 90743	2558 02189	49654	18
39.9	326 88990 16240	2525 73419	48779	18
40.0	.(4)324 41400 15156	2493 93428	47921	17

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
40.0	.(4)324 41400 15156	2493 93428	47921	17
40.1	321 96304 07501	2462 61359	47081	17
40.2	319 53670 61205	2431 76371	46258	16
40.3	317 13468 91279	2401 37640	45450	16
40.4	314 75668 58993	2371 44359	44659	16
40.5	312 40239 71066	2341 95738	43884	15
40.6	310 07152 78877	2312 91001	43124	15
40.7	307 76378 77688	2284 29387	42379	15
40.8	305 47889 05887	2256 10153	41648	14
40.9	303 21655 44239	2228 32566	40932	14
41.0	300 97650 15156	2200 95912	40230	14
41.1	298 75845 81987	2173 99489	39542	13
41.2	296 56215 48306	2147 42608	38867	13
41.3	294 38732 57233	2121 24594	38206	13
41.4	292 23370 90754	2095 44786	37557	13
41.5	290 10104 69062	2070 02534	36921	12
41.6	287 98908 49903	2044 97203	36296	12
41.7	285 89757 27948	2020 28168	35684	12
41.8	283 82626 34162	1995 94819	35084	11
41.9	281 77491 35195	1971 96553	34496	11
42.0	279 74328 32781	1948 32783	33918	11
42.1	277 73113 63151	1925 02931	33352	11
42.2	275 73823 96451	1902 06430	32796	11
42.3	273 76436 36181	1879 42726	32251	10
42.4	271 80928 18638	1857 11272	31716	10
42.5	269 87277 12366	1835 11535	31192	10
42.6	267 95461 17631	1813 42989	30677	10
42.7	266 05458 65884	1792 05120	30172	9
42.8	264 17248 19258	1770 97423	29676	9
42.9	262 30808 70054	1750 19402	29190	9
43.0	260 46119 40252	1729 70570	28713	9
43.1	258 63159 81020	1709 50451	28244	9
43.2	256 81909 72239	1689 58576	27785	9
43.3	255 02349 22035	1669 94486	27334	8
43.4	253 24458 66316	1650 57730	26891	8
43.5	251 48218 68327	1631 47864	26456	8
43.6	249 73610 18202	1612 64454	26030	8
43.7	248 00614 32531	1594 07074	25611	8
43.8	246 29212 53934	1575 75305	25200	7
43.9	244 59386 50642	1557 68736	24796	8
44.0	242 91118 16086	1539 86963	24400	7
44.1	241 24389 68493	1522 29591	24011	7
44.2	239 59183 50490	1504 96229	23629	7
44.3	237 95482 28717	1487 86497	23254	7
44.4	236 33268 93441	1471 00020	22886	7
44.5	234 72526 58185	1454 36429	22525	6
44.6	233 13238 59359	1437 95363	22170	7
44.7	231 55388 55895	1421 76467	21821	6
44.8	229 98960 28899	1405 79392	21479	6
44.9	228 43937 81294	1390 03796	21143	6
45.0	.(4)226 90305 37485	1374 49342	20812	6

x	$\Psi^{(3)}(x)$	δ^2	δ^4	δ^6
45.0	.(4)226 90305 37485	1374 49342	20812	6
45.1	225 38047 43017	1359 15701	20488	6
45.2	223 87148 64250	1344 02547	20169	6
45.3	222 37593 88031	1329 09564	19857	5
45.4	220 89368 21376	1314 36436	19549	6
45.5	219 42456 91156	1299 82858	19247	5
45.6	217 96845 43795	1285 48527	18950	5
45.7	216 52519 44961	1271 33146	18659	5
45.8	215 09464 79273	1257 36424	18373	5
45.9	213 67667 50009	1243 58075	18091	5
46.0	212 27113 78820	1229 97817	17815	5
46.1	210 87790 05448	1216 55374	17543	5
46.2	209 49682 87451	1203 30475	17276	5
46.3	208 12778 99928	1190 22851	17014	5
46.4	206 77065 35256	1177 32242	16756	4
46.5	205 42529 02827	1164 58389	16503	4
46.6	204 09157 28787	1152 01040	16254	4
46.7	202 76937 55787	1139 59945	16010	4
46.8	201 45857 42732	1127 34859	15769	4
46.9	200 15904 64536	1115 25543	15533	4
47.0	198 87067 11884	1103 31760	15301	4
47.1	197 59332 90992	1091 53278	15072	4
47.2	196 32690 23378	1079 89868	14848	4
47.3	195 07127 45631	1068 41306	14627	4
47.4	193 82633 09190	1057 07371	14411	4
47.5	192 59195 80121	1045 87847	14197	3
47.6	191 36804 38899	1034 82521	13988	4
47.7	190 15447 80197	1023 91182	13782	3
47.8	188 95115 12677	1013 13624	13579	3
47.9	187 75795 58781	1002 49646	13380	3
48.0	186 57478 54532	991 99048	13184	3
48.1	185 40153 49331	981 61634	12991	3
48.2	184 23810 05764	971 37211	12802	3
48.3	183 08437 99407	961 25590	12616	3
48.4	181 94027 18640	951 26584	12433	3
48.5	180 80567 64458	941 40012	12252	3
48.6	179 68049 50287	931 65691	12075	3
48.7	178 56463 01808	922 03446	11901	3
48.8	177 45798 56775	912 53102	11730	3
48.9	176 36046 64845	903 14488	11561	3
49.0	175 27197 87403	893 87435	11395	3
49.1	174 19242 97395	884 71777	11232	3
49.2	173 12172 79165	875 67351	11072	3
49.3	172 05978 28285	866 73998	10914	3
49.4	171 00650 51404	857 91558	10759	2
49.5	169 96180 66080	849 19877	10606	2
49.6	168 92560 00633	840 58802	10456	2
49.7	167 89779 93987	832 08183	10308	2
49.8	166 87831 95525	823 67872	10163	2
49.9	165 86707 64935	815 37724	10019	2
50.0	.(4)164 86398 72068	807 17595	9879	2

x	$\Psi^{(3)}(x)$	δ^2	δ^4	x	$\Psi^{(3)}(x)$	δ^2	δ^4
	.(4)				.(4)		
50.0	1648 63987 20682	8071 75945	98787	55.0	1235 28565 12915	4989 41557	50377
50.1	1638 68969 67960	7990 73443	97402	55.1	1228 51154 73343	4943 89947	49735
50.2	1628 81942 88680	7910 68342	96038	55.2	1221 78688 23718	4898 88071	49101
50.3	1619 02826 77743	7831 59279	94697	55.3	1215 11120 62164	4854 35297	48478
50.4	1609 31542 26085	7753 44913	93376	55.4	1208 48407 35906	4810 31000	47864
50.5	1599 68011 19339	7676 23923	92078	55.5	1201 90504 40648	4766 74568	47258
50.6	1590 12156 36517	7599 95012	90799	55.6	1195 37368 19959	4723 65393	46660
50.7	1580 63901 48707	7524 56899	89541	55.7	1188 88955 64662	4681 02878	46073
50.8	1571 23171 17796	7450 08326	88303	55.8	1182 45224 12243	4638 86436	45492
50.9	1561 89890 95211	7376 48057	87083	55.9	1176 06131 46261	4597 15486	44921
51.0	1552 63987 20682	7303 74870	85884	56.0	1169 71635 95765	4555 89458	44357
51.1	1543 45387 21024	7231 87568	84703	56.1	1163 41696 34726	4515 07786	43802
51.2	1534 34019 08934	7160 84969	83541	56.2	1157 16271 81473	4474 69916	43255
51.3	1525 29811 81812	7090 65910	82398	56.3	1150 95321 98137	4434 75302	42715
51.4	1516 32695 20600	7021 29248	81271	56.4	1144 78806 90102	4395 23402	42183
51.5	1507 42599 88636	6952 73857	80162	56.5	1138 66687 05469	4356 13685	41659
51.6	1498 59457 30528	6884 98627	79070	56.6	1132 58923 34521	4317 45627	41141
51.7	1489 83199 71048	6818 02468	77996	56.7	1126 55477 09200	4279 18711	40633
51.8	1481 13760 14035	6751 84304	76938	56.8	1120 56310 02590	4241 32427	40130
51.9	1472 51072 41326	6686 43079	75897	56.9	1114 61384 28406	4203 86273	39634
52.0	1463 95071 11697	6621 77751	74871	57.0	1108 70662 40496	4166 79753	39146
52.1	1455 45691 59818	6557 87293	73862	57.1	1102 84107 32339	4130 12380	38665
52.2	1447 02869 95232	6494 70698	72867	57.2	1097 01682 36561	4093 83671	38190
52.3	1438 66543 01343	6432 26970	71889	57.3	1091 23351 24453	4057 93152	37722
52.4	1430 36648 34425	6370 55131	70925	57.4	1085 49078 05498	4022 40355	37260
52.5	1422 13124 22637	6309 54217	69976	57.5	1079 78827 26897	3987 24818	36806
52.6	1413 95909 65066	6249 23279	69041	57.6	1074 12563 73113	3952 46086	36356
52.7	1405 84944 30774	6189 61383	68121	57.7	1068 50252 65416	3918 03711	35914
52.8	1397 80168 57864	6130 67608	67215	57.8	1062 91859 61429	3883 97249	35478
52.9	1389 81523 52563	6072 41047	66322	57.9	1057 37350 54692	3850 26266	35047
53.0	1381 88950 88308	6014 80809	65443	58.0	1051 86691 74221	3816 90329	34623
53.1	1374 02393 04863	5957 86014	64578	58.1	1046 39849 84078	3783 89016	34204
53.2	1366 21793 07432	5901 55797	63724	58.2	1040 96791 82952	3751 21907	33792
53.3	1358 47094 65798	5845 89305	62885	58.3	1035 57485 03732	3718 88589	33385
53.4	1350 78242 13469	5790 85697	62058	58.4	1030 21897 13102	3686 88656	32983
53.5	1343 15180 46837	5736 44147	61243	58.5	1024 89996 11128	3655 21707	32583
53.6	1335 57855 24351	5682 63840	60440	58.6	1019 61750 30860	3623 87345	32196
53.7	1328 06212 65704	5629 43972	59650	58.7	1014 37128 37937	3592 85179	31812
53.8	1320 60199 51030	5576 83755	58871	58.8	1009 16099 30193	3562 14824	31432
53.9	1313 19763 20111	5524 82408	58103	58.9	1003 98632 37273	3531 75902	31056
54.0	1305 84851 71600	5473 39166	57348	59.0	998 84697 20255	3501 68036	30688
54.1	1298 55413 62254	5422 53270	56603	59.1	993 74263 71274	3471 90858	30323
54.2	1291 31398 06178	5372 23978	55870	59.2	988 67302 13150	3442 44002	29963
54.3	1284 12754 74079	5322 50555	55147	59.3	983 63782 99027	3413 27109	29608
54.4	1276 99433 92535	5273 32279	54435	59.4	978 63677 12014	3384 39824	29258
54.5	1269 91386 43271	5224 68438	53734	59.5	973 66955 64824	3355 81797	28913
54.6	1262 88563 62444	5176 58331	53041	59.6	968 73589 99431	3327 52683	28571
54.7	1255 90917 39948	5129 01264	52361	59.7	963 83551 86720	3299 52140	28236
54.8	1248 98400 18716	5081 96559	51690	59.8	958 96813 26150	3271 79834	27905
54.9	1242 10964 94044	5035 43544	51028	59.9	954 13346 45413	3244 35433	27577
55.0	1235 28565 12915	4989 41557	50377	60.0	949 33124 00109	3217 18608	27255
	.(4)				.(5)		

x	$\Psi^{(3)}(x)$	δ^2	δ^4	x	$\Psi^{(3)}(x)$	δ^2	δ^4
	.(5)				.(5)		
60.0	949 33124 00109	3217 18608	27255	65.0	745 24430 14773	2149 23953	15495
60.1	944 56118 73414	3190 29038	26936	65.1	741 78904 49899	2132 65791	15326
60.2	939 82303 75756	3163 66404	26622	65.2	738 35511 50816	2116 22955	15166
60.3	935 11652 44502	3137 30393	26312	65.3	734 94234 74690	2099 95285	14998
60.4	930 44138 43641	3111 20692	26006	65.4	731 55057 93850	2083 82613	14838
60.5	925 79735 63472	3085 36998	25705	65.5	728 17964 95622	2067 84779	14680
60.6	921 18418 20302	3059 79009	25406	65.6	724 82939 82174	2052 01625	14522
60.7	916 60160 56141	3034 46425	25113	65.7	721 49966 70351	2036 32993	14367
60.8	912 04937 38404	3009 38954	24822	65.8	718 19029 91521	2020 78728	14214
60.9	907 52723 59623	2984 56306	24536	65.9	714 90113 91419	2005 38677	14062
61.0	903 03494 37146	2959 98193	24254	66.0	711 63203 29994	1990 12688	13914
61.1	898 57225 12864	2935 64335	23975	66.1	708 38282 81256	1975 00613	13765
61.2	894 13891 52915	2911 54451	23700	66.2	705 15337 33132	1960 02302	13619
61.3	889 73469 47418	2887 68268	23429	66.3	701 94351 87309	1945 17610	13475
61.4	885 35935 10188	2864 05512	23161	66.4	698 75311 59097	1930 46394	13332
61.5	881 01264 78471	2840 65918	22896	66.5	695 58201 77279	1915 88510	13192
61.6	876 69435 12671	2817 49219	22635	66.6	692 43007 83970	1901 43818	13052
61.7	872 40422 96091	2794 55156	22378	66.7	689 29715 34479	1887 12178	12915
61.8	868 14205 34666	2771 83470	22124	66.8	686 18309 97166	1872 93453	12780
61.9	863 90759 56712	2749 33908	21872	66.9	683 08777 53306	1858 87509	12644
62.0	859 70063 12666	2727 06218	21625	67.0	680 01103 96954	1844 94209	12514
62.1	855 52093 74838	2705 00153	21380	67.1	676 95275 34812	1831 13423	12382
62.2	851 36829 37162	2683 15468	21138	67.2	673 91277 86092	1817 45019	12252
62.3	847 24248 14954	2661 51921	20901	67.3	670 89097 82392	1803 88867	12125
62.4	843 14328 44668	2640 09275	20666	67.4	667 88721 67558	1790 44841	11998
62.5	839 07048 83656	2618 87295	20433	67.5	664 90135 97565	1777 12813	11874
62.6	835 02388 09940	2597 85748	20204	67.6	661 93327 40385	1763 92659	11750
62.7	831 00325 21972	2577 04405	19978	67.7	658 98282 75863	1750 84255	11629
62.8	827 00839 38409	2556 43040	19755	67.8	656 04988 95596	1737 87480	11508
62.9	823 03909 97886	2536 01430	19533	67.9	653 13433 02810	1725 02214	11389
63.0	819 09516 58792	2515 79353	19317	68.0	650 23602 12237	1712 28337	11272
63.1	815 17638 99051	2495 76592	19101	68.1	647 35483 50002	1699 65731	11156
63.2	811 28257 15903	2475 92933	18889	68.2	644 49064 53498	1687 14282	11041
63.3	807 41351 25687	2456 28163	18680	68.3	641 64332 71275	1674 73873	10928
63.4	803 56901 63635	2436 82073	18473	68.4	638 81275 62926	1662 44391	10815
63.5	799 74888 83656	2417 54456	18269	68.5	635 99880 98968	1650 25725	10705
63.6	795 95293 58133	2398 45107	18066	68.6	633 20136 60735	1638 17763	10595
63.7	792 18096 77716	2379 53825	17868	68.7	630 42030 40264	1626 20396	10486
63.8	788 43279 51123	2360 80410	17672	68.8	627 65550 40189	1614 33515	10380
63.9	784 70823 04941	2342 24667	17476	68.9	624 90684 73629	1602 57014	10273
64.0	781 00708 83426	2323 86401	17285	69.0	622 17421 64082	1590 90786	10170
64.1	777 32918 48311	2305 65419	17096	69.1	619 45749 45321	1579 34728	10066
64.2	773 67433 78616	2287 61534	16909	69.2	616 75656 61288	1567 88736	9964
64.3	770 04236 70455	2269 74558	16724	69.3	614 07131 65992	1556 52708	9863
64.4	766 43309 36852	2252 04306	16542	69.4	611 40163 23403	1545 26543	9763
64.5	762 84634 07556	2234 50596	16362	69.5	608 74740 07356	1534 10141	9665
64.6	759 28193 28856	2217 13249	16184	69.6	606 10851 01451	1523 03403	9567
64.7	755 73969 63404	2199 92085	16009	69.7	603 48484 98949	1512 06233	9471
64.8	752 21945 90038	2182 86929	15836	69.8	600 87631 02679	1501 18534	9376
64.9	748 72105 03601	2165 97610	15663	69.9	598 28278 24944	1490 40210	9281
65.0	745 24430 14773	2149 23953	15495	70.0	595 70415 87418	1479 71167	9188
	.(5)				.(5)		

x	$\Psi^{(3)}(x)$	δ^2	δ^4	x	$\Psi^{(3)}(x)$	δ^2	δ^4
	.(5)				.(5)		
70.0	595 70415 87418	1479 71167	9188	75.0	483 63982 79012	1045 52408	5651
70.1	593 14033 21060	1469 11313	9097	75.1	481 69761 78346	1038 53593	5597
70.2	590 59119 66015	1458 60556	9005	75.2	479 76579 31272	1031 60374	5545
70.3	588 05664 71526	1448 18803	8916	75.3	477 84428 44573	1024 72701	5494
70.4	585 53657 95840	1437 85967	8826	75.4	475 93302 30575	1017 90521	5442
70.5	583 03089 06121	1427 61956	8739	75.5	474 03194 07098	1011 13784	5392
70.6	580 53947 78358	1417 46685	8652	75.6	472 14096 97404	1004 42438	5341
70.7	578 06223 97280	1407 40065	8566	75.7	470 26004 30149	997 76434	5292
70.8	575 59907 56266	1397 42011	8482	75.8	468 38909 39327	991 15722	5244
70.9	573 14988 57264	1387 52439	8396	75.9	466 52805 64228	984 60254	5194
71.0	570 71457 10700	1377 71263	8315	76.0	464 67686 49382	978 09980	5147
71.1	568 29303 35400	1367 98401	8232	76.1	462 83545 44516	971 64853	5100
71.2	565 88517 58500	1358 33772	8151	76.2	461 00376 04503	965 24825	5052
71.3	563 49090 15372	1348 77293	8071	76.3	459 18171 89314	958 89849	5006
71.4	561 11011 49537	1339 28885	7991	76.4	457 36926 63975	952 59880	4960
71.5	558 74272 12587	1329 88468	7914	76.5	455 56633 98515	946 34870	4915
71.6	556 38862 64105	1320 55964	7834	76.6	453 77287 67926	940 14775	4869
71.7	554 04773 71586	1311 31295	7759	76.7	451 98881 52110	933 99548	4825
71.8	551 71996 10362	1302 14385	7683	76.8	450 21409 35844	927 89147	4781
71.9	549 40520 63523	1293 05157	7607	76.9	448 44865 08724	921 83527	4737
72.0	547 10338 21841	1284 03537	7534	77.0	446 69242 65131	915 82643	4694
72.1	544 81439 83696	1275 09451	7460	77.1	444 94536 04182	909 86454	4652
72.2	542 53816 55001	1266 22825	7388	77.2	443 20739 29686	903 94916	4608
72.3	540 27459 49132	1257 43586	7316	77.3	441 47846 50106	898 07986	4568
72.4	538 02359 86849	1248 71664	7245	77.4	439 75851 78513	892 25624	4525
72.5	535 78508 96229	1240 06987	7171	77.5	438 04749 32543	886 47787	4485
72.6	533 55898 12596	1231 49484	7105	77.6	436 34533 34361	880 74436	4444
72.7	531 34518 78447	1222 99087	7037	77.7	434 65198 10615	875 05528	4404
72.8	529 14362 43386	1214 55727	6969	77.8	432 96737 92396	869 41024	4365
72.9	526 95420 64052	1206 19337	6902	77.9	431 29147 15202	863 80885	4324
73.0	524 77685 04054	1197 89848	6836	78.0	429 62420 18892	858 25070	4287
73.1	522 61147 33904	1189 67195	6770	78.1	427 96551 47653	852 73542	4247
73.2	520 45799 30949	1181 51312	6705	78.2	426 31535 49956	847 26261	4210
73.3	518 31632 79306	1173 42134	6641	78.3	424 67366 78520	841 83190	4172
73.4	516 18639 69797	1165 39597	6577	78.4	423 04039 90273	836 44290	4134
73.5	514 06811 99885	1157 43637	6515	78.5	421 41549 46316	831 09525	4098
73.6	511 96141 73610	1149 54193	6451	78.6	419 79890 11884	825 78857	4061
73.7	509 86621 01528	1141 71200	6393	78.7	418 19056 56309	820 52250	4022
73.8	507 78242 00646	1133 94599	6331	78.8	416 59043 52984	815 29667	3990
73.9	505 70996 94363	1126 24328	6269	78.9	414 99845 79326	810 11074	3953
74.0	503 64878 12408	1118 60327	6211	79.0	413 41458 16742	804 96433	3918
74.1	501 59877 90781	1111 02537	6152	79.1	411 83875 50591	799 85711	3884
74.2	499 55988 71690	1103 50899	6094	79.2	410 27092 70151	794 78873	3849
74.3	497 53203 03498	1096 05354	6036	79.3	408 71104 68584	789 75883	3815
74.4	495 51513 40660	1088 65846	5980	79.4	407 15906 42900	784 76708	3781
74.5	493 50912 43669	1081 32317	5922	79.5	405 61492 93924	779 81315	3748
74.6	491 51392 78994	1074 04710	5868	79.6	404 07859 26264	774 89670	3714
74.7	489 52947 19029	1066 82971	5812	79.7	402 55000 48273	770 01739	3682
74.8	487 55568 42036	1059 67044	5758	79.8	401 02911 72020	765 17490	3651
74.9	485 59249 32086	1052 56875	5703	79.9	399 51588 13259	760 36892	3615
75.0	483 63982 79012	1045 52408	5651	80.0	398 01024 91389	755 59909	3590
	.(5)				.(5)		

x	$\Psi^{(3)}(x)$	δ^2	δ^4	x	$\Psi^{(3)}(x)$	δ^2	δ^4
	.(5)				.(5)		
80.0	398 01024 91389	755 59909	3590	85.0	331 45872 82887	556 99743	2343
80.1	396 51217 29430	750 86516	3552	85.1	330 28478 61597	553 71350	2319
80.2	395 02160 53986	746 16675	3524	85.2	329 11638 11657	550 45275	2302
80.3	393 53849 95218	741 50358	3493	85.3	327 95348 06991	547 21502	2283
80.4	392 06280 86808	736 87534	3462	85.4	326 79605 23827	544 00012	2264
80.5	390 59448 65932	732 28172	3432	85.5	325 64406 40675	540 80785	2246
80.6	389 13348 73229	727 72242	3402	85.6	324 49748 38308	537 63804	2226
80.7	387 67976 52767	723 19714	3373	85.7	323 35627 99745	534 49050	2209
80.8	386 23327 52020	718 70559	3344	85.8	322 22042 10231	531 36504	2191
80.9	384 79397 21831	714 24748	3311	85.9	321 08987 57222	528 26150	2170
81.0	383 36181 16389	709 82248	3290	86.0	319 96461 30363	525 17965	2159
81.1	381 93674 93196	705 43038	3254	86.1	318 84460 21469	522 11940	2135
81.2	380 51874 13040	701 07082	3230	86.2	317 72981 24514	519 08049	2121
81.3	379 10774 39965	696 74356	3201	86.3	316 62021 35608	516 06279	2103
81.4	377 70371 41246	692 44831	3174	86.4	315 51577 52981	513 06611	2086
81.5	376 30660 87359	688 18480	3146	86.5	314 41646 76966	510 09030	2069
81.6	374 91638 51950	683 95275	3119	86.6	313 32226 09980	507 13517	2052
81.7	373 53300 11816	679 75188	3092	86.7	312 23312 56512	504 20056	2036
81.8	372 15641 46871	675 58195	3067	86.8	311 14903 23099	501 28631	2020
81.9	370 78658 40120	671 44268	3037	86.9	310 06995 18317	498 39225	2000
82.0	369 42346 77637	667 33378	3018	87.0	308 99585 52759	495 51819	1990
82.1	368 06702 48532	663 25505	2985	87.1	307 92671 39020	492 66403	1968
82.2	366 71721 44931	659 20618	2963	87.2	306 86249 91684	489 82956	1955
82.3	365 37399 61948	655 18693	2937	87.3	305 80318 27304	487 01464	1939
82.4	364 03732 97659	651 19706	2912	87.4	304 74873 64388	484 21910	1923
82.5	362 70717 53075	647 23630	2888	87.5	303 69913 23382	481 44280	1908
82.6	361 38349 32122	643 30443	2862	87.6	302 65434 26657	478 68559	1892
82.7	360 06624 41612	639 40117	2838	87.7	301 61433 98490	475 94729	1878
82.8	358 75538 91219	635 52630	2816	87.8	300 57909 65052	473 22777	1864
82.9	357 45038 93456	631 67959	2788	87.9	299 54858 54391	470 52689	1844
83.0	356 15270 63653	627 86075	2771	88.0	298 52277 96420	467 84445	1837
83.1	354 86080 19924	624 06963	2741	88.1	297 50165 22894	465 18039	1816
83.2	353 57513 83159	620 30591	2721	88.2	296 48517 67407	462 53448	1805
83.3	352 29567 76984	616 56940	2698	88.3	295 47332 65368	459 90662	1790
83.4	351 02238 27749	612 85987	2675	88.4	294 46607 53991	457 29665	1774
83.5	349 75521 64502	609 17709	2653	88.5	293 46339 72279	454 70442	1765
83.6	348 49414 18965	605 52084	2630	88.6	292 46526 61010	452 12984	1744
83.7	347 23912 25511	601 89088	2608	88.7	291 47165 62724	449 57271	1734
83.8	345 99012 21146	598 28701	2587	88.8	290 48254 21709	447 03291	1721
83.9	344 74710 45481	594 70901	2562	88.9	289 49789 83986	444 51032	1704
84.0	343 51003 40718	591 15663	2547	89.0	288 51769 97294	442 00477	1697
84.1	342 27887 51618	587 62971	2520	89.1	287 54192 11080	439 51619	1677
84.2	341 05359 25488	584 12802	2501	89.2	286 57053 76485	437 04437	1667
84.3	339 83415 12159	580 65130	2480	89.3	285 60352 46327	434 58922	1653
84.4	338 62051 63959	577 19940	2460	89.4	284 64085 75091	432 15061	1640
84.5	337 41265 35699	573 77210	2439	89.5	283 68251 18916	429 72840	1628
84.6	336 21052 84649	570 36919	2419	89.6	282 72846 35581	427 32246	1614
84.7	335 01410 70518	566 99046	2399	89.7	281 77868 84492	424 93266	1602
84.8	333 82335 55432	563 63573	2380	89.8	280 83316 26669	422 55889	1591
84.9	332 63824 03920	560 30480	2357	89.9	279 89186 24734	420 20102	1574
85.0	331 45872 82887	556 99743	2343	90.0	278 95476 42901	417 85889	1569
	.(5)				.(5)		

x	$\Psi^{(3)}(x)$	δ^2	δ^4	x	$\Psi^{(3)}(x)$	δ^2	δ^4
	.(5)				.(5)		
90.0	278 95476 42901	417 85889	1569	95.0	236 97921 46659	318 41434	1072
90.1	278 02184 46957	415 53245	1550	95.1	236 22851 70399	316 73505	1060
90.2	277 09308 04258	413 22151	1541	95.2	235 48098 67644	315 06636	1054
90.3	276 16844 83709	410 92597	1529	95.3	234 73660 71524	313 40821	1046
90.4	275 24792 55757	408 64573	1516	95.4	233 99536 16225	311 76052	1038
90.5	274 33148 92378	406 38065	1506	95.5	233 25723 36978	310 12321	1031
90.6	273 41911 67065	404 13062	1493	95.6	232 52220 70053	308 49622	1023
90.7	272 51078 54814	401 89553	1481	95.7	231 79026 52748	306 87945	1016
90.8	271 60647 32115	399 67525	1472	95.8	231 06139 23389	305 27284	1008
90.9	270 70615 76941	397 46968	1456	95.9	230 33557 21313	303 67632	999
91.0	269 80981 68736	395 27867	1452	96.0	229 61278 86870	302 08979	996
91.1	268 91742 88397	393 10218	1434	96.1	228 89302 61405	300 51322	984
91.2	268 02897 18277	390 94003	1426	96.2	228 17626 87262	298 94650	980
91.3	267 14442 42160	388 79214	1415	96.3	227 46250 07769	297 38957	972
91.4	266 26376 45257	386 65840	1403	96.4	226 75170 67233	295 84236	965
91.5	265 38697 14194	384 53869	1393	96.5	226 04387 10933	294 30481	958
91.6	264 51402 37000	382 43291	1382	96.6	225 33897 85114	292 77683	950
91.7	263 64490 03097	380 34096	1372	96.7	224 63701 36978	291 25836	945
91.8	262 77958 03290	378 26272	1362	96.8	223 93796 14678	289 74934	937
91.9	261 91804 29755	376 19810	1348	96.9	223 24180 67312	288 24968	929
92.0	261 06026 76030	374 14696	1344	97.0	222 54853 44914	286 75932	927
92.1	260 20623 37000	372 10926	1328	97.1	221 85812 98447	285 27822	915
92.2	259 35592 08897	370 08485	1321	97.2	221 17057 79802	283 80627	911
92.3	258 50930 89278	368 07364	1310	97.3	220 48586 41783	282 34343	904
92.4	257 66637 77023	366 07553	1300	97.4	219 80397 38108	280 88963	897
92.5	256 82710 72321	364 09042	1290	97.5	219 12489 23395	279 44480	891
92.6	255 99147 76662	362 11822	1281	97.6	218 44860 53163	278 00888	884
92.7	255 15946 92825	360 15883	1271	97.7	217 77509 83819	276 58181	878
92.8	254 33106 24870	358 21214	1262	97.8	217 10435 72655	275 16352	872
92.9	253 50623 78129	356 27807	1249	97.9	216 43636 77844	273 75395	865
93.0	252 68497 59195	354 35649	1246	98.0	215 77111 58427	272 35302	862
93.1	251 86725 75909	352 44737	1231	98.1	215 10858 74312	270 96071	851
93.2	251 05306 37362	350 55056	1224	98.2	214 44876 86267	269 57691	848
93.3	250 24237 53870	348 66599	1215	98.3	213 79164 55914	268 20159	841
93.4	249 43517 36978	346 79357	1205	98.4	213 13720 45720	266 83469	835
93.5	248 63143 99442	344 93320	1197	98.5	212 48543 18995	265 47613	830
93.6	247 83115 55227	343 08479	1187	98.6	211 83631 39882	264 12586	823
93.7	247 03430 19490	341 24826	1179	98.7	211 18983 73355	262 78383	818
93.8	246 24086 08579	339 42350	1171	98.8	210 54598 85212	261 44998	811
93.9	245 45081 40018	337 61046	1158	98.9	209 90475 42066	260 12423	805
94.0	244 66414 32504	335 80899	1156	99.0	209 26612 11343	258 80654	803
94.1	243 88083 05888	334 01909	1142	99.1	208 63007 61274	257 49687	792
94.2	243 10085 81182	332 24060	1136	99.2	207 99660 60893	256 19513	790
94.3	242 32420 80535	330 47346	1127	99.3	207 36569 80025	254 90128	783
94.4	241 55086 27234	328 71759	1118	99.4	206 73733 89285	253 61527	778
94.5	240 78080 45693	326 97290	1110	99.5	206 11151 60072	252 33703	773
94.6	240 01401 61442	325 23933	1102	99.6	205 48821 64562	251 06652	767
94.7	239 25048 01125	323 51676	1094	99.7	204 86742 75705	249 80368	762
94.8	238 49017 92482	321 80513	1086	99.8	204 24913 67215	248 54845	756
94.9	237 73309 64353	320 10435	1076	99.9	203 63333 13569	247 30078	750
95.0	236 97921 46659	318 41434	1072	100.0	203 01999 90002	246 06061	744
	.(5)				.(5)		

TABLE 25

The Hexagamma Function

Description: $\Psi^{(4)}(x)$ to 12 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1.

$\mathrm{Log}_{10}|\Psi^{(4)}(x)|$ to 10 decimal places from $x = 0.0$ to $x = -10.0$ by increments of .1.

$\Psi^{(4)}(x)$ to 12 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

$\mathrm{Log}_{10}|\Psi^{(4)}(x)|$ to 10 decimal places from $x = 0.00$ to $x = 1.00$ by increments of .01.

x	$\Psi^{(4)}(-x)$	$\mathrm{Log}_{10}\lvert\Psi^{(4)}(-x)\rvert$	x	$\Psi^{(4)}(-x)$	$\mathrm{Log}_{10}\lvert\Psi^{(4)}(-x)\rvert$
0.0	∞	∞	5.0	∞	∞
0.1	+2399958.227553 827466	6.38020 36826	5.1	+2399973.828812 191441	6.38020 65058
0.2	+ 74925.301133 760609	4.87462 84972	5.2	+ 74935.508088 349735	4.87468 76563
0.3	+ 9731.834956 233518	3.98819 47351	5.3	+ 9738.755126 667707	3.98850 34460
0.4	+ 2032.557268 070738	3.30804 27907	5.4	+ 2037.393704 563007	3.30907 49598
0.5	− 3.474249 826667	0.54086 10445	5.5	− 0.004525 930280	7.65570 78595*
0.6	− 2039.949298 179348	3.30961 93733	5.6	− 2037.402780 984180	3.30907 68946
0.7	− 9740.670911 658111	3.98858 88710	5.7	− 9738.764277 048672	3.98850 38540
0.8	− 74936.970310 472660	4.87469 61306	5.8	− 74935.517362 928909	4.87468 77100
0.9	−2399974.962992 455631	6.38020 67110	5.9	−2399973.838262 623984	6.38020 65075
1.0	∞	∞	6.0	∞	∞
1.1	+2399973.129665 580885	6.38020 63793	6.1	+2399973.831653 785699	6.38020 65063
1.2	+ 74934.946195 489004	4.87468 43998	6.2	+ 74935.510708 057180	4.87468 76715
1.3	+ 9738.298854 017748	3.98848 30983	6.3	+ 9738.757544 958344	3.98850 35538
1.4	+ 2037.019694 440703	3.30899 52278	6.4	+ 2037.395939 737186	3.30907 54363
1.5	− 0.313755 999507	9.49659 20389*	6.5	− 0.002457 482989	7.39049 05202*
1.6	− 2037.660479 819973	3.30913 18224	6.6	− 2037.400864 560343	3.30907 64861
1.7	− 9738.980600 591574	3.98851 35007	6.7	− 9738.762499 435629	3.98850 37747
1.8	− 74935.700178 887028	4.87468 87696	6.8	− 74935.515712 234508	4.87468 77005
1.9	−2399973.993725 879457	6.38020 65356	6.9	−2399973.836728 127888	6.38020 65072
2.0	∞	∞	7.0	∞	∞
2.1	+2399973.717310 205736	6.38020 64856	7.1	+2399973.832983 993524	6.38020 65065
2.2	+ 74935.411886 481298	4.87468 70987	7.2	+ 74935.511948 420056	4.87468 76786
2.3	+ 9738.671736 569222	3.98849 97272	7.3	+ 9738.758702 660764	3.98850 36054
2.4	+ 2037.321102 619715	3.30905 94836	7.4	+ 2037.397021 302232	3.30907 56668
2.5	− 0.067995 999507	8.83248 33621*	7.5	− 0.001446 124965	7.16020 58234*
2.6	− 2037.458483 014216	3.30908 87679	7.6	− 2037.399918 010951	3.30907 62843
2.7	− 9738.813340 465071	3.98850 60420	7.7	− 9738.761612 774610	3.98850 37352
2.8	− 74935.560728 062967	4.87468 79614	7.8	− 74935.514880 971933	4.87468 76956
2.9	−2399973.876716 344788	6.38020 65144	7.9	−2399973.835948 161683	6.38020 65070
3.0	∞	∞	8.0	∞	∞
3.1	+2399973.801140 843955	6.38020 65007	8.1	+2399973.833672 306802	6.38020 65066
3.2	+ 74935.483412 055029	4.87468 75133	8.2	+ 74935.512595 774271	4.87468 76824
3.3	+ 9738.733062 131993	3.98850 24620	8.3	+ 9738.759311 946176	3.98850 36326
3.4	+ 2037.373924 840545	3.30907 07434	8.4	+ 2037.397595 173936	3.30907 57892
3.5	− 0.022300 753478	8.34831 95368*	8.5	− 0.000905 225423	6.95675 67425*
3.6	− 2037.418791 402165	3.30908 03073	8.6	− 2037.399407 836173	3.30907 61756
3.7	− 9738.778730 383619	3.98850 44985	8.7	− 9738.761131 253891	3.98850 37137
3.8	− 74935.530438 482461	4.87468 77858	8.8	− 74935.514426 195573	4.87468 76930
3.9	−2399973.850115 942384	6.38020 65096	8.9	−2399973.835518 366832	6.38020 65070
4.0	∞	∞	9.0	∞	∞
4.1	+2399973.821856 178821	6.38020 65045	9.1	+2399973.834056 902374	6.38020 65067
4.2	+ 74935.501775 949555	4.87468 76197	9.2	+ 74935.512959 917387	4.87468 76845
4.3	+ 9738.749387 724939	3.98850 31900	9.3	+ 9738.759656 928225	3.98850 36480
4.4	+ 2037.388477 684054	3.30907 38456	9.4	+ 2037.397922 192173	3.30907 58589
4.5	− 0.009294 606041	7.96823 09866*	9.5	− 0.000595 060119	6.77456 08447*
4.6	− 2037.407138 822431	3.30907 78235	9.6	− 2037.399113 492248	3.30907 61128
4.7	− 9738.768265 800014	3.98850 40318	9.7	− 9738.760851 772710	3.98850 37012
4.8	− 74935.521019 476867	4.87468 77312	9.8	− 74935.514160 685585	4.87468 76915
4.9	−2399973.841619 622765	6.38020 65081	9.9	−2399973.835265 998261	6.38020 65069
5.0	∞	∞	10.0	∞	∞

*Ten should be subtracted from these logarithms.

x	$\Psi^{(4)}(x)$	$\mathrm{Log}_{10}\lvert\Psi^{(4)}(x)\rvert$	x	$\Psi^{(4)}(x)$	$\mathrm{Log}_{10}\lvert\Psi^{(4)}(x)\rvert$
.00	$-\infty$	∞	.50	−771.474249 826667	2.88732 14348
.01	−240000 000023.700928 1341	11.38021 12417	.51	−698.966516 252942	2.84445 63715
.02	−7500 000022.583357 6371	9.87506 12634	.52	−634.500428 374195	2.80243 19196
.03	−987 654342.516711 5995	8.99469 49776	.53	−577.053809 746340	2.76121 63126
.04	−234 375020.533868 312853	8.36991 13230	.54	−525.750341 771756	2.72077 95637
.05	−76 800019.593942 159721	7.88536 13308	.55	−479.836560 107956	2.68109 33350
.06	−30 864216.236578 792130	7.48945 52530	.56	−438.662812 423165	2.64213 08186
.07	−14 279782.249769 968715	7.15472 15848	.57	−401.667439 882787	2.60386 66282
.08	−7 324235.821381 565144	6.86476 23190	.58	−368.363592 076236	2.56627 66993
.09	−4 064437.393392 128468	6.60900 04383	.59	−338.328201 065979	2.52933 81995
.10	−2 400015.607203 195865	6.38021 40659	.60	−311.192731 929262	2.49302 94452
.11	−1 490226.107771 981965	6.17325 21675	.61	−286.635399 956592	2.45732 98254
.12	−964520.465605 975475	5.98431 14471	.62	−264.374602 697234	2.42221 97319
.13	−646403.464511 029983	5.81050 36755	.63	−244.163361 477184	2.38768 04953
.14	−446255.747412 807859	5.64958 38225	.64	−225.784604 314884	2.35369 43250
.15	−316061.946623 672831	5.49977 22106	.65	−209.047152 230779	2.32024 42556
.16	−228893.880780 426293	5.35963 41824	.66	−193.782295 275274	2.28731 40956
.17	−169042.658272 046558	5.22799 63134	.67	−179.840864 349461	2.25488 83812
.18	−127024.241299 787326	5.10388 66093	.68	−167.090720 978829	2.22295 23329
.19	−96937.294416 358398	4.98649 08941	.69	−155.414600 343709	2.19149 18157
.20	−75010.212497 972660	4.87512 03958	.70	−144.708253 643025	2.16049 33023
.21	−58774.271096 390130	4.76918 72518	.71	−134.878844 724061	2.12994 39258
.22	−46578.523224 123080	4.66818 57151	.72	−125.843563 212836	2.09983 10064
.23	−37297.312724 528165	4.57167 75420	.73	−117.528422 416972	2.07014 29065
.24	−30149.526256 521022	4.47928 04924	.74	−109.867215 278524	2.04086 81166
.25	−24584.375388 637934	4.39065 91786	.75	−102.800605 815417	2.01199 56739
.26	−20207.738371 015849	4.30551 77103	.76	−96.275336 958386	1.98351 50473
.27	−16733.767398 107882	4.22359 37278	.77	−90.243538 588302	1.95541 61163
.28	−13952.547875 535086	4.14465 35211	.78	−84.662122 006206	1.92768 91496
.29	−11708.142697 332997	4.06848 80068	.79	−79.492249 106550	1.90032 47848
.30	−9883.468555 496988	3.99490 93849	.80	−74.698866 239391	1.87331 40102
.31	−8389.736992 117329	3.92374 83464	.81	−70.250294 194157	1.84664 81473
.32	−7158.989471 115674	3.85485 17237	.82	−66.117866 959725	1.82031 88342
.33	−6138.757835 849817	3.78808 05015	.83	−62.275612 958946	1.79431 80109
.34	−5288.203098 012376	3.72330 81262	.84	−58.699973 310154	1.76863 79037
.35	−4575.294564 109911	3.66041 90598	.85	−55.369552 466876	1.74327 10132
.36	−3974.729125 648942	3.59930 75372	.86	−52.264897 161802	1.71821 01002
.37	−3466.382588 736638	3.53987 64947	.87	−49.368300 176145	1.69344 81740
.38	−3034.147156 771941	3.48203 66403	.88	−46.663625 896426	1.66897 84817
.39	−2665.051749 171137	3.42570 56464	.89	−44.136155 027465	1.64479 44964
.40	−2348.591273 487990	3.37080 74428	.90	−41.772446 172534	.162088 99080
.41	−2076.211530 453112	3.31727 15986	.91	−39.560212 286875	1.59725 86134
.42	−1840.910935 659818	3.26503 27775	.92	−37.488210 265104	1.57389 47072
.43	−1636.930567 035002	3.21403 02583	.93	−35.546142 142672	1.55079 24732
.44	−1459.511462 104247	3.16420 75099	.94	−33.724566 581391	1.52794 63769
.45	−1304.703458 047607	3.11551 18134	.95	−32.014819 473485	1.50535 10572
.46	−1169.213785 842036	3.06789 39272	.96	−30.408942 641287	1.48300 13194
.47	−1050.286510 870922	3.02130 77876	.97	−28.899619 733577	1.46089 21282
.48	−945.606045 993736	2.97571 02403	.98	−27.480118 527433	1.43901 86016
.49	−853.219553 939511	2.93106 08000	.99	−26.144238 938325	1.41737 60040
.50	−771.474249 826667	2.88732 14348	1.00	−24.886266 123441	1.39595 97409

TABLE 26

THE HEXAGAMMA FUNCTION

Description: $\Psi^{(4)}(x)$ to 14 decimal places with central differences from $x = 1.00$ to $x = 4.00$ by increments of .01.

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
1.00	—24.88626612344088	7263482554086	40461548219	364259530
1.01	—23.70092813407076	6776749239351	36997929667	326482281
1.02	—22.58335763709416	6327013854284	33860793397	292939512
1.03	—21.52905727866040	5911139262614	31016596639	263121776
1.04	—20.53386831285278	5526281267582	28435521658	236585008
1.05	—19.59394215972098	5169858794208	26091031683	212941648
1.06	—18.70571459453126	4839527352518	23959483357	191852989
1.07	—17.86588230286672	4533155394184	22019788020	173022645
1.08	—17.07138156514402	4248803223871	20253115328	156190931
1.09	—16.31936885966003	3984704168887	18642633567	141130041
1.10	—15.60720319586491	3739247747469	17173281846	127639887
1.11	—14.93243000954448	3510964607897	15831570013	115544522
1.12	—14.29276646930302	3298513038338	14605402701	104689040
1.13	—13.68608805944494	3100666871481	13483924429	94936908
1.14	—13.11041631830168	2916304629053	12457383065	86167647
1.15	—12.56390762344895	2744399769691	11517009348	78274829
1.16	—12.04484292629312	2584011919676	10654910460	71164335
1.17	—11.55161834833406	2434278980122	9863975907	64752840
1.18	—11.08273656017621	2294410016474	9137794193	58966501
1.19	—10.63679887218312	2163678847020	8470578980	53739809
1.20	—10.21249797266021	2041418256545	7857103577	49014586
1.21	— 9.80861125570276	1927014769647	7292642759	44739119
1.22	— 9.42399468644178	1819903925508	6772921061	40867392
1.23	— 9.05757715643588	1719566002430	6294066754	37358418
1.24	— 8.70835528645428	1625522146105	5852570865	34175659
1.25	— 8.37538863793373	1537330860646	5445250636	31286510
1.26	— 8.05779529801965	1454584825822	5069216917	28661847
1.27	— 7.75474780636378	1376908007915	4721845044	26275636
1.28	— 7.46546939478706	1303953035052	4400748807	24104580
1.29	— 7.18923051356087	1235398810997	4103757150	22127817
1.30	— 6.92534562044464	1170948344091	3828893310	20326646
1.31	— 6.67317021076932	1110326770496	3574356116	18684292
1.32	— 6.43209806879897	1053279553017	3338503214	17185693
1.33	— 6.20155872235878	999570838751	3119836005	15817312
1.34	— 5.98101508430611	948981960490	2916986107	14566979
1.35	— 5.76996126585833	901310068336	2728703189	13423736
1.36	— 5.56792054809391	856366879371	2553844006	12377717
1.37	— 5.37444349912321	813977534413	2391362541	11420025
1.38	— 5.18910622549664	773979551995	2240301100	10542635
1.39	— 5.01150874739001	736221870676	2099782293	9738301
1.40	— 4.84127348799015	700563971652	1969001788	9000478
1.41	— 4.67804386830680	666875074415	1847221761	8323247
1.42	— 4.52148299936760	635033398939	1733764981	7.701252
1.43	— 4.37127246441779	604925488444	1628009453	7.129645
1.44	— 4.22711118435242	576445587402	1529383570	6604032
1.45	— 4.08871436016107	549495069929	1437361719	6120430
1.46	— 3.95581248666900	523981914175	1351460299	5675224
1.47	— 3.82815043231869	499820218721	1271234102	5265131
1.48	— 3.70548658015559	476929757367	1196273036	4887172
1.49	— 3.58759202556615	455235569050	1126199142	4538634
1.50	— 3.47424982666723	434667579875	1060663883	4217053

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
1.50	−3.47424982 666723	434667 579875	1060 663883	4 217053
1.51	−3.36525430 356705	415160 254582	999 345676	3 920183
1.52	−3.26041038 301269	396652 274966	941 947652	3 645979
1.53	−3.15953298 520799	379086 243001	888 195608	3 392579
1.54	−3.06244644 983330	362408 406643	837 836141	3 158281
1.55	−2.96898399 852504	346568 406426	790 634956	2 941536
1.56	−2.87898723 128104	331519 041165	746 375307	2 740928
1.57	−2.79230565 444870	317216 051211	704 856586	2 555161
1.58	−2.70879623 812846	303617 917843	665 893026	2 383055
1.59	−2.62832300 098666	290685 677502	629 312521	2 223525
1.60	−2.55075662 061988	278382 749681	594 955541	2 075584
1.61	−2.47597406 774990	266674 777402	562 674145	1 938323
1.62	−2.40385826 265395	255529 479269	532 331073	1 810912
1.63	−2.33429775 235068	244916 512208	503 798912	1 692590
1.64	−2.26718640 716949	234807 344058	476 959341	1 582658
1.65	−2.20242313 542888	225175 135251	451 702429	1 480475
1.66	−2.13991161 504078	215994 628872	427 925992	1 385453
1.67	−2.07956004 094140	207242 048485	405 535008	1 297050
1.68	−2.02128088 732687	198895 003106	384 441074	1 214770
1.69	−1.96499068 374341	190932 398801	364 561910	1 138157
1.70	−1.91060980 414795	183334 356405	345 820903	1 066788
1.71	−1.85806226 811654	176082 134912	328 146684	1 000278
1.72	−1.80727555 343426	169158 060103	311 472743	938270
1.73	−1.75818041 935300	162545 458037	295 737072	880435
1.74	−1.71071073 985212	156228 593044	280 881837	826471
1.75	−1.66480334 628167	150192 609887	266 853072	776099
1.76	−1.62039787 881009	144423 479802	253 600407	729060
1.77	−1.57743664 613653	138907 950124	241 076801	685117
1.78	−1.53586449 296421	133633 497248	229 238313	644051
1.79	−1.49562867 476437	128588 282685	218 043876	605658
1.80	−1.45667873 939138	123761 111998	207 455096	569750
1.81	−1.41896641 513837	119141 396407	197 436067	536155
1.82	−1.38244550 484943	114719 116883	187 953193	504711
1.83	−1.34707178 572931	110484 790553	178 975031	475271
1.84	−1.31280291 451473	106429 439253	170 472139	447696
1.85	−1.27959833 769268	102544 560093	162 416943	421859
1.86	−1.24741920 647156	98822 097875	154 783606	397642
1.87	−1.21622829 622920	95254 419264	147 547911	374935
1.88	−1.18598993 017947	91834 288564	140 687150	353638
1.89	−1.15666990 701538	88554 845014	134 180027	333653
1.90	−1.12823543 230144	85409 581491	128 006556	314897
1.91	−1.10065505 340240	82392 324524	122 147982	297286
1.92	−1.07389859 774861	79497 215539	116 586693	280745
1.93	−1.04793711 425020	76718 693246	111 306150	265205
1.94	−1.02274281 768426	74051 477104	106 290811	250599
1.95	− .99828903 588934	71490 551772	101 526071	236867
1.96	− .97455015 961215	69031 152510	96 998198	223954
1.97	− .95150159 486006	66668 751447	92 694279	211805
1.98	− .92911971 762244	64399 044663	88 602165	200373
1.99	− .90738183 083144	62217 940043	84 710424	189611
2.00	− .88626612 344088	60121 545848	81 008295	179478

x	$\psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
2.00	—.88626612 344088	60121 545848	81 008295	179478
2.01	—.86575163 150879	58106 159947	77 485643	169934
2.02	—.84581820 117618	56168 259690	74 132926	160941
2.03	—.82644645 344047	54304 492358	70 941149	152465
2.04	—.80761775 062834	52511 666175	67 901837	144475
2.05	—.78931416 447796	50786 741830	65 007001	136941
2.06	—.77151844 574589	49126 824486	62 249106	129833
2.07	—.75421399 525867	47529 156248	59 621043	123126
2.08	—.73738483 633394	45991 109053	57 116107	116796
2.09	—.72101558 849974	44510 177965	54 727967	110821
2.10	—.70509144 244519	43083 974843	52 450647	105177
2.11	—.68959813 613907	41710 222369	50 278505	99846
2.12	—.67452193 205663	40386 748399	48 206208	94809
2.13	—.65984959 545819	39111 480637	46 228721	90048
2.14	—.64556837 366611	37882 441597	44 341282	85548
2.15	—.63166597 628999	36697 743839	42 539391	81292
2.16	—.61813055 635226	35555 585472	40 818792	77266
2.17	—.60495069 226925	34454 245897	39 175458	73457
2.18	—.59211537 064521	33392 081780	37 605581	69852
2.19	—.57961396 983896	32367 523244	36 105557	66440
2.20	—.56743624 426515	31379 070266	34 671973	63209
2.21	—.55557230 939400	30425 289261	33 301599	60149
2.22	—.54401262 741545	29504 809854	31 991373	57251
2.23	—.53274799 353545	28616 321821	30 738399	54504
2.24	—.52176952 287366	27758 572187	29 539928	51900
2.25	—.51106863 793373	26930 362480	28 393357	49432
2.26	—.50063705 661861	26130 546130	27 296219	47092
2.27	—.49046678 076480	25358 025999	26 246172	44872
2.28	—.48055008 517097	24611 752040	25 240998	42766
2.29	—.47087950 709756	23890 719079	24 278590	40768
2.30	—.46144783 621493	23193 964708	23 356949	38871
2.31	—.45224810 497938	22520 567285	22 474179	37070
2.32	—.44327357 941668	21869 644041	21 628479	35360
2.33	—.43451775 029439	21240 349276	20 818140	33736
2.34	—.42597432 466486	20631 872652	20 041537	32193
2.35	—.41763721 776184	20043 437564	19 297127	30727
2.36	—.40950054 523447	19474 299604	18 583445	29334
2.37	—.40155861 570314	18923 745089	17 899096	28009
2.38	—.39380592 362270	18391 089669	17 242756	26750
2.39	—.38623714 243895	17875 677005	16 613166	25551
2.40	—.37884711 802526	17376 877507	16 009127	24412
2.41	—.37163086 238662	16894 087136	15 429501	23328
2.42	—.36458354 761935	16426 726266	14 873203	22296
2.43	—.35770050 011474	15974 238599	14 339201	21314
2.44	—.35097719 499613	15536 090134	13 826514	20379
2.45	—.34440925 077885	15111 768183	13 334206	19489
2.46	—.33799242 424340	14700 780438	12 861388	18641
2.47	—.33172260 551234	14302 654081	12 407210	17833
2.48	—.32559581 332207	13916 934933	11 970866	17064
2.49	—.31960819 048115	13543 186651	11 551585	16330
2.50	—.31375599 950673	13180 989955	11 148635	15631

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
2.50	—.31375599 950673	13180 989955	11 148635	15631
2.51	—.30803561 843186	12829 941893	10 761316	14965
2.52	—.30244353 677593	12489 655147	10 388961	14329
2.53	—.29697635 167147	12159 757363	10 030936	13723
2.54	—.29163076 414063	11839 890514	9 686633	13145
2.55	—.28640357 551494	11529 710298	9 355475	12593
2.56	—.28129168 399223	11228 885556	9 036909	12066
2.57	—.27629208 132508	10937 097724	8 730410	11564
2.58	—.27140184 963516	10654 040301	8 435474	11084
2.59	—.26661815 834826	10379 418352	8 151623	10626
2.60	—.26193826 124488	10112 948026	7 878397	10189
2.61	—.25735949 362176	9854 356098	7 615361	9771
2.62	—.25287926 955962	9603 379530	7 362095	9372
2.63	—.24849507 929278	9359 765057	7 118200	8990
2.64	—.24420448 667651	9123 268785	6 883296	8626
2.65	—.24000512 674809	8893 655808	6 657018	8277
2.66	—.23589470 337775	8670 699850	6 439017	7944
2.67	—.23187098 700591	8454 182908	6 228961	7626
2.68	—.22793181 246315	8243 894928	6 026530	7321
2.69	—.22407507 686967	8039 633478	5 831421	7030
2.70	—.22029873 761097	7841 203449	5 643343	6752
2.71	—.21660081 038676	7648 416763	5 462015	6485
2.72	—.21297936 733017	7461 092092	5 287173	6230
2.73	—.20943253 519450	7279 054593	5 118560	5986
2.74	—.20595849 360476	7102 135655	4 955933	5752
2.75	—.20255547 337157	6930 172650	4 799058	5528
2.76	—.19922175 486488	6763 008702	4 647710	5314
2.77	—.19595566 644520	6600 492464	4 501677	5108
2.78	—.19275558 295017	6442 477904	4 360751	4912
2.79	—.18961992 423417	6288 824094	4 224738	4723
2.80	—.18654715 375911	6139 395022	4 093448	4543
2.81	—.18353577 723427	5994 059398	3 966700	4370
2.82	—.18058434 130341	5852 690474	3 844323	4204
2.83	—.17769143 227729	5715 165872	3 726149	4045
2.84	—.17485567 490988	5581 367420	3 612021	3893
2.85	—.17207573 121668	5451 180990	3 501786	3747
2.86	—.16935029 933337	5324 496345	3 395297	3606
2.87	—.16667811 241352	5201 206998	3 292415	3472
2.88	—.16405793 756365	5081 210065	3 193005	3343
2.89	—.16148857 481442	4964 406137	3 096937	3219
2.90	—.15896885 612658	4850 699147	3 004090	3101
2.91	—.15649764 443020	4739 996246	2 914342	2987
2.92	—.15407383 269629	4632 207688	2 827582	2877
2.93	—.15169634 303925	4527 246711	2 743698	2772
2.94	—.14936412 584934	4425 029433	2 662587	2671
2.95	—.14707615 895375	4325 474741	2 584147	2574
2.96	—.14483144 680556	4228 504196	2 508281	2481
2.97	—.14262901 969934	4134 041931	2 434897	2392
2.98	—.14046793 301242	4042 014564	2 363904	2306
2.99	—.13834726 647115	3952 351100	2 295218	2224
3.00	—.13626612 344088	3864 982856	2 228756	2144

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
3.00	—.13626612 344088	3864 982856	2 228756	2144
3.01	—.13422363 023916	3779 843367	2 164438	2068
3.02	—.13221893 547112	3696 868316	2 102188	1995
3.03	—.13025120 938624	3615 995454	2 041934	1925
3.04	—.12831964 325590	3537 164526	1 983604	1857
3.05	—.12642344 877083	3460 317202	1 927131	1792
3.06	—.12456185 745776	3385 397008	1 872449	1729
3.07	—.12273412 011478	3312 349263	1 819497	1669
3.08	—.12093950 626443	3241 121015	1 768214	1611
3.09	—.11917730 362423	3171 660982	1 718542	1555
3.10	—.11744681 759384	3103 919490	1 670425	1502
3.11	—.11574737 075836	3037 848422	1 623810	1450
3.12	—.11407830 240709	2973 401164	1 578644	1400
3.13	—.11243896 806747	2910 532551	1 534879	1353
3.14	—.11082873 905336	2849 198817	1 492467	1306
3.15	—.10924700 202741	2789 357549	1 451361	1262
3.16	—.10769315 857695	2730 967643	1 411517	1219
3.17	—.10616662 480293	2673 989255	1 372893	1178
3.18	—.10466683 092145	2618 383759	1 335447	1139
3.19	—.10319322 087756	2564 113711	1 299140	1101
3.20	—.10174525 197078	2511 142802	1 263933	1064
3.21	—.10032239 449202	2459 435827	1 229790	1028
3.22	—.09892413 137153	2408 958642	1 196676	994
3.23	—.09754995 783745	2359 678133	1 164556	961
3.24	—.09619938 108471	2311 562179	1 133397	930
3.25	—.09487191 995375	2264 579623	1 103168	899
3.26	—.09356710 461903	2218 700235	1 073838	870
3.27	—.09228447 628665	2173 894684	1 045377	841
3.28	—.09102358 690111	2130 134511	1 017758	814
3.29	—.08978399 886068	2087 392096	990953	787
3.30	—.08856528 474121	2045 640634	964935	762
3.31	—.08736702 702809	2004 854107	939679	737
3.32	—.08618881 785603	1965 007259	915160	714
3.33	—.08503025 875657	1926 075572	891355	691
3.34	—.08389096 041283	1888 035240	868241	669
3.35	—.08277054 242149	1850 863148	845795	647
3.36	—.08166863 306163	1814 536852	823996	627
3.37	—.08058486 907028	1779 034551	802825	607
3.38	—.07951889 542445	1744 335076	782260	588
3.39	—.07847036 512938	1710 417860	762283	569
3.40	—.07743893 901291	1677 262927	742874	551
3.41	—.07642428 552571	1644 850869	724018	534
3.42	—.07542608 054720	1613 162828	705695	517
2.43	—.07444400 719697	1582 180481	687890	501
3.44	—.07347775 565155	1551 886025	670585	486
3.45	—.07252702 296639	1522 262154	653767	471
3.46	—.07159151 290276	1493 292050	637419	456
3.47	—.07067093 575964	1464 959366	621528	442
3.48	—.06976500 821018	1437 248209	606078	429
3.49	—.06887345 314281	1410 143130	591057	415
3.50	—.06799599 950673	1383 629109	576452	403

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
3.50	—.06799599 950673	1383 629109	576452	403
3.51	—.06713238 216174	1357 691539	562249	391
3.52	—.06628234 173215	1332 316218	548437	379
3.53	—.06544562 446473	1307 489335	535003	367
3.54	—.06462198 209067	1283 197454	521937	356
3.55	—.06381117 169114	1259 427511	509227	345
3.56	—.06301295 556672	1236 166794	496862	335
3.57	—.06222710 111025	1213 402940	484833	325
3.58	—.06145338 068319	1191 123919	473128	315
3.59	—.06069157 149531	1169 318026	461739	306
3.60	—.05994145 548770	1147 973873	450656	297
3.61	—.05920281 921881	1127 080375	439870	288
3.62	—.05847545 375368	1106 626748	429372	280
3.63	—.05775915 455603	1086 602493	419154	271
3.64	—.05705372 138332	1066 997393	409207	264
3.65	—.05635895 818453	1047 801500	399524	256
3.66	—.05567467 300074	1029 005131	390097	248
3.67	—.05500067 786826	1010 598859	380918	241
3.68	—.05433678 872436	992 573504	371980	234
3.69	—.05368282 531551	974 920130	363276	227
3.70	—.05303861 110795	957 630032	354800	221
3.71	—.05240397 320072	940 694733	346544	214
3.72	—.05177874 224081	924 105979	338503	208
3.73	—.05116275 234071	907 855729	330671	202
3.74	—.05055584 099789	891 936149	323040	197
3.75	—.04995784 901656	876 339609	315607	191
3.76	—.04936862 043132	861 058676	308364	186
3.77	—.04878800 243285	846 086108	301307	180
3.78	—.04821584 529545	831 414846	294430	175
3.79	—.04765200 230651	817 038014	287729	170
3.80	—.04709632 969771	802 948911	281198	166
3.81	—.04654868 657802	789 141007	274832	161
3.82	—.04600893 486841	775 607934	268628	156
3.83	—.04547693 923813	762 343489	262580	152
3.84	—.04495256 704275	749 341624	256684	148
3.85	—.04443568 826361	736 596443	250936	144
3.86	—.04392617 544890	724 102197	245331	140
3.87	—.04342390 365617	711 853283	239867	136
3.88	—.04292875 039626	699 844235	234538	132
3.89	—.04244059 557870	688 069725	229342	129
3.90	—.04195932 145838	676 524556	224274	125
3.91	—.04148481 258362	665 203662	219331	122
3.92	—.04101695 574549	654 102099	214510	118
3.93	—.04055563 992833	643 215046	209808	115
3.94	—.04010075 626164	632 537801	205220	112
3.95	—.03965219 797295	622 065776	200745	109
3.96	—.03920986 034202	611 794496	196379	106
3.97	—.03877364 065604	601 719595	192118	103
3.98	—.03834343 816602	591 836812	187961	101
3.99	—.03791915 404411	582 141991	183905	98
4.00	—.03750069 134211	572 631074	179947	95

TABLE 27

THE HEXAGAMMA FUNCTION

Description: $\Psi^{(4)}(x)$ to 16 decimal places with central differences from $x = 4.00$ to $x = 20.00$ by increments of .02.

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$	$-\delta^8$
4.00	−.(1)37500 69134 21128	22 90704 24383	2879 90405	609717	1941
4.02	36680 85156 06456	22 16753 18251	2757 71747	577807	1821
4.04	35883 17931 10035	21 45559 83868	2641 30897	547718	1709
4.06	35106 96265 97480	20 77007 80378	2530 37764	519338	1603
4.08	34351 51608 65304	20 10986 14654	2424 63970	492562	1506
4.10	33616 17937 47783	19 47389 12900	2323 82737	467291	1414
4.12	32900 31655 43161	18 86115 93883	2227 68796	443435	1329
4.14	32203 31489 32422	18 27070 43662	2135 98290	420907	1248
4.16	31524 58393 65345	17 70160 91731	2048 48690	399627	1174
4.18	30863 55458 90000	17 15299 88491	1964 98717	379521	1103
4.20	30219 67824 03145	16 62403 83967	1885 28264	360518	1038
4.22	29592 42593 00258	16 11393 07708	1809 18329	342553	977
4.24	28981 28755 05078	15 62191 49778	1736 50947	325565	919
4.26	28385 77108 59676	15 14726 42794	1667 09130	309496	866
4.28	27805 40188 57068	14 68928 44941	1600 76808	294292	815
4.30	27239 72196 99402	14 24731 23896	5137 38779	279903	768
4.32	26688 28936 65631	13 82071 41629	1476 80653	266282	723
4.34	26150 67747 73489	13 40888 40016	1418 88809	253384	682
4.36	25626 47447 21363	13 01124 27211	1363 50349	241168	642
4.38	25115 28270 96447	12 62723 64755	1310 53058	229595	606
4.40	24616 71818 36287	12 25633 55358	1259 85362	218628	571
4.42	24130 40999 31485	11 89803 31323	1211 36293	208232	540
4.44	23655 99983 58006	11 55184 43581	1164 95456	198376	509
4.46	23193 14152 28108	11 21730 51295	1120 52996	189029	481
4.48	22741 50051 49506	10 89397 12005	1077 99564	180163	454
4.50	22300 75347 82908	10 58141 72280	1037 26296	171751	429
4.52	21870 58785 88591	10 27923 58851	998 24778	163767	405
4.54	21450 70147 53125	9 98703 70199	960 87027	156189	383
4.56	21040 80212 87857	9 70444 68575	925 05466	148993	362
4.58	20640 60722 91165	9 43110 72417	890 72897	142160	342
4.60	20249 84343 66889	9 16667 49155	857 82488	135668	324
4.62	19868 24631 91768	8 91082 08382	826 27747	129500	306
4.64	19495 56002 25030	8 66322 95357	796 02507	123639	290
4.66	19131 53695 53649	8 42359 84838	767 00905	118067	274
4.68	18775 93748 67106	8 19163 75225	739 17370	112769	260
4.70	18428 52965 55788	7 96706 82982	712 46604	107731	245
4.72	18089 08889 27452	7 74962 37342	686 83568	102938	233
4.74	17757 39775 36458	7 53904 75271	662 23471	98379	220
4.76	17433 24566 20734	7 33509 36670	638 61752	94040	209
4.78	17116 42866 41681	7 13752 59820	615 94073	89910	198
4.80	16806 74919 22447	6 94611 77044	594 16303	85978	188
4.82	16504 01583 80257	6 76065 10570	573 24511	82233	178
4.84	16208 04313 48637	6 58091 68607	553 14952	78668	169
4.86	15918 65134 85624	6 40671 41596	533 84061	75270	161
4.88	15635 66627 64207	6 23784 98646	515 28440	72033	152
4.90	15358 91905 41436	6 07413 84137	497 44853	68949	144
4.92	15088 24597 02802	5 91540 14480	480 30214	66008	137
4.94	14823 48828 78648	5 76146 75037	463 81583	63205	130
4.96	14564 49207 29531	5 61217 17178	447 96157	60531	124
4.98	14311 10802 97592	5 46735 55475	432 71261	57981	117
5.00	−.(1)14063 19134 21128	5 32686 65033	418 04347	55549	111

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$	$-\delta^8$
5.00	−.(1)14063 19134 21128	5 32686 65033	418 04347	55549	111
5.02	13820 60152 09696	5 19055 78938	403 92981	53227	106
5.04	13583 20225 77203	5 05828 85824	390 34843	51012	101
5.06	13350 86128 30534	4 92992 27554	377 27717	48898	96
5.08	13123 45023 11418	4 80532 97000	364 69489	46879	91
5.10	12900 84450 89302	4 68438 35936	352 58141	44952	87
5.12	12682 92317 03122	4 56696 33013	340 91744	43111	82
5.14	12469 56879 49955	4 45295 21834	329 68459	41352	77
5.16	12260 66737 18622	4 34223 79113	318 86525	39672	75
5.18	12056 10818 66402	4 23471 22918	308 44264	38067	71
5.20	11855 78371 37100	4 13027 10987	298 40069	36532	68
5.22	11659 58951 18785	4 02881 39126	288 72407	35065	64
5.24	11467 42412 39596	3 93024 39671	279 39809	33663	61
5.26	11279 18898 00077	3 83446 80025	270 40874	32321	58
5.28	11094 78830 40584	3 74139 61253	261 74260	31038	56
5.30	10914 12902 42343	3 65094 16742	253 38685	29811	53
5.32	10737 12068 60845	3 56302 10915	245 32921	28637	51
5.34	10563 67536 90262	3 47755 38009	237 55793	27513	48
5.36	10393 70760 57688	3 39446 20897	230 06180	26438	46
5.38	10227 13430 46011	3 31367 09964	222 83004	25409	44
5.40	−.(1)10063 87467 44298	3 23510 82034	215 85237	24423	42
5.42	−.(2) 9903 85015 24619	3 15870 39342	209 11892	23479	40
5.44	9746 98433 44282	3 08439 08541	202 62027	22575	38
5.46	9593 20290 72486	3 01210 39768	196 34737	21709	36
5.48	9442 43358 40459	2 94178 05732	190 29156	20880	35
5.50	9294 60604 14164	2 87336 00852	184 44455	20085	33
5.52	9149 65185 88720	2 80678 40426	178 79838	19323	31
5.54	9007 50446 03704	2 74199 59839	173 34544	18592	30
5.56	8868 09905 78525	2 67894 13795	168 07842	17892	29
5.58	8731 37259 67142	2 61756 75594	162 99033	17221	28
5.60	8597 26370 31352	2 55782 36426	158 07445	16577	26
5.62	8465 71263 31988	2 49966 04703	153 32435	15960	25
5.64	8336 66122 37327	2 44303 05416	148 73385	15368	24
5.66	8210 05284 48081	2 38788 79513	144 29702	14800	23
5.68	8085 83235 38348	2 33418 83312	140 00819	14254	22
5.70	7963 94605 11928	2 28188 87931	135 86190	13731	21
5.72	7844 34163 73438	2 23094 78740	131 85293	13229	20
5.74	7726 96817 13688	2 18132 54841	127 97624	12747	19
5.76	7611 77603 08780	2 13298 28567	124 22701	12284	18
5.78	7498 71687 32438	2 08588 24993	120 60063	11839	18
5.80	7387 74359 81089	2 03998 81482	117 09263	11411	17
5.82	7278 81031 11223	1 99526 47234	113 69875	11002	16
5.84	7171 87228 88591	1 95167 82862	110 41489	10608	15
5.86	7066 88594 48820	1 90919 59978	107 23712	10229	15
5.88	6963 80879 69028	1 86778 60806	104 16163	9866	14
5.90	6862 59943 50041	1 82741 77797	101 18480	9516	13
5.92	6763 21749 08852	1 78806 13269	98 30313	9180	13
5.94	6665 62360 80932	1 74968 79053	95 51326	8857	12
5.96	6569 77941 32064	1 71226 96164	92 81196	8546	12
5.98	6475 64748 79361	1 67577 94471	90 19611	8247	11
6.00	−.(2) 6383 19134 21128	1 64019 12389	87 66274	7960	11

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$	$-\delta^8$
6.00	−.(2) 6383 19134 21128	1 64019 12389	87 66274	7960	11
6.02	6292 37538 75284	1 60547 96581	85 20898	7684	10
6.04	6203 16491 26021	1 57162 01671	82 83204	7418	10
6.06	6115 52605 78429	1 53858 89965	80 52929	7162	10
6.08	6029 42579 20803	1 50636 31188	78 29815	6916	9
6.10	5944 83188 94365	1 47492 02226	76 13617	6679	9
6.12	5861 71290 70153	1 44423 86882	74 04099	6451	9
6.14	5780 03816 32823	1 41429 75636	72 01031	6231	8
6.16	5699 77771 71129	1 38507 65421	70 04194	6020	8
6.18	5620 90234 74855	1 35655 59400	68 13378	5817	8
6.20	5543 38353 37982	1 32871 66757	66 28378	5621	7
6.22	5467 19343 67865	1 30154 02492	64 48999	5432	7
6.24	5392 30488 00241	1 27500 87227	62 75052	5250	7
6.26	5318 69133 19844	1 24910 47013	61 06355	5075	6
6.28	5246 32688 86460	1 22381 13154	59 42733	4906	6
6.30	5175 18625 66230	1 19911 22028	57 84017	4744	6
6.32	5105 24473 68028	1 17499 14919	56 30044	4587	6
6.34	5036 47820 84745	1 15143 37854	54 80659	4436	5
6.36	4968 86311 39316	1 12842 41448	53 35710	4290	5
6.38	4902 37644 35335	1 10594 80752	51 95050	4150	5
6.40	4836 99572 12105	1 08399 15106	50 58541	4015	5
6.42	4772 69899 03981	1 06254 08000	49 26046	3884	5
6.44	4709 46480 03857	1 04158 26941	47 97435	3758	4
6.46	4647 27219 30675	1 02110 43317	46 72583	3637	4
6.48	4586 10069 00810	1 00109 32276	45 51367	3520	4
6.50	4525 93028 03221	98153 72601	44 33670	3407	4
6.52	4466 74140 78233	96242 46597	43 19381	3298	4
6.54	4408 51495 99842	94374 39974	42 08389	3192	4
6.56	4351 23225 61425	92548 41739	41 00589	3091	3
6.58	4294 87503 64747	90763 44094	39 95881	2993	3
6.60	4239 42545 12164	89018 42330	38 94165	2898	3
6.62	4184 86605 01910	87312 34731	37 95348	2807	3
6.64	4131 17977 26387	85644 22480	36 99338	2719	3
6.66	4078 34993 73345	84013 09567	36 06046	2634	3
6.68	4026 36023 29869	82418 02700	35 15389	2552	3
6.70	3975 19470 89094	80858 11223	34 27283	2472	3
6.72	3924 83776 59541	79332 47028	33 41649	2395	3
6.74	3875 27414 77016	77840 24481	32 58410	2321	3
6.76	3826 48893 18972	76380 60346	31 77493	2250	2
6.78	3778 46752 21274	74952 73703	30 98826	2181	2
6.80	3731 19563 97278	73555 85886	30 22339	2114	2
6.82	3684 65931 59168	72189 20407	29 47966	2049	2
6.84	3638 84488 41466	70852 02894	28 75641	1987	2
6.86	3593 73897 26657	69543 61023	28 05304	1926	2
6.88	3549 32849 72872	68263 24456	27 36893	1868	2
6.90	3505 60065 43542	67010 24781	26 70350	1812	2
6.92	3462 54291 38994	65783 95457	26 05619	1757	2
6.94	3420 14301 29902	64583 71751	25 42644	1704	2
6.96	3378 38894 92562	63408 90690	24 81374	1653	2
6.98	3337 26897 45912	62258 91002	24 21757	1604	2
7.00	−.(2) 3296 77158 90264	61133 13072	23 63744	1556	2

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
7.00	—.(2) 3296 77158 90264	61133 13072	23 63744	1556
7.02	3256 88553 47688	60030 98886	22 07288	1510
7.04	3217 59979 03998	58951 91987	22 52341	1465
7.06	3178 90356 52295	57895 37430	21 98860	1422
7.08	3140 78629 38022	56860 81732	21 46800	1380
7.10	3103 23763 05480	55847 72834	20 96121	1340
7.12	3066 24744 45773	54855 60058	20 46782	1301
7.14	3029 80581 46123	53883 94064	19 98744	1263
7.16	2993 90302 40536	52932 26814	19 51968	1226
7.18	2958 52955 61764	52000 11532	19 06419	1190
7.20	2923 67608 94524	51087 02670	18 62059	1156
7.22	2889 33349 29953	50192 55866	18 18856	1123
7.24	2855 49282 21249	49316 27919	17 76776	1090
7.26	2822 14531 40463	48457 76747	17 35786	1059
7.28	2789 28238 36424	47616 61361	16 95855	1029
7.30	2756 89561 93747	46792 41829	16 56953	1000
7.32	2724 97677 92899	45984 79251	16 19050	971
7.34	2693 51778 71301	45193 35722	15 82119	944
7.36	2662 51072 85426	44417 74312	15 46131	917
7.38	2631 94784 73863	43657 59034	15 11060	891
7.40	2601 82154 21334	42912 54816	14 76881	866
7.42	2572 12436 23621	42182 27479	14 43567	842
7.44	2542 84900 53387	41466 43709	14 11095	818
7.46	2513 98831 26863	40764 71035	13 79442	795
7.48	2485 53526 71373	40076 77802	13 48584	773
7.50	2457 48298 93686	39402 33154	13 18499	752
7.52	2429 82473 49152	38741 07004	12 89166	731
7.54	2402 55389 11622	38092 70020	12 60564	711
7.56	2375 66397 44111	37456 93599	12 32673	691
7.58	2349 14862 70200	36833 49852	12 05473	672
7.60	2323 00161 46140	36222 11578	11 78946	654
7.62	2297 21682 33659	35622 52250	11 53073	636
7.64	2271 78825 73428	35034 45995	11 27836	619
7.66	2246 71003 59192	34457 67575	11 03217	602
7.68	2221 97639 12531	33891 92373	10 79201	586
7.70	2197 58166 58242	33336 96372	10 55771	570
7.72	2173 52031 00325	32792 56141	10 32910	555
7.74	2149 78687 98550	32258 48821	10 10604	540
7.76	2126 37603 45596	31734 52106	9 88838	525
7.78	2103 28253 44748	31220 44229	9 67597	511
7.80	2080 50123 88128	30716 03949	9 46868	498
7.82	2058 02710 35457	30221 10537	9 26636	484
7.84	2035 85517 93323	29735 43761	9 06888	472
7.86	2013 98060 94950	29258 83873	8 87612	459
7.88	1992 39862 80449	28791 11597	8 68795	447
7.90	1971 10455 77546	28332 08116	8 50425	435
7.92	1950 09380 82758	27881 55061	8 32491	424
7.94	1929 36187 43032	27439 34497	8 14980	413
7.96	1908 90433 37803	27005 28912	7 97882	402
7.98	1888 71684 61485	26579 21209	7 81186	392
8.00	—.(2) 1868 79515 06376	26160 94691	7 64881	381

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
8.00	−.(2) 1868 79515 06376	26160 94691	7 64881	381
8.02	1849 13506 45958	25750 33055	7 48959	371
8.04	1829 73248 18595	25347 20376	7 33406	362
8.06	1810 58337 11608	24951 41103	7 18216	353
8.08	1791 68377 45724	24562 80046	7 03379	344
8.10	1773 02980 59886	24181 22369	6 88885	335
8.12	1754 61764 96417	23806 53576	6 74726	326
8.14	1736 44355 86524	23438 59509	6 60893	318
8.16	1718 50385 36140	23077 26336	6 47378	310
8.18	1700 79492 12091	22722 40540	6 34173	302
8.20	1683 31321 28583	22373 88917	6 21269	294
8.22	1666 05524 33993	22031 58564	6 08660	287
8.24	1649 01758 97967	21695 36871	5 96338	280
8.26	1632 19688 98811	21365 11515	5 84295	273
8.28	1615 58984 11170	21040 70455	5 72525	266
8.30	1599 19319 93985	20722 01919	5 61021	259
8.32	1583 00377 78719	20408 94404	5 49775	253
8.34	1567 01844 57857	20101 36665	5 38783	246
8.36	1551 23412 73660	19799 17707	5 28036	240
8.38	1535 64780 07170	19502 26787	5 17530	234
8.40	1520 25649 67467	19210 53396	5 07259	229
8.42	1505 05729 81159	18923 87264	4 97215	223
8.44	1490 04733 82116	18642 18347	4 87395	217
8.46	1475 22380 01419	18365 36825	4 77792	212
8.48	1460 58391 57548	18093 33095	4 68401	207
8.50	1446 12496 46772	17825 97767	4 59217	202
8.52	1431 84427 33763	17563 21656	4 50235	197
8.54	1417 73921 42409	17304 95780	4 41450	192
8.56	1403 80720 46836	17051 11354	4 32857	188
8.58	1390 04570 62617	16801 59786	4 24452	183
8.60	1376 45222 38183	16556 32669	4 16229	179
8.62	1363 02430 46418	16315 21781	4 08185	174
8.64	1349 75953 76434	16078 19079	4 00316	170
8.66	1336 65555 25529	15845 16692	3 92616	166
8.68	1323 71001 91316	15616 06922	3 85083	162
8.70	1310 92064 64026	15390 82234	3 77711	158
8.72	1298 28518 18968	15169 35257	3 70498	154
8.74	1285 80141 09168	14951 58778	3 63439	151
8.76	1273 46715 58145	14737 45738	3 56531	147
8.78	1261 28027 52860	14526 89228	3 49770	144
8.80	1249 23866 36804	14319 82489	3 43153	140
8.82	1237 34025 03236	14116 18902	3 36676	137
8.84	1225 58299 88571	13915 91992	3 30336	134
8.86	1213 96490 65899	13718 95419	3 24131	131
8.88	1202 48400 38645	13525 22975	3 18055	128
8.90	1191 13835 34367	13334 68587	3 12108	125
8.92	1179 92604 98676	13147 26307	3 06285	122
8.94	1168 84521 89293	12962 90312	3 00584	119
8.96	1157 89401 70222	12781 54902	2 95002	116
8.98	1147 07063 06053	12603 14493	2 89536	114
9.00	−.(2) 1136 37327 56376	12427 63620	2 84184	111

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
9.00	—.(2)1136 37327 56376	12427 63620	2 84184	111
9.02	1125 80019 70320	12254 96932	2 78943	108
9.04	1115 34966 81196	12085 09187	2 73810	106
9.06	1105 01999 01259	11917 95251	2 68783	103
9.08	1094 80949 16573	11753 50098	2 63859	101
9.10	1084 71652 81984	11591 68804	2 59037	99
9.12	1074 73948 16200	11432 46548	2 54313	97
9.14	1064 87675 96963	11275 78604	2 49686	94
9.16	1055 12679 56331	11121 60348	2 45154	92
9.18	1045 48804 76046	10969 87245	2 40714	90
9.20	1035 95899 83006	10820 54855	2 36363	88
9.22	1026 53815 44821	10673 58829	2 32101	86
9.24	1017 22404 65465	10528 94904	2 27926	84
9.26	1008 01522 81014	10386 58905	2 23834	82
9.28	—.(3) 998 91027 55468	10246 46741	2 19825	81
9.30	989 90778 76663	10108 54401	2 15896	79
9.32	981 00638 52258	9972 77957	2 12046	77
9.34	972 20471 05811	9839 13559	2 08273	75
9.36	963 50142 72922	9707 57435	2 04576	74
9.38	954 89521 97469	9578 05887	2 00952	72
9.40	946 38479 27903	9450 55290	1 97400	70
9.42	937 96887 13627	9325 02094	1 93918	69
9.44	929 64620 01445	9201 42815	1 90505	67
9.46	921 41554 32078	9079 74041	1 87160	66
9.48	913 27568 36752	8959 92428	1 83880	64
9.50	905 22542 33854	8841 94695	1 80665	63
9.52	897 26358 25650	8725 77627	1 77513	62
9.54	889 38899 95074	8611 38073	1 74423	60
9.56	881 60053 02571	8498 72941	1 71393	59
9.58	873 89704 83009	8387 79203	1 68422	58
9.60	866 27744 42650	8278 53887	1 65509	57
9.62	858 74062 56177	8170 94079	1 62652	55
9.64	851 28551 63784	8064 96924	1 59851	54
9.66	843 91105 68315	7960 59619	1 57103	53
9.68	836 61620 32466	7857 79418	1 54409	52
9.70	829 39992 76034	7756 53626	1 51767	51
9.72	822 26121 73228	7656 79601	1 49175	50
9.74	815 19907 50024	7558 54750	1 46633	49
9.76	808 21251 81569	7461 76532	1 44139	48
9.78	801 30057 89647	7366 42454	1 41693	47
9.80	794 46230 40179	7272 50069	1 39294	46
9.82	787 69675 40780	7179 96977	1 36940	45
9.84	781 00300 38357	7088 80825	1 34631	44
9.86	774 38014 16760	6998 99304	1 32365	43
9.88	767 82726 94467	6910 50149	1 30142	42
9.90	761 34350 22322	6823 31135	1 27961	41
9.92	754 92796 81313	6737 40083	1 25822	40
9.94	748 57980 80387	6652 74853	1 23722	39
9.96	742 29817 54314	6569 33344	1 21661	38
9.98	736 08223 61585	6487 13499	1 19639	38
10.00	—.(3) 729 93116 82353	6406 13288	1 17655	37

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
10.00	—.(3) 729 93116 82353	6406 13288	1 17655	37
10.02	723 84416 16409	6326 30735	1 15708	36
10.04	717 82041 81201	6247 63890	1 13796	35
10.06	711 85915 09882	6170 10840	1 11921	35
10.08	705 95958 49403	6093 69712	1 10079	34
10.10	700 12095 58637	6018 38663	1 08272	33
10.12	694 34251 06533	5944 15886	1 06498	33
10.14	688 62350 70315	5870 99608	1 04757	32
10.16	682 96321 33705	5798 88086	1 03048	31
10.18	677 36090 85181	5727 79613	1 01370	31
10.20	671 81588 16270	5657 72510	99722	30
10.22	666 32743 19869	5588 65129	98105	29
10.24	660 89486 88597	5520 55853	96517	29
10.26	655 51751 13178	5453 43094	94958	28
10.28	650 19468 80853	5387 25294	93427	28
10.30	644 92573 73821	5322 00920	91924	27
10.32	639 71000 67709	5257 68471	90448	27
10.34	634 54685 30069	5194 26467	88999	26
10.36	629 43564 18898	5131 73467	87575	26
10.38	624 37574 81195	5070 08040	86177	25
10.40	619 36655 51532	5009 28790	84805	25
10.42	614 40745 50659	4949 34345	83456	24
10.44	609 49784 84132	4890 23357	82132	24
10.46	604 63714 40961	4831 94500	80832	23
10.48	599 82475 92290	4774 46475	79554	23
10.50	595 06011 90094	4717 78004	78299	22
10.52	590 34265 65902	4661 87832	77066	22
10.54	585 67181 29541	4606 74726	75856	21
10.56	581 04703 67907	4552 37476	74666	21
10.58	576 46778 43749	4498 74892	73497	21
10.60	571 93351 94482	4445 85805	72349	20
10.62	567 44371 31021	4393 69067	71221	20
10.64	562 99784 36627	4342 23551	70113	19
10.66	558 59539 65784	4291 48148	69024	19
10.68	554 23586 43089	4241 41768	67954	19
10.70	549 91874 62162	4192 03343	66903	18
10.72	545 64354 84578	4143 31821	65870	18
10.74	541 40978 38815	4095 26168	64855	18
10.76	537 21697 19220	4047 85367	63857	17
10.78	533 06463 84995	4001 08429	62877	17
10.80	528 95231 59198	3954 94364	61913	17
10.82	524 87954 27766	3909 42213	60966	16
10.84	520 84586 38546	3864 51027	60035	16
10.86	516 85083 00353	3820 19877	59121	16
10.88	512 89399 82038	3776 47848	58222	15
10.90	508 97493 11570	3733 34040	57338	15
10.92	505 09319 75142	3690 77570	56469	15
10.94	501 24837 16284	3648 77569	55615	14
10.96	497 44003 34994	3607 33183	54776	14
10.98	493 66776 86887	3566 43572	53950	14
11.00	—.(3) 489 93116 82353	3526 07912	53139	14

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
11.00	−.(3)489 93116 82353	3526 07912	53139	14
11.02	486 22982 85731	3486 25392	52342	13
11.04	482 56335 14500	3446 95212	51557	13
11.06	478 93134 38482	3408 16590	50786	13
11.08	475 33341 79054	3369 88755	50028	13
11.10	471 76919 08381	3332 10947	49283	12
11.12	468 23828 48655	3294 82423	48550	12
11.14	464 74032 71352	3258 02449	47829	12
11.16	461 27494 96498	3221 70304	47121	12
11.18	457 84178 91948	3185 85280	46424	12
11.20	454 44048 72679	3150 46680	45739	11
11.22	451 07069 00089	3115 53818	45065	11
11.24	447 73204 81317	3081 06021	44402	11
11.26	444 42421 68566	3047 02626	43750	11
11.28	441 14685 58442	3013 42981	43109	11
11.30	437 89962 91299	2980 26445	42478	10
11.32	434 68220 50600	2947 52387	41858	10
11.34	431 49425 62288	2915 20186	41248	10
11.36	428 33545 94163	2883 29234	40648	10
11.38	425 20549 55271	2851 78930	40058	10
11.40	422 10404 95310	2820 68683	39477	9
11.42	419 03081 04031	2789 97913	38906	9
11.44	415 98547 10666	2759 66049	38344	9
11.46	412 96772 83349	2729 72528	37791	9
11.48	409 97728 28560	2700 16798	37247	9
11.50	407 01383 90570	2670 98315	36712	9
11.52	404 07710 50894	2642 16544	36185	8
11.54	401 16679 27763	2613 70958	35667	8
11.56	398 28261 75591	2585 61040	35158	8
11.58	395 42429 84458	2557 86279	34656	8
11.60	392 59155 79604	2530 46174	34163	8
11.62	389 78412 20924	2503 40232	33677	8
11.64	387 00172 02477	2476 67967	33199	8
11.66	384 24408 51996	2450 28901	32729	7
11.68	381 51095 30417	2424 22564	32266	7
11.70	378 80206 31401	2398 48493	31811	7
11.72	376 11715 80879	2373 06233	31363	7
11.74	373 45598 36590	2347 95335	30921	7
11.76	370 81828 87636	2323 15359	30487	7
11.78	368 20382 54042	2298 65870	30060	7
11.80	365 61234 86317	2274 46441	29639	7
11.82	363 04361 65033	2250 56651	29225	7
11.84	360 49739 00401	2226 96087	28818	6
11.86	357 97343 31855	2203 64340	28417	6
11.88	355 47151 27650	2180 61010	28022	6
11.90	352 99139 84454	2157 85702	27633	6
11.92	350 53286 26961	2135 38027	27250	6
11.94	348 09568 07494	2113 17603	26874	6
11.96	345 67963 05630	2091 24052	26503	6
11.98	343 28449 27818	2069 57004	26138	6
12.00	−.(3)340 91005 07011	2048 16094	25778	6

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
12.00	−.(3)340 91005 07011	2048 16094	25778	6
12.02	338 55609 02298	2027 00963	25424	5
12.04	336 22239 98547	2006 11256	25076	5
12.06	333 90877 06053	1985 46625	24733	5
12.08	331 61499 60183	1965 06727	24395	5
12.10	329 34087 21040	1944 91223	24062	5
12.12	327 08619 73120	1924 99783	23735	5
12.14	324 85077 24984	1905 32077	23412	5
12.16	322 63440 08923	1885 87783	23095	5
12.18	320 43688 80647	1866 66584	22782	5
12.20	318 25804 18954	1847 68167	22474	5
12.22	316 09767 25429	1828 92224	22170	5
12.24	313 95559 24127	1810 38451	21872	5
12.26	311 83161 61276	1792 06550	21577	4
12.28	309 72556 04975	1773 96226	21288	4
12.30	307 63724 44900	1756 07190	21002	4
12.32	305 56648 92015	1738 39156	20721	4
12.34	303 51311 78285	1720 91843	20444	4
12.36	301 47695 56398	1703 64974	20171	4
12.38	299 45782 99484	1686 58276	19903	4
12.40	297 45557 00846	1669 71481	19638	4
12.42	295 47000 73689	1653 04324	19377	4
12.44	293 50097 50856	1636 56544	19120	4
12.46	291 54830 84568	1620 27885	18867	4
12.48	289 61184 46164	1604 18093	18618	4
12.50	287 69142 25854	1588 26920	18373	4
12.52	285 78688 32463	1572 54119	18131	4
12.54	283 89806 93191	1556 99448	17892	4
12.56	282 02482 53367	1541 62670	17658	3
12.58	280 16699 76213	1526 43550	17426	3
12.60	278 32443 42610	1511 41856	17198	3
12.62	276 49698 50862	1496 57360	16974	3
12.64	274 68450 16475	1481 89838	16752	3
12.66	272 88683 71926	1467 39068	16534	3
12.68	271 10384 66444	1453 04832	16319	3
12.70	269 33538 65795	1438 86916	16108	3
12.72	267 58131 52062	1424 85107	15899	3
12.74	265 84149 23435	1410 99197	15693	3
12.76	264 11577 94006	1397 28981	15491	3
12.78	262 40403 93558	1383 74254	15291	3
12.80	260 70613 67364	1370 34819	15094	3
12.82	259 02193 75989	1357 10478	14900	3
12.84	257 35130 95093	1344 01037	14709	3
12.86	255 69412 15234	1331 06305	14520	3
12.88	254 05024 41680	1318 26094	14335	3
12.90	252 41954 94220	1305 60217	14152	3
12.92	250 80191 06976	1293 08492	13971	3
12.94	249 19720 28224	1280 70738	13793	3
12.96	247 60530 20210	1268 46777	13618	3
12.98	246 02608 58973	1256 36435	13445	2
13.00	−.(3)244 45943 34171	1244 39538	13275	2

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
13.00	—.(3)244 45943 34171	1244 39538	13275	2
13.02	242 90522 48908	1232 55916	13107	2
13.04	241 36334 19560	1220 85401	12942	2
13.06	239 83366 75614	1209 27828	12778	2
13.08	238 31608 59495	1197 83033	12617	2
13.10	236 81048 26409	1186 50855	12459	2
13.12	235 31674 44179	1175 31137	12303	2
13.14	233 83475 93085	1164 23720	12148	2
13.16	232 36441 65712	1153 28453	11996	2
13.18	230 90560 66791	1142 45181	11846	2
13.20	229 45822 13051	1131 73756	11699	2
13.22	228 02215 33068	1121 14030	11553	2
13.24	226 59729 67115	1110 65857	11409	2
13.26	225 18354 67019	1100 29093	11268	2
13.28	223 78079 96016	1090 03597	11128	2
13.30	222 38895 28610	1079 89229	10990	2
13.32	221 00790 50434	1069 85852	10855	2
13.34	219 63755 58109	1059 93329	10721	2
13.36	218 27780 59114	1050 11527	10589	2
13.38	216 92855 71644	1040 40313	10458	2
13.40	215 58971 24488	1030 79557	10330	2
13.42	214 26117 56889	1021 29132	10203	2
13.44	212 94285 18422	1011 88909	10078	2
13.46	211 63464 68864	1002 58765	9955	2
13.48	210 33646 78071	993 38576	9834	2
13.50	209 04822 25854	984 28220	9714	2
13.52	207 76982 01857	975 27578	9595	2
13.54	206 50117 05439	966 36532	9479	2
13.56	205 24218 45553	957 54964	9364	2
13.58	203 99277 40631	948 82760	9250	2
13.60	202 75285 18470	940 19807	9138	2
13.62	201 52233 16115	931 65992	9028	1
13.64	200 30112 79752	923 21205	8919	1
13.66	199 08915 64594	914 85337	8812	1
13.68	197 88633 34774	906 58281	8706	1
13.70	196 69257 63235	898 39931	8601	1
13.72	195 50780 31628	890 30183	8498	1
13.74	194 33193 30203	882 28932	8396	1
13.76	193 16488 57710	874 36078	8296	1
13.78	192 00658 21296	866 51520	8197	1
13.80	190 85694 36402	858 75160	8099	1
13.82	189 71589 26668	851 06898	8003	1
13.84	188 58335 23832	843 46640	7908	1
13.86	187 45924 67635	835 94289	7814	1
13.88	186 34350 05728	828 49753	7722	1
13.90	185 23603 93573	821 12938	7630	1
13.92	184 13678 94357	813 83753	7540	1
13.94	183 04567 78894	806 62109	7451	1
13.96	181 96263 25540	799 47915	7363	1
13.98	180 88758 20102	792 41085	7277	1
14.00	—.(3)179 82045 55748	785 41532	7191	1

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
14.00	−.(3) 179 82045 55748	785 41532	7191	15.00	−.(3) 135 19619 18752	512 20294	4069
14.02	178 76118 32927	778 49170	7107	15.02	134 45430 28254	507 99591	4024
14.04	177 70969 59276	771 63915	7024	15.04	133 71749 37347	503 82912	3980
14.06	176 66592 49540	764 85684	6942	15.06	132 98572 29352	499 70213	3937
14.08	175 62980 25488	758 14394	6861	15.08	132 25894 91569	495 61451	3894
14.10	174 60126 15830	751 49965	6781	15.10	131 53713 15238	491 56584	3852
14.12	173 58023 56138	744 92317	6702	15.12	130 82022 95491	487 55569	3810
14.14	172 56665 88762	738 41370	6624	15.14	130 10820 31313	483 58363	3769
14.16	171 56046 62757	731 97048	6547	15.16	129 40101 25498	479 64927	3728
14.18	170 56159 33799	725 59272	6471	15.18	128 69861 84609	475 75218	3688
14.20	169 56997 64113	719 27967	6396	15.20	128 00098 18938	471 89197	3648
14.22	168 58555 22394	713 03058	6322	15.22	127 30806 42465	468 06824	3609
14.24	167 60825 83732	706 84471	6249	15.24	126 61982 72815	464 28059	3570
14.26	166 63803 29542	700 72134	6177	15.26	125 93623 31224	460 52864	3531
14.28	165 67481 47486	694 65974	6106	15.28	125 25724 42498	456 81201	3493
14.30	164 71854 31404	688 65920	6036	15.30	124 58282 34973	453 13031	3456
14.32	163 76915 81241	682 71902	5967	15.32	123 91293 40479	449 48316	3419
14.34	162 82660 02981	676 83851	5898	15.34	123 24753 94301	445 87021	3382
14.36	161 89081 08572	671 01698	5831	15.36	122 58660 35144	442 29108	3346
14.38	160 96173 15860	665 25376	5764	15.38	121 93009 05096	438 74541	3311
14.40	160 03930 48524	659 54818	5698	15.40	121 27796 49588	435 23285	3275
14.42	159 12347 36006	653 89959	5634	15.42	120 63019 17366	431 75304	3240
14.44	158 21418 13446	648 30733	5569	15.44	119 98673 60447	428 30563	3206
14.46	157 31137 21619	642 77076	5506	15.46	119 34756 34092	424 89029	3172
14.48	156 41499 06869	637 28926	5444	15.48	118 71263 96766	421 50666	3138
14.50	155 52498 21044	631 86220	5382	15.50	118 08193 10106	418 15442	3105
14.52	154 64129 21439	626 48895	5321	15.52	117 45540 38888	414 83323	3072
14.54	153 76386 70730	621 16892	5261	15.54	116 83302 50992	411 54276	3040
14.56	152 89265 36912	615 90149	5202	15.56	116 21476 17372	408 28269	3008
14.58	152 02759 93243	610 68609	5143	15.58	115 60058 12022	405 05270	2976
14.60	151 16865 18184	605 52211	5085	15.60	114 99045 11941	401 85247	2945
14.62	150 31575 95335	600 40899	5028	15.62	114 38433 97107	398 68169	2914
14.64	149 46887 13386	595 34615	4972	15.64	113 78221 50443	395 54005	2883
14.66	148 62793 66052	590 33302	4916	15.66	113 18404 57784	392 42725	2853
14.68	147 79290 52020	585 36906	4861	15.68	112 58980 07849	389 34297	2823
14.70	146 96372 74893	580 45370	4807	15.70	111 99944 92212	386 28694	2794
14.72	146 14035 43137	575 58641	4753	15.72	111 41296 05269	383 25884	2765
14.74	145 32273 70022	570 76666	4700	15.74	110 83030 44209	380 25838	2736
14.76	144 51082 73572	565 99390	4648	15.76	110 25145 08988	377 28529	2708
14.78	143 70457 76513	561 26762	4596	15.78	109 67637 02296	374 33928	2679
14.80	142 90394 06215	556 58730	4545	15.80	109 10503 29532	371 42005	2652
14.82	142 10886 94648	551 95244	4495	15.82	108 53740 98772	368 52734	2624
14.84	141 31931 78324	547 36252	4445	15.84	107 97347 20747	365 66088	2597
14.86	140 53523 98253	542 81706	4396	15.86	107 41319 08810	362 82038	2570
14.88	139 75658 99888	538 31555	4348	15.88	106 85653 78911	360 00558	2544
14.90	138 98332 33078	533 85752	4298	15.90	106 30348 49570	357 21622	2517
14.92	138 21539 52021	529 44249	4252	15.92	105 75400 41851	354 45203	2491
14.94	137 45276 15212	525 06998	4206	15.94	105 20806 79335	351 71276	2466
14.96	136 69537 85402	520 73952	4159	15.96	104 66564 88095	348 99814	2440
14.98	135 94320 29544	516 45066	4114	15.98	104 12671 96669	346 30793	2415
15.00	−.(3) 135 19619 18752	512 20294	4069	16.00	−.(3) 103 59125 36036	343 64187	2391

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
16.00	−.(3)103 59125 36036	343 64187	2391	17.00	−.(4) 80 70307 00098	236 35273	1452
16.02	103 05922 39590	340 99972	2366	17.02	80 31355 66316	234 64468	1438
16.04	102 53060 43116	338 38123	2342	17.04	79 92638 97003	232 95102	1424
16.06	102 00536 84766	335 78616	2318	17.06	79 54155 22791	231 27159	1410
16.08	101 48349 05031	333 21428	2295	17.08	79 15902 75738	229 60627	1397
16.10	100 96494 46725	330 66533	2271	17.10	78 77879 89312	227 95492	1384
16.12	−.(3)100 44970 54951	328 13910	2248	17.12	78 40084 98378	226 31740	1370
16.14	−.(4) 99 93774 77088	325 63535	2225	17.14	78 02516 39184	224 69359	1357
16.16	99 42904 62761	323 15386	2203	17.16	77 65172 49350	223 08335	1344
16.18	98 92357 63818	320 69438	2180	17.18	77 28051 67850	221 48655	1331
16.20	98 42131 34315	318 25672	2158	17.20	76 91152 35005	219 90307	1319
16.22	97 92223 30483	315 84063	2136	17.22	76 54472 92467	218 33277	1306
16.24	97 42631 10714	313 44591	2115	17.24	76 18011 83207	216 77554	1294
16.26	96 93352 35536	311 07234	2094	17.26	75 81767 51500	215 23125	1282
16.28	96 44384 67592	308 71971	2073	17.28	75 45738 42919	213 69977	1269
16.30	95 95725 71620	306 38780	2052	17.30	75 09923 04314	212 18099	1257
16.32	95 47373 14427	304 07641	2031	17.32	74 74319 83808	210 67478	1246
16.34	94 99324 64875	301 78533	2011	17.34	74 38927 30780	209 18103	1234
16.36	94 51577 93856	299 51436	1991	17.36	74 03743 95855	207 69961	1222
16.38	94 04130 74273	297 26329	1971	17.38	73 68768 30891	206 23042	1211
16.40	93 56980 81019	295 03193	1951	17.40	73 33998 88970	204 77334	1199
16.42	93 10125 90958	292 82008	1932	17.42	72 99434 24382	203 32824	1188
16.44	92 63563 82905	290 62755	1912	17.44	72 65072 92618	201 89503	1177
16.46	92 17292 37607	288 45414	1893	17.46	72 30913 50358	200 47359	1166
16.48	91 71309 37723	286 29966	1874	17.48	71 96954 55457	199 06381	1155
16.50	91 25612 67805	284 16393	1856	17.50	71 63194 66937	197 66557	1144
16.52	90 80200 14280	282 04675	1837	17.52	71 29632 44974	196 27878	1133
16.54	90 35069 65430	279 94795	1819	17.54	70 96266 50888	194 90332	1123
16.56	89 90219 11376	277 86735	1801	17.56	70 63095 47135	193 53909	1112
16.58	89 45646 44056	275 80475	1783	17.58	70 30117 97291	192 18599	1102
16.60	89 01349 57212	273 75999	1766	17.60	69 97332 66046	190 84390	1092
16.62	88 57326 46367	271 73289	1748	17.62	69 64738 19191	189 51274	1082
16.64	88 13575 08811	269 72327	1731	17.64	69 32333 23609	188 19239	1072
16.66	87 70093 43582	267 73097	1714	17.66	69 00116 47267	186 88276	1062
16.68	87 26879 51451	265 75581	1697	17.68	68 68086 59199	185 58374	1052
16.70	86 83931 34900	263 79762	1681	17.70	68 36242 29506	184 29524	1042
16.72	86 41246 98111	261 85624	1664	17.72	68 04582 29338	183 01717	1033
16.74	85 98824 46946	259 93150	1648	17.74	67 73105 30887	181 74943	1023
16.76	85 56661 88932	258 02325	1632	17.76	67 41810 07379	180 49191	1014
16.78	85 14757 33242	256 13131	1616	17.78	67 10695 33061	179 24453	1004
16.80	84 73108 90682	254 25553	1600	17.80	66 79759 83196	178 00719	995
16.82	84 31714 73676	252 39575	1585	17.82	66 49002 34050	176 77980	986
16.84	83 90572 96245	250 55182	1569	17.84	66 18421 62884	175 56228	977
16.86	83 49681 73996	248 72359	1554	17.86	65 88016 47947	174 35452	968
16.88	83 09039 24107	246 91089	1539	17.88	65 57785 68461	173 15644	959
16.90	82 68643 65306	245 11358	1524	17.90	65 27728 04619	171 96795	950
16.92	82 28493 17864	243 33152	1509	17.92	64 97842 37572	170 78897	942
16.94	81 88586 03573	241 56454	1495	17.94	64 68127 49422	169 61940	933
16.96	81 48920 45738	239 81252	1480	17.96	64 38582 23213	168 45917	925
16.98	81 09494 69153	238 07529	1466	17.98	64 09205 42920	167 30818	916
17.00	−.(4) 80 70307 00098	236 35273	1452	18.00	−.(4) 63 79995 93445	166 16635	908

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
18.00	−.(4) 63 79995 93445	166 16635	908	19.00	−.(4) 51 09864 34881	119 12417	583
18.02	63 50952 60604	165 03360	900	19.02	50 87853 84689	118 35568	578
18.04	63 22074 31124	163 90985	891	19.04	50 65961 70064	117 59297	573
18.06	62 93359 92628	162 79501	883	19.06	50 44187 14737	116 83599	568
18.08	62 64808 33633	161 68900	875	19.08	50 22529 43009	116 08468	563
18.10	62 36418 43539	160 59175	868	19.10	50 00987 79748	115 33901	558
18.12	62 08189 12620	159 50318	860	19.12	49 79561 50389	114 59891	553
18.14	61 80119 32018	158 42320	852	19.14	49 58249 80921	113 86435	549
18.16	61 52207 93737	157 35174	844	19.16	49 37051 97888	113 13527	544
18.18	61 24453 90630	156 28872	837	19.18	49 15967 28382	112 41164	539
18.20	60 96856 16395	155 23407	829	19.20	48 94995 00040	111 69339	535
18.22	60 69413 65567	154 18772	822	19.22	48 74134 41038	110 98050	530
18.24	60 42125 33512	153 14958	814	19.24	48 53384 80085	110 27290	526
18.26	60 14990 16414	152 11958	807	19.26	48 32745 46422	109 57056	521
18.28	59 88007 11273	151 09765	800	19.28	48 12215 69816	108 87344	517
18.30	59 61175 15898	150 08372	793	19.30	47 91794 80553	108 18148	512
18.32	59 34493 28895	149 07772	786	19.32	47 71482 09438	107 49464	508
18.34	59 07960 49664	148 07958	779	19.34	47 51276 87788	106 81289	504
18.36	58 81575 78391	147 08922	772	19.36	47 31178 47426	106 13618	500
18.38	58 55338 16039	146 10657	763	19.38	47 11186 20683	105 46446	495
18.40	58 29246 64345	145 13158	758	19.40	46 91299 40385	104 79769	491
18.42	58 03300 25809	144 16417	751	19.42	46 71517 39855	104 13583	487
18.44	57 77498 03689	143 20427	745	19.44	46 51839 52909	103 47885	483
18.46	57 51839 01996	142 25181	738	19.46	46 32265 13848	102 82669	479
18.48	57 26322 25484	141 30674	732	19.48	46 12793 57457	102 17933	475
18.50	57 00946 79646	140 36898	725	19.50	45 93424 18998	101 53671	471
18.52	56 75711 70706	139 43847	719	19.52	45 74156 34211	100 89880	467
18.54	56 50616 05613	138 51515	712	19.54	45 54989 39304	100 26557	463
18.56	56 25658 92035	137 59895	706	19.56	45 35922 70953	99 63696	459
18.58	56 00839 38352	136 68982	700	19.58	45 16955 66298	99 01294	455
18.60	55 76156 53651	135 78768	694	19.60	44 98087 62938	98 39348	452
18.62	55 51609 47718	134 89248	688	19.62	44 79317 98925	97 77853	448
18.64	55 27197 31032	134 00415	682	19.64	44 60646 12765	97 16807	444
18.66	55 02919 14762	133 12264	676	19.66	44 42071 43412	96 56204	440
18.68	54 78774 10755	132 24789	670	19.68	44 23593 30263	95 96042	437
18.70	54 54761 31537	131 37983	664	19.70	44 05211 13156	95 36316	433
18.72	54 30879 90303	130 51841	658	19.72	43 86924 32364	94 77024	430
18.74	54 07129 00909	129 66358	652	19.74	43 68732 28596	94 18161	426
18.76	53 83507 77873	128 81526	647	19.76	43 50634 42989	93 59724	422
18.78	53 60015 36363	127 97341	641	19.78	43 32630 17106	93 01710	419
18.80	53 36650 92195	127 13798	635	19.80	43 14718 92933	92 44114	416
18.82	53 13413 61824	126 30889	630	19.82	42 96900 12874	91 86934	412
18.84	52 90302 62342	125 48611	624	19.84	42 79173 19749	91 30167	409
18.86	52 67317 11471	124 66957	619	19.86	42 61537 56791	90 73808	405
18.88	52 44456 27557	123 85922	614	19.88	42 43992 67641	90 17854	402
18.90	52 21719 29566	123 05501	608	19.90	42 26537 96345	89 62303	399
18.92	51 99105 37075	122 25688	603	19.92	42 09172 87352	89 07150	395
18.94	51 76613 70272	121 46479	598	19.94	41 91896 85509	88 52393	392
18.96	51 54243 49948	120 67867	593	19.96	41 74709 36059	87 98028	389
18.98	51 31993 97491	119 89848	588	19.98	41 57609 84636	87 44051	386
19.00	−.(4) 51 09864 34881	119 12417	583	20.00	−.(4) 41 40597 77264	86 90461	383

TABLE 28

THE HEXAGAMMA FUNCTION

Description: $\Psi^{(4)}(x)$ to 19 decimal places with central differences from $x = 20.0$ to $x = 100.0$ by increments of .1.

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
20.0	—.(4) 41405 97772 63967	21 72806 68771	2392 64903	451363
20.1	40568 30388 51272	21 07230 96395	2296 89161	428904
20.2	39751 70235 34972	20 43952 13180	2205 42323	407669
20.3	38955 54034 31852	19 82878 72289	2118 03154	387584
20.4	38179 20712 01021	19 23923 34551	2034 51569	368582
20.5	37422 11313 04741	18 67002 48382	1954 68566	350600
20.6	36683 68916 56844	18 12036 30779	1878 36163	333578
20.7	35963 38556 39725	17 58948 49339	1805 37338	317460
20.8	35260 67144 71946	17 07666 05237	1735 55972	302195
20.9	34575 03399 09403	16 58119 17107	1668 76802	287733
21.0	33905 97772 63967	16 10241 05778	1604 85365	274029
21.1	33253 02387 24309	15 63967 79815	1543 67956	261039
21.2	32615 70969 64466	15 19238 21807	1485 11586	248723
21.3	31993 58790 26429	14 75993 75385	1429 03939	237043
21.4	31386 22604 63778	14 34178 32902	1375 33335	225964
21.5	30793 20597 34028	13 93738 23754	1323 88695	215451
21.6	30214 12328 28031	13 54622 03301	1274 59506	205474
21.7	29648 58681 25336	13 16780 42354	1227 35792	196003
21.8	29096 21814 64994	12 80166 17199	1182 08080	187009
21.9	28556 65114 21852	12 44734 00123	1138 67377	178468
22.0	28029 53147 78832	12 10440 50425	1097 05142	170353
22.1	27514 51621 86238	11 77244 05869	1057 13261	162643
22.2	27011 27339 99511	11 45104 74573	1018 84022	155314
22.3	26519 48162 87358	11 13984 27299	982 10097	148347
22.4	26038 82970 02502	10 83845 90122	946 84520	141723
22.5	25569 01623 07769	10 54654 37464	913 00665	135422
22.6	25109 74930 50500	10 26375 85472	880 52232	129428
22.7	24660 74613 78703	9 98977 85712	849 33227	123724
22.8	24221 73274 92618	9 72429 19178	819 37945	118296
22.9	23792 44365 25712	9 46699 90590	790 60960	113128
23.0	23372 62155 49396	9 21761 22961	762 97102	108208
23.1	22962 01706 96040	8 97585 52434	736 41452	103522
23.2	22560 38843 95119	8 74146 23360	710 89324	99058
23.3	22167 50127 17558	8 51417 83610	686 36254	94805
23.4	21783 12828 23607	8 29375 80114	662 77989	90752
23.5	21407 04905 09771	8 07996 54607	640 10476	86889
23.6	21039 04978 50541	7 87257 39577	618 29851	83205
23.7	20678 92309 30888	7 67136 54397	597 32432	79693
23.8	20326 46776 65632	7 47613 01649	577 14706	76343
23.9	19981 48857 02025	7 28666 63607	557 73323	73148
24.0	19643 79604 02024	7 10277 98890	539 05088	70099
24.1	19313 20629 00911	6 92428 39257	521 06952	67189
24.2	18989 54082 39054	6 75099 86579	503 76005	64411
24.3	18672 62635 63777	6 58275 09906	487 09469	61759
24.4	18362 29463 98405	6 41937 42702	471 04693	59227
24.5	18058 38229 75735	6 26070 80191	455 59144	56809
24.6	17760 73066 33257	6 10659 76824	440 70403	54498
24.7	17469 18562 67602	5 95689 43859	426 36161	52291
24.8	17183 59748 45807	5 81145 47056	412 54210	50182
24.9	16903 82079 71067	5 67014 04462	399 22440	48165
25.0	—.(4) 16629 71425 00789	5 53281 84308	386 38836	46238

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
25.0	—.(4)16629 71425 00789	5 53281 84308	386 38836	46238
25.1	16361 14052 14820	5 39936 02990	374 01469	44395
25.2	16097 96615 31839	5 26964 23140	362 08497	42632
25.3	15840 06142 71999	5 14354 51788	350 58157	40946
25.4	15587 30024 63948	5 02095 38593	339 48764	39333
25.5	15339 56001 94489	4 90175 74163	328 78704	37792
25.6	15096 72154 99193	4 78584 88436	318 46435	36311
25.7	14858 66892 92333	4 67312 49145	308 50478	34901
25.8	14625 28943 34617	4 56348 60331	298 89422	33546
25.9	14396 47342 37233	4 45683 60941	289 61912	32251
26.0	14172 11425 00789	4 35308 23462	280 66652	31009
26.1	13952 10815 87807	4 25213 52635	272 02402	29821
26.2	13736 35420 27461	4 15390 84211	263 67973	28683
26.3	13524 75415 51326	4 05831 83760	255 62226	27587
26.4	13317 21242 58951	3 96528 45534	247 84066	26551
26.5	13113 63598 12109	3 87472 91375	240 32457	25540
26.6	12913 93426 56643	3 78657 69674	233 06389	24581
26.7	12718 01912 70851	3 70075 54361	226 04901	23662
26.8	12525 80474 39419	3 61719 43950	219 27076	22777
26.9	12337 20755 51938	3 53582 60615	212 72029	21931
27.0	12152 14619 25071	3 45658 49308	206 38911	21118
27.1	11970 54141 47513	3 37940 76913	200 26912	20238
27.2	11792 31604 46868	3 30423 31429	194 35150	19990
27.3	11617 39490 77651	3 23100 21096	188 63379	18272
27.4	11445 70477 29631	3 15965 74142	183 09880	18582
27.5	11277 17429 55753	3 09014 37067	177 74963	17423
27.6	11111 73396 18942	3 02240 74955	172 57468	16884
27.7	10949 31603 57085	2 95639 70311	167 56858	16278
27.8	10789 85450 65541	2 89206 22526	162 72526	15680
27.9	10633 28503 96522	2 82935 47267	158 03873	15148
28.0	10479 54492 74770	2 76822 75891	153 50369	14577
28.1	10328 57304 28909	2 70863 54884	149 11441	14068
28.2	10180 30979 37932	2 65053 45317	144 86581	13569
28.3	—.(4)10034 69707 92272	2 59388 22332	140 75290	13089
28.4	—.(5) 9891 67824 68944	2 53863 74636	136 77087	12627
28.5	9751 19805 20251	2 48476 04027	132 91512	12186
28.6	9613 20261 75586	2 43221 24930	129 18122	11756
28.7	9477 63939 55850	2 38095 63955	125 56489	11350
28.8	9344 45713 00068	2 33095 59468	122 06205	10953
28.9	9213 60582 03756	2 28217 61187	118 66875	10575
29.0	9085 03668 68630	2 23458 29780	115 38119	10210
29.1	8958 70213 63284	2 18814 36493	112 19573	9859
29.2	8834 55572 94431	2 14282 62778	109 10886	9521
29.3	8712 55214 88356	2 09859 99949	106 11720	9196
29.4	8592 64716 82230	2 05543 48840	103 21749	8882
29.5	8474 79762 24944	2 01330 19480	100 40661	8583
29.6	8358 96137 87138	1 97217 30782	97 68156	8290
29.7	8245 09730 80115	1 93202 10239	95 03940	8014
29.8	8133 16525 83330	1 89281 93638	92 47738	7742
29.9	8023 12602 80183	1 85454 24774	89 99279	7484
30.0	—.(5)7914 94134 01810	1 81716 55189	87 58303	7234

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^{6'}$
30.0	−.(5)7914 94134 01810	81716 55189	87 58303	7234
30.1	7808 57381 78627	78066 43908	85 24562	6994
30.2	7703 98695 99351	74501 57188	82 97815	6762
30.3	7601 14511 77264	71019 68283	80 77829	6540
30.4	7500 01347 23460	67618 57208	78 64384	6320
30.5	7400 55801 26864	64296 10516	76 57258	6122
30.6	7302 74551 40784	61050 21082	74 56255	5911
30.7	7206 54351 75785	57878 87903	72 61162	5726
30.8	7111 92030 98689	54780 15886	70 71795	5537
30.9	7018 84490 37479	51752 15664	68 87965	5358
31.0	6927 28701 91934	48793 03408	67 09494	5185
31.1	6837 21706 49796	45901 00645	65 36207	5018
31.2	6748 60612 08304	43074 34090	63 67938	4857
31.3	6661 42592 00901	40311 35472	62 04527	4703
31.4	6575 64883 28970	37610 41381	60 45819	4548
31.5	6491 24784 98422	34969 93110	58 91660	4414
31.6	6408 19656 60982	32388 36498	57 41915	4259
31.7	6326 46916 60041	29864 21801	55 96428	4142
31.8	6246 04040 80900	27396 03532	54 55084	3998
31.9	6166 88561 05290	24982 40347	53 17738	3879
32.0	6088 98063 70027	22621 94900	51 84271	3757
32.1	6012 30188 29664	20313 33725	50 54561	3621
32.2	5936 82626 23026	18055 27111	49 28472	3562
32.3	5862 53119 43499	15846 48987	48 05945	3400
32.4	5789 39459 12959	13685 76808	46 86818	3308
32.5	5717 39484 59227	11571 91446	45 70999	3215
32.6	5646 51081 96941	09503 77084	44 58395	3106
32.7	5576 72183 11739	07480 21118	43 48897	3007
32.8	5508 00764 47655	05500 14048	42 42406	2983
32.9	5440 34845 97619	03562 49385	41 38898	2715
33.0	5373 72489 96980	01666 23619	40 38104	2869
33.1	5308 11800 19949	99810 35958	39 40180	2606
33.2	5243 50920 78876	97993 88477	38 44861	2597
33.3	5179 88035 26279	96215 85856	37 52140	2509
33.4	5117 21365 59539	94475 35376	36 61928	2429
33.5	5055 49171 28176	92771 46823	35 74144	2365
33.6	4994 69748 43635	91103 32415	34 88726	2286
33.7	4934 81428 91508	89470 06732	34 05594	2224
33.8	4875 82579 46114	87870 86644	33 24685	2155
33.9	4817 71600 87364	86304 91240	32 45931	2092
34.0	4760 46927 19853	84771 41767	31 69270	2031
34.1	4704 07024 94109	83269 61564	30 94640	1971
34.2	4648 50392 29929	81798 76001	30 21980	1913
34.3	4593 75558 41750	80358 12419	29 51234	1859
34.4	4539 81082 65991	78947 00071	28 82348	1800
34.5	4486 65553 90303	77564 70071	28 15260	1757
34.6	4434 27589 84686	76210 55331	27 49930	1695
34.7	4382 65836 34399	74883 90521	26 86296	1654
34.8	4331 78966 74634	73584 12007	26 24315	1603
34.9	4281 65681 26875	72310 57808	25 63938	1558
35.0	.(5)4232 24706 36925	71062 67547	25 05119	1514

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
35.0	−.(5)4232 24706 36925	71062 67547	25 05119	1514
35.1	4183 54794 14521	69839 82406	24 47814	1470
35.2	4135 54721 74523	68641 45079	23 91979	1429
35.3	4088 23290 79604	67466 99731	23 37573	1388
35.4	4041 59326 84416	66315 91957	22 84555	1348
35.5	3995 61678 81184	65187 68738	22 32886	1312
35.6	3950 29218 46690	64081 78404	21 82528	1272
35.7	3905 60839 90601	62997 70600	21 33443	1240
35.8	3861 55459 05111	61934 96238	20 85598	1203
35.9	3818 12013 15859	60893 07474	20 38955	1170
36.0	3775 29460 34081	59871 57665	19 93483	1138
36.1	3733 06779 09968	58870 01340	19 49149	1106
36.2	3691 42967 87194	57887 94163	19 05920	1075
36.3	3650 37044 58584	56924 92906	18 63767	1047
36.4	3609 88046 22880	55980 55417	18 22661	1013
36.5	3569 95028 42594	55054 40589	17 82569	995
36.6	3530 57065 02896	54146 08330	17 43471	957
36.7	3491 73247 71529	53255 19542	17 05331	939
36.8	3453 42685 59704	52381 36085	16 68129	910
36.9	3415 64504 83964	51524 20756	16 31837	886
37.0	3378 37848 28980	50683 37265	15 96430	862
37.1	3341 61875 11260	49858 50203	15 61886	839
37.2	3305 35760 43744	49049 25028	15 28180	816
37.3	3269 58695 01255	48255 28032	14 95290	796
37.4	3234 29884 86799	47476 26327	14 63196	769
37.5	3199 48550 98670	46711 87818	14 31870	758
37.6	3165 13928 98359	45961 81179	14 01303	727
37.7	3131 25268 79227	45225 75844	13 71463	716
37.8	3097 81834 35939	44503 41971	13 42338	693
37.9	3064 82903 34622	43794 50437	13 13907	676
38.0	3032 27766 83742	43098 72810	12 86151	658
38.1	3000 15729 05673	42415 81334	12 59053	640
38.2	2968 46107 08937	41745 48912	12 32596	624
38.3	2937 18230 61114	41087 49085	12 06762	609
38.4	2906 31441 62375	40441 56021	11 81537	588
38.5	2875 85094 19658	39807 44494	11 56900	582
38.6	2845 78554 21434	39184 89868	11 32845	556
38.7	2816 11199 13079	38573 68087	11 09347	554
38.8	2786 82417 72811	37973 55653	10 86401	516
38.9	2757 91609 88195	37384 29620	10 63972	543
39.0	2729 38186 33196	36805 67559	10 42086	490
39.1	2701 21568 45755	36237 47584	10 20689	497
39.2	2673 41188 05899	35679 48299	9 99790	480
39.3	2645 96487 14342	35131 48803	9 79370	469
39.4	2618 86917 71587	34593 28677	9 59420	452
39.5	2592 11941 57508	34064 67970	9 39921	451
39.6	2565 71030 11400	33545 47185	9 20873	428
39.7	2539 63664 12477	33035 47273	9 02254	425
39.8	2513 89333 60827	32534 49615	8 84059	411
39.9	2488 47537 58791	32042 36016	8 66275	402
40.0	−.(5)2463 37783 92772	31558 88692	8 48893	391

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
40.0	—.(5)2463 37783 92772	31558 88692	8 48893	391
40.1	2438 59589 15445	31083 90261	8 31902	381
40.2	2414 12478 28379	30617 23733	8 15293	372
40.3	2389 95984 65045	30158 72497	7 99056	363
40.4	2366 09649 74209	29708 20318	7 83182	353
40.5	2342 53023 03691	29265 51320	7 67661	347
40.6	2319 25661 84494	28830 49984	7 52487	335
40.7	2296 27131 15280	28403 01134	7 37647	330
40.8	2273 57003 47201	27982 89932	7 23138	320
40.9	2251 14858 69053	27570 01867	7 08948	313
41.0	2229 00283 92772	27164 22749	6 95070	305
41.1	2207 12873 39241	26765 38702	6 81498	297
41.2	2185 52228 24411	26373 36152	6 68222	290
41.3	2164 17956 45734	25988 01825	6 55237	285
41.4	2143 09672 68881	25609 22734	6 42537	272
41.5	2122 26998 14762	25236 86180	6 30108	276
41.6	2101 69560 46824	24870 79735	6 17956	258
41.7	2081 36993 58621	24510 91245	6 06061	260
41.8	2061 28937 61662	24157 08816	5 94426	250
41.9	2041 45038 73519	23809 20812	5 83041	245
42.0	2021 84949 06187	23467 15849	5 71900	239
42.1	2002 48326 54706	23130 82787	5 60999	233
42.2	1983 34834 86011	22800 10723	5 50330	228
42.3	1964 44143 28039	22474 88990	5 39890	224
42.4	1945 75926 59056	22155 07146	5 29673	213
42.5	1927 29864 97220	21840 54974	5 19668	218
42.6	1909 05643 90357	21531 22471	5 09882	202
42.7	1891 02954 05966	21226 99851	5 00298	205
42.8	1873 21491 21426	20927 77528	4 90918	197
42.9	1855 60956 14414	20633 46124	4 81735	193
43.0	1838 21054 53527	20343 96456	4 72745	188
43.1	1821 01496 89095	20059 19532	4 63944	184
43.2	1804 01998 44196	19779 06553	4 55326	180
43.3	1787 22279 05849	19503 48899	4 46887	177
43.4	1770 62063 16401	19232 38132	4 38626	168
43.5	1754 21079 65086	18965 65991	4 30532	173
43.6	1737 99061 79761	18703 24382	4 22611	158
43.7	1721 95747 18819	18445 05385	4 14849	163
43.8	1706 10877 63261	18191 01236	4 07250	155
43.9	1690 44199 08940	17941 04338	3 99806	153
44.0	1674 95461 58956	17695 07245	3 92515	149
44.1	1659 64419 16218	17453 02668	3 85373	146
44.2	1644 50829 76148	17214 83463	3 78377	142
44.3	1629 54455 19541	16980 42635	3 71523	141
44.4	1614 75061 05569	16749 73331	3 64810	132
44.5	1600 12416 64928	16522 68837	3 58230	139
44.6	1585 66294 93124	16299 22572	3 51788	125
44.7	1571 36472 43893	16079 28096	3 45471	130
44.8	1557 22729 22757	15862 79091	3 39285	124
44.9	1543 24848 80712	15649 69370	3 33221	122
45.0	—.(5)1529 42618 08036	15439 92871	3 27280	119

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
45.0	—.(5)1529 42618 08036	15439 92871	3 27280	119
45.1	1515 75827 28232	15233 43652	3 21457	116
45.2	1502 24269 92079	15030 15890	3 15751	114
45.3	1488 87742 71816	14830 03879	3 10158	113
45.4	1475 66045 55432	14633 02026	3 04678	105
45.5	1462 58981 41074	14439 04852	2 99303	112
45.6	1449 66356 31568	14248 06981	2 94040	99
45.7	1436 87979 29043	14060 03149	2 88875	105
45.8	1424 23662 29666	13874 88192	2 83815	99
45.9	1411 73220 18482	13692 57051	2 78854	97
46.0	1399 36470 64349	13513 04765	2 73991	95
46.1	1387 13234 14981	13336 26469	2 69222	93
46.2	1375 03333 92081	13162 17395	2 64547	91
46.3	1363 06595 86575	12990 72868	2 59962	91
46.4	1351 22848 53938	12821 88303	2 55469	83
46.5	1339 51923 09603	12655 59207	2 51058	91
46.6	1327 93653 24475	12491 81169	2 46739	78
46.7	1316 47875 20516	12330 49870	2 42498	85
46.8	1305 14427 66426	12171 61069	2 38342	79
46.9	1293 93151 73405	12015 10609	2 34264	78
47.0	1282 83890 90994	11860 94414	2 30266	77
47.1	1271 86491 02996	11709 08484	2 26343	75
47.2	1261 00800 23483	11559 48898	2 22496	73
47.3	1250 26668 92868	11412 11808	2 18722	73
47.4	1239 63949 74061	11266 93440	2 15021	66
47.5	1229 12497 48693	11123 90093	2 11387	75
47.6	1218 72169 13419	10982 98133	2 07828	62
47.7	1208 42823 76278	10844 14001	2 04330	69
47.8	1198 24322 53138	10707 34199	2 00901	64
47.9	1188 16528 64198	10572 55298	1 97537	63
48.0	1178 19307 30555	10439 73934	1 94235	62
48.1	1168 32525 70846	10308 86805	1 90996	61
48.2	1158 56052 97943	10179 90671	1 87817	59
48.3	1148 89760 15711	10052 82355	1 84697	60
48.4	1139 33520 15833	9927 58735	1 81637	53
48.5	1129 87207 74690	9804 16752	1 78630	62
48.6	1120 50699 50300	9682 53399	1 75684	49
48.7	1111 23873 79308	9562 65730	1 72788	56
48.8	1102 06610 74046	9444 50850	1 69949	52
48.9	1092 98792 19634	9328 05919	1 67161	51
49.0	1084 00301 71141	9213 28148	1 64424	50
49.1	1075 11024 50796	9100 14801	1 61738	49
49.2	1066 30847 45252	8988 63192	1 59101	48
49.3	1057 59659 02901	8878 70684	1 56512	49
49.4	1048 97349 31233	8770 34687	1 53971	42
49.5	1040 43809 94251	8663 52661	1 51473	51
49.6	1031 98934 09932	8558 22109	1 49027	39
49.7	1023 62616 47721	8454 40584	1 46619	46
49.8	1015 34753 26094	8352 05678	1 44258	42
49.9	—.(5)1007 15242 10146	8251 15031	1 41939	42
50.0	—.(6) 999 03982 09227	8151 66323	1 39662	41

	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$	$-\delta^6$
50.0	.(6)999 03982 09227	8151 66323	1 39662	41
50.1	991 00873 74632	8053 57277	1 37426	40
50.2	983 05818 97313	7956 35657	1 35230	39
50.3	975 18721 05651	7861 49267	1 33073	40
50.4	967 39484 63255	7767 45951	1 30957	34
50.5	959 68015 66811	7674 73591	1 28874	43
50.6	952 04221 43957	7583 30105	1 26834	31
50.7	944 48010 51208	7493 13453	1 24825	38
50.8	936 99292 71912	7404 21626	1 22855	34
50.9	929 57979 14242	7316 52655	1 20919	34
51.0	922 23982 09227	7230 04602	1 19017	34
51.1	914 97215 08815	7144 75567	1 17149	33
51.2	907 77592 83969	7060 63680	1 15314	32
51.3	900 65031 22803	6977 67107	1 13510	33
51.4	893 59447 28744	6895 84044	1 11740	27
51.5	886 60759 18729	6815 12722	1 09997	36
51.6	879 68886 21436	6735 51397	1 08291	24
51.7	872 83748 75540	6656 98362	1 06608	32
51.8	866 05268 28006	6579 51936	1 04958	28
51.9	859 33367 32408	6503 10468	1 03336	28
52.0	852 67969 47278	6427 72337	1 01742	28
52.1	846 08999 34485	6353 35947	1 00176	27
52.2	839 56382 57639	6279 99734	98637	26
52.3	833 10045 80527	6207 62157	97124	28
52.4	826 69916 65572	6136 21704	95638	22
52.5	820 35923 72322	6065 76890	94175	31
52.6	814 07996 55961	5996 26250	92742	19
52.7	807 86065 65850	5927 68351	91328	27
52.8	801 70062 44090	5860 01781	89942	23
52.9	795 59919 24112	5793 25153	88578	23
53.0	789 55569 29287	5727 37102	87237	23
53.1	783 56946 71564	5662 36289	85920	22
53.2	777 63986 50130	5598 21395	84624	22
53.3	771 76624 50091	5534 91126	83351	24
53.4	765 94797 41179	5472 44208	82100	16
53.5	760 18442 76474	5410 79389	80867	27
53.6	754 47498 91159	5349 95438	79660	15
53.7	748 81905 01281	5289 91146	78468	23
53.8	743 21601 02549	5230 65322	77299	19
53.9	737 66527 69139	5172 16797	76149	19
54.0	732 16626 52526	5114 44420	75018	19
54.1	726 71839 80333	5057 47062	73906	18
54.2	721 32110 55202	5001 23608	72812	18
54.3	715 97382 53679	4945 72967	71736	19
54.4	710 67600 25124	4890 94062	70680	13
54.5	705 42708 90630	4836 85837	69637	23
54.6	700 22654 41975	4783 47250	68617	12
54.7	695 07383 40569	4730 77279	67609	19
54.8	689 96843 16442	4678 74918	66621	15
54.9	684 90981 67234	4627 39178	65647	16
55.0	.(6)679 89747 57204	4576 69085	64690	16

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
55.0	−.(6)679 89747 57204	4576 69085	64690
55.1	674 93090 16259	4526 63682	63749
55.2	670 00959 38995	4477 22028	62822
55.3	665 13305 83760	4428 43195	61911
55.4	660 30080 71719	4380 26275	61015
55.5	655 51235 85954	4332 70369	60133
55.6	650 76723 70557	4285 74596	59267
55.7	646 06497 29756	4239 38090	58412
55.8	641 40510 27046	4193 59997	57573
55.9	636 78716 84332	4148 39476	56747
56.0	632 21071 81094	4103 75703	55935
56.1	627 67530 53560	4059 67865	55135
56.2	623 18048 93890	4016 15161	54348
56.3	618 72583 49382	3973 16806	53574
56.4	614 31091 21681	3930 72025	52813
56.5	609 93529 66004	3888 80058	52061
56.6	605 59856 90386	3847 40151	51327
56.7	601 30031 54918	3806 51571	50599
56.8	597 04012 71022	3766 13590	49885
56.9	592 81760 00715	3726 25494	49182
57.0	588 63233 55903	3686 86579	48490
57.1	584 48393 97669	3647 96155	47809
57.2	580 37202 35590	3609 53539	47139
57.3	576 29620 27049	3571 58062	46479
57.4	572 25609 76571	3534 09064	45831
57.5	568 25133 35157	3497 05896	45189
57.6	564 28153 99638	3460 47917	44563
57.7	560 34635 12037	3424 34501	43941
57.8	556 44540 58937	3388 65026	43332
57.9	552 57834 70863	3353 38884	42732
58.0	548 74482 21674	3318 55474	42141
58.1	544 94448 27959	3284 14206	41560
58.2	541 17698 48451	3250 14498	40987
58.3	537 44198 83440	3216 55776	40423
58.4	533 73915 74205	3183 37478	39869
58.5	530 06816 02448	3150 59049	39320
58.6	526 42866 89740	3118 19941	38785
58.7	522 82035 96974	3086 19618	38254
58.8	519 24291 23825	3054 57548	37732
58.9	515 69601 08225	3023 33211	37219
59.0	512 17934 25836	2992 46093	36713
59.1	508 69259 89540	2961 95687	36215
59.2	505 23547 48930	2931 81496	35724
59.3	501 80766 89817	2902 03030	35241
59.4	498 40888 33733	2872 59804	34766
59.5	495 03882 37454	2843 51345	34295
59.6	491 69719 92519	2814 77181	33837
59.7	488 38372 24764	2786 36854	33381
59.8	485 09810 93864	2758 29907	32934
59.9	481 84007 92870	2730 55894	32493
60.0	−.(6)478 60935 47771	2703 14374	32058

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
60.0	−.(6)478 60935 47771	2703 14374	32058
60.1	475 40566 17045	2676 04912	31631
60.2	472 22872 91232	2649 27080	31209
60.3	469 07828 92498	2622 80458	30794
60.4	465 95407 74223	2596 64631	30387
60.5	462 85583 20579	2570 79189	29981
60.6	459 78329 46124	2545 23729	29588
60.7	456 73620 95398	2519 97857	29195
60.8	453 71432 42529	2495 01179	28810
60.9	450 71738 90840	2470 33312	28431
61.0	447 74515 72462	2445 93876	28057
61.1	444 79738 47961	2421 82497	27689
61.2	441 87383 05956	2397 98807	27326
61.3	438 97425 62757	2374 42443	26969
61.4	436 09842 62003	2351 13049	26618
61.5	433 24610 74297	2328 10272	26268
61.6	430 41706 96862	2305 33763	25929
61.7	427 61108 53190	2282 83183	25590
61.8	424 82792 92702	2260 58193	25259
61.9	422 06737 90406	2238 58463	24932
62.0	419 32921 46574	2216 83664	24609
62.1	416 61321 86405	2195 33475	24292
62.2	413 91917 59712	2174 07577	23978
62.3	411 24687 40595	2153 05657	23670
62.4	408 59610 27136	2132 27407	23367
62.5	405 96665 41084	2111 72524	23064
62.6	403 35832 27556	2091 40705	22772
62.7	400 77090 54734	2071 31659	22479
62.8	398 20420 13571	2051 45090	22192
62.9	395 65801 17497	2031 80715	21910
63.0	393 13214 02139	2012 38249	21631
63.1	390 62639 25029	1993 17413	21356
63.2	388 14057 65333	1974 17934	21085
63.3	385 67450 23570	1955 39539	20818
63.4	383 22798 21346	1936 81962	20556
63.5	380 80083 01084	1918 44941	20293
63.6	378 39286 25763	1900 28213	20041
63.7	376 00389 78654	1882 31525	19786
63.8	373 63375 63070	1864 54624	19538
63.9	371 28226 02111	1846 97262	19293
64.0	368 94923 38414	1829 59193	19052
64.1	366 63450 33910	1812 40175	18813
64.2	364 33789 69581	1795 39971	18578
64.3	362 05924 45224	1778 58345	18346
64.4	359 79837 79211	1761 95066	18121
64.5	357 55513 08265	1745 49906	17891
64.6	355 32933 87224	1729 22637	17672
64.7	353 12083 88819	1713 13040	17452
64.8	350 92947 03456	1697 20896	17236
64.9	348 75507 38987	1681 45987	17023
65.0	−.(6)346 59749 20506	1665 88102	16813

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
65.0	—.(6)346 59749 20506	1665 88102	16813
65.1	344 45656 90127	1650 47030	16606
65.2	342 33215 06778	1635 22564	16402
65.3	340 22408 45992	1620 14500	16200
65.4	338 13221 99707	1605 22637	16003
65.5	336 05640 76058	1590 46776	15804
65.6	333 99649 99186	1575 86721	15614
65.7	331 95235 09034	1561 42279	15422
65.8	329 92381 61161	1547 13260	15235
65.9	327 91075 26549	1532 99475	15049
66.0	325 91301 91411	1519 00740	14866
66.1	323 93047 57014	1505 16871	14686
66.2	321 96298 39487	1491 47688	14508
66.3	320 01040 69649	1477 93014	14333
66.4	318 07260 92825	1464 52672	14161
66.5	316 14945 68672	1451 26492	13987
66.6	314 24081 71012	1438 14298	13822
66.7	312 34655 87650	1425 15927	13654
66.8	310 46655 20216	1412 31210	13491
66.9	308 60066 83991	1399 59984	13329
67.0	306 74878 07751	1387 02087	13170
67.1	304 91076 33598	1374 57360	13012
67.2	303 08649 16804	1362 25645	12857
67.3	301 27584 25656	1350 06787	12704
67.4	299 47869 41295	1338 00633	12554
67.5	297 69492 57567	1326 07033	12402
67.6	295 92441 80871	1314 25834	12258
67.7	294 16705 30010	1302 56894	12111
67.8	292 42271 36042	1291 00065	11968
67.9	290 69128 42140	1279 55204	11827
68.0	288 97265 03442	1268 22171	11688
68.1	287 26669 86915	1257 00824	11550
68.2	285 57331 71212	1245 91028	11414
68.3	283 89239 46538	1234 92647	11280
68.4	282 22382 14510	1224 05545	11149
68.5	280 56748 88027	1213 29592	11016
68.6	278 92328 91137	1202 64656	10890
68.7	277 29111 58902	1192 10609	10761
68.8	275 67086 37277	1181 67324	10636
68.9	274 06242 82976	1171 34676	10513
69.0	272 46570 63351	1161 12540	10391
69.1	270 88059 56266	1151 00795	10270
69.2	269 30699 49975	1140 99320	10151
69.3	267 74480 43004	1131 07995	10033
69.4	266 19392 44029	1121 26705	9919
69.5	264 65425 71758	1111 55333	9802
69.6	263 12570 54820	1101 93762	9692
69.7	261 60817 31644	1092 41884	9578
69.8	260 10156 50353	1082 99584	9469
69.9	258 60578 68645	1073 66753	9360
70.0	—.(6)257 12074 53691	1064 43283	9253

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
70.0	-.(6)257 12074 53691	1064 43283	9253
70.1	255 64634 82019	1055 29065	9147
70.2	254 18250 39413	1046 23996	9043
70.3	252 72912 20802	1037 27969	8940
70.4	251 28611 30160	1028 40881	8839
70.5	249 85338 80400	1019 62633	8736
70.6	248 43085 93272	1010 93121	8640
70.7	247 01843 99266	1002 32248	8540
70.8	245 61604 37509	993 79916	8444
70.9	244 22358 55667	985 36027	8348
71.0	242 84098 09852	977 00487	8254
71.1	241 46814 64524	968 73200	8161
71.2	240 10499 92396	960 54074	8069
71.3	238 75145 74342	952 43018	7978
71.4	237 40743 99306	944 39940	7890
71.5	236 07286 64210	936 44751	7799
71.6	234 74765 73866	928 57362	7714
71.7	233 43173 40882	920 77686	7626
71.8	232 12501 85586	913 05637	7542
71.9	230 82743 35926	905 41130	7458
72.0	229 53890 27396	897 84080	7375
72.1	228 25935 02946	890 34405	7293
72.2	226 98870 12902	882 92023	7212
72.3	225 72688 14880	875 56852	7131
72.4	224 47381 73710	868 28812	7054
72.5	223 22943 61352	861 07827	6973
72.6	221 99366 56822	853 93814	
72.7	220 76643 46105	846 86701	6821
72.8	219 54767 22089	839 86408	6747
72.9	218 33730 84481	832 92863	6672
73.0	217 13527 39737	826 05990	6599
73.1	215 94150 00982	819 25716	6527
73.2	214 75591 87942	812 51968	6455
73.3	213 57846 26872	805 84677	6385
73.4	212 40906 50477	799 23769	6316
73.5	211 24765 97852	792 69178	6244
73.6	210 09418 14405	786 20831	6180
73.7	208 94856 51789	779 78664	6110
73.8	207 81074 67836	773 42607	6045
73.9	206 68066 26490	767 12594	5979
74.0	205 55824 97739	760 88561	5914
74.1	204 44344 57549	754 70442	5850
74.2	203 33618 87801	748 58173	5787
74.3	202 23641 76226	742 51692	5724
74.4	201 14407 16343	736 50935	5664
74.5	200 05909 07394	730 55841	5600
74.6	198 98141 54287	724 66349	5543
74.7	197 91098 67529	718 82399	5482
74.8	196 84774 63169	713 03931	5424
74.9	195 79163 62741	707 30887	5366
75.0	-.(6)194 74259 93200	701 63209	5308

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
75.0	−.(6)194 74259 93200	701 63209	5308
75.1	193 70057 86869	696 00839	5252
75.2	192 66551 81376	690 43721	5196
75.3	191 63736 19604	684 91798	5140
75.4	190 61605 49630	679 45015	5087
75.5	189 60154 24672	674 03320	5030
75.6	188 59377 03033	668 66654	4980
75.7	187 59268 48048	663 34968	4925
75.8	186 59823 28031	658 08207	4873
75.9	185 61036 16221	652 86320	4823
76.0	184 62901 90731	647 69255	4771
76.1	183 65415 34496	642 56961	4721
76.2	182 68571 35222	637 49388	4671
76.3	181 72364 85336	632 46487	4622
76.4	180 76790 81938	627 48208	4575
76.5	179 81844 26747	622 54503	4524
76.6	178 87520 26060	617 65323	4480
76.7	177 93813 90696	612 80623	4432
76.8	177 00720 35955	608 00354	4385
76.9	176 08234 81568	603 24470	4340
77.0	175 16352 51652	598 52927	4294
77.1	174 25068 74662	593 85678	4250
77.2	173 34378 83350	589 22679	4206
77.3	172 44278 14716	584 63885	4162
77.4	171 54762 09968	580 09254	4120
77.5	170 65826 14474	575 58743	4075
77.6	169 77465 77724	571 12307	4036
77.7	168 89676 53280	566 69907	3993
77.8	168 02453 98743	562 31499	3951
77.9	167 15793 75705	557 97042	3911
78.0	166 29691 49710	553 66497	3871
78.1	165 44142 90211	549 39823	3831
78.2	164 59143 70536	545 16980	3792
78.3	163 74689 67841	540 97929	3753
78.4	162 90776 63074	536 82630	3716
78.5	162 07400 40938	532 71048	3675
78.6	161 24556 89849	528 63140	3643
78.7	160 42242 01900	524 58875	3598
78.8	159 60451 72824	520 58208	3567
78.9	158 79182 01956	516 61108	3530
79.0	157 98428 92196	512 67538	3493
79.1	157 18188 49974	508 77461	3458
79.2	156 38456 85214	504 90843	3423
79.3	155 59230 11296	501 07648	3388
79.4	154 80504 45027	497 27841	3355
79.5	154 02276 06598	493 51389	3319
79.6	153 24541 19559	489 78257	3288
79.7	152 47296 10776	486 08413	3254
79.8	151 70537 10406	482 41822	3221
79.9	150 94260 51859	478 78453	3190
80.0	−.(6)150 18462 71764	475 18273	3157

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
80.0	−.(6)150 18462 71764	475 18273	3157
80.1	149 43140 09943	471 61251	3126
80.2	148 68289 09373	468 07353	3094
80.3	147 93906 16156	464 56550	3063
80.4	147 19987 79489	461 08811	3034
80.5	146 46530 51634	457 64105	3001
80.6	145 73530 87883	454 22400	2974
80.7	145 00985 46532	450 83670	2943
80.8	144 28890 88852	447 47882	2914
80.9	143 57243 79054	444 15009	2886
81.0	142 86040 84264	440 85021	2857
81.1	142 15278 74496	437 57889	2829
81.2	141 44954 22617	434 33586	2801
81.3	140 75064 04324	431 12084	2773
81.4	140 05604 98115	427 93355	2747
81.5	139 36573 85261	424 77372	2717
81.6	138 67967 49779	421 64106	2693
81.7	137 99782 78403	418 53533	2665
81.8	137 32016 60560	415 45626	2639
81.9	136 64665 88344	412 40358	2614
82.0	135 97727 56485	409 37704	2588
82.1	135 31198 62331	406 37638	2563
82.2	134 65076 05815	403 40135	2538
82.3	133 99356 89433	400 45169	2513
82.4	133 34038 18220	397 52717	2490
82.5	132 69116 99724	394 62754	2463
82.6	132 04590 43981	391 75254	2442
82.7	131 40455 63493	388 90196	2417
82.8	130 76709 73200	386 07555	2393
82.9	130 13349 90462	383 27306	2371
83.0	129 50373 35030	380 49429	2347
83.1	128 87777 29027	377 73899	2325
83.2	128 25558 96922	375 00694	2302
83.3	127 63715 65513	372 29792	2280
83.4	127 02244 63895	369 61169	2259
83.5	126 41143 23446	366 94806	2235
83.6	125 80408 77803	364 30679	2217
83.7	125 20038 62839	361 68767	2194
83.8	124 60030 16643	359 09050	2173
83.9	124 00380 79497	356 51506	2153
84.0	123 41087 93856	353 96114	2132
84.1	122 82149 04330	351 42854	2112
84.2	122 23561 57658	348 91706	2091
84.3	121 65323 02692	346 42649	2071
84.4	121 07430 90374	343 95663	2053
84.5	120 49882 73720	341 50730	2031
84.6	119 92676 07796	339 07828	2015
84.7	119 35808 49700	336 66941	1994
84.8	118 79277 58546	334 28048	1975
84.9	118 23080 95439	331 91129	1957
85.0	−.(6)117 67216 23461	329 56167	1938

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
85.0	—.(6)117 67216 23461	329 56167	1938
85.1	117 11681 07650	327 23144	1920
85.2	116 56473 14983	324 92040	1902
85.3	116 01590 14356	322 62839	1884
85.4	115 47029 76568	320 35521	1867
85.5	114 92789 74300	318 10070	1847
85.6	114 38867 82102	315 86467	1833
85.7	113 85261 76372	313 64696	1814
85.8	113 31969 35338	311 44740	1797
85.9	112 78988 39044	309 26581	1781
86.0	112 26316 69332	307 10203	1764
86.1	111 73952 09822	304 95589	1748
86.2	111 21892 45902	302 82723	1731
86.3	110 70135 64704	300 71588	1715
86.4	110 18679 55094	298 62168	1700
86.5	109 67522 07653	296 54449	1682
86.6	109 16661 14660	294 48412	1669
86.7	108 66094 70079	292 44044	1653
86.8	108 15820 69542	290 41329	1637
86.9	107 65837 10334	288 40251	1623
87.0	107 16141 91376	286 40795	1607
87.1	106 66733 13214	284 42947	1593
87.2	106 17608 77999	282 46692	1578
87.3	105 68766 89476	280 52014	1563
87.4	105 20205 52968	278 58900	1550
87.5	104 71922 75359	276 67336	1533
87.6	104 23916 65088	274 77306	1522
87.7	103 76185 32122	272 88798	1507
87.8	103 28726 87953	271 01796	1493
87.9	102 81539 45581	269 16288	1480
88.0	102 34621 19496	267 32259	1466
88.1	101 87970 25671	265 49697	1453
88.2	101 41584 81542	263 68587	1440
88.3	100 95463 06000	261 88918	1426
88.4	100 49603 19376	260 10674	1415
88.5	—.(6) 100 04003 43426	258 33845	1399
88.6	—.(7) 99 58662 01322	256 58416	1390
88.7	99 13577 17633	254 84376	1376
88.8	98 68747 18321	253 11712	1363
88.9	98 24170 30721	251 40410	1351
89.0	97 79844 83530	249 70460	1339
89.1	97 35769 06800	248 01848	1327
89.2	96 91941 31917	246 34564	1315
89.3	96 48359 91599	244 68594	1303
89.4	96 05023 19874	243 03927	1292
89.5	95 61929 52076	241 40552	1278
89.6	95 19077 24831	239 78456	1270
89.7	94 76464 76040	238 17629	1257
89.8	94 34090 44879	236 58059	1245
89.9	93 91952 71777	234 99734	1235
90.0	—.(7) 93 50049 98410	233 42645	1224

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
90.0	—.(7)93 50049 98410	233 42645	1224
90.1	93 08380 67687	231 86779	1213
90.2	92 66943 23744	230 32126	1202
90.3	92 25736 11927	228 78675	1191
90.4	91 84757 78785	227 26416	1182
90.5	91 44006 72059	225 75338	1169
90.6	91 03481 40671	224 25429	1161
90.7	90 63180 34712	222 76681	1150
90.8	90 23102 05435	221 29083	1139
90.9	89 83245 05240	219 82624	1130
91.0	89 43607 87669	218 37295	1120
91.1	89 04189 07394	216 93086	1110
91.2	88 64987 20205	215 49986	1100
91.3	88 26000 83001	214 07987	1090
91.4	87 87228 53785	212 67078	1081
91.5	87 48668 91646	211 27250	1071
91.6	87 10320 56758	209 88492	1063
91.7	86 72182 10361	208 50798	1053
91.8	86 34252 14763	207 14156	1043
91.9	85 96529 33321	205 78557	1035
92.0	85 59012 30436	204 43993	1025
92.1	85 21699 71545	203 10455	1017
92.2	84 84590 23108	201 77933	1008
92.3	84 47682 52604	200 46418	999
92.4	84 10975 28519	199 15903	991
92.5	83 74467 20336	197 86378	980
92.6	83 38156 98531	196 57834	975
92.7	83 02043 34561	195 30264	965
92.8	82 66125 00854	194 03659	956
92.9	82 30400 70807	192 78010	949
93.0	81 94869 18769	191 53309	940
93.1	81 59529 20039	190 29548	932
93.2	81 24379 50858	189 06719	924
93.3	80 89418 88397	187 84815	916
93.4	80 54646 10750	186 63826	909
93.5	80 20059 96929	185 43746	899
93.6	79 85659 26853	184 24565	894
93.7	79 51442 81342	183 06277	885
93.8	79 17409 42108	181 88875	877
93.9	78 83557 91749	180 72350	870
94.0	78 49887 13740	179 56695	862
94.1	78 16395 92426	178 41902	855
94.2	77 83083 13013	177 27965	848
94.3	77 49947 61565	176 14875	841
94.4	77 16988 24993	175 02627	835
94.5	76 84203 91047	173 91212	825
94.6	76 51593 48313	172 80623	821
94.7	76 19155 86202	171 70854	812
94.8	75 86889 94945	170 61898	805
94.9	75 54794 65587	169 53747	799
95.0	—.(7)75 22868 89977	168 46396	792

x	$\Psi^{(4)}(x)$	$-\delta^2$	$-\delta^4$
95.0	−.(7)75 22868 89977	168 46396	792
95.1	74 91111 60762	167 39836	786
95.2	74 59521 71383	166 34063	779
95.3	74 28098 16068	165 29068	772
95.4	73 96839 89819	164 24845	767
95.5	73 65745 88416	163 21389	758
95.6	73 34815 08402	162 18691	754
95.7	73 04046 47078	161 16747	747
95.8	72 73439 02501	160 15550	740
95.9	72 42991 73474	159 15093	735
96.0	72 12703 59539	158 15370	728
96.1	71 82573 60974	157 16376	721
96.2	71 52600 78786	156 18104	717
96.3	71 22784 14701	155 20548	709
96.4	70 93122 71164	154 23702	705
96.5	70 63615 51329	153 27561	697
96.6	70 34261 59055	152 32117	694
96.7	70 05059 98898	151 37367	687
96.8	69 76009 76108	150 43304	681
96.9	69 47109 96622	149 49921	676
97.0	69 18359 67057	148 57215	670
97.1	68 89757 94707	147 65178	665
97.2	68 61303 87534	146 73806	659
97.3	68 32996 54167	145 83092	653
97.4	68 04835 03893	144 93032	649
97.5	67 76818 46650	144 03622	641
97.6	67 48945 93030	143 14852	639
97.7	67 21216 54261	142 26722	632
97.8	66 93629 42214	141 39223	627
97.9	66 66183 69391	140 52351	622
98.0	66 38878 48919	139 66102	617
98.1	66 11712 94548	138 80470	612
98.2	65 84686 20648	137 95449	607
98.3	65 57797 42196	137 11036	602
98.4	65 31045 74780	136 27224	598
98.5	65 04430 34588	135 44010	591
98.6	64 77950 38406	134 61387	589
98.7	64 51605 03611	133 79352	582
98.8	64 25393 48168	132 97900	577
98.9	63 99314 90625	132 17026	574
99.0	63 73368 50109	131 36725	569
99.1	63 47553 46317	130 56992	564
99.2	63 21868 99518	129 77824	559
99.3	62 96314 30542	128 99215	555
99.4	62 70888 60782	128 21161	551
99.5	62 45591 12183	127 43658	544
99.6	62 204210 7242	126 66700	543
99.7	61 95377 69001	125 90284	537
99.8	61 70460 21044	125 14406	533
99.9	61 45667 87492	124 39060	530
100.0	−.(7)61 20999 93001	123 64244	526

THE
BERNOULLI POLYNOMIALS
AND
BERNOULLI NUMBERS

BERNOULLI POLYNOMIALS AND BERNOULLI NUMBERS.

1. *Definition of the Bernoulli Polynomials.* By the Bernoulli polynomials of the nth degree and mth order we shall mean the coefficients of $t^n/n!$ in the following development:

$$t^m e^{xt}/(e^t-1)^m = \sum_{n=0}^{\infty} B_n^{(m)}(x)\, t^n/n! \,, \qquad |t| < 2\pi \,.$$

For $m = 1$ we get the Bernoulli polynomials of first order (commonly referred to as the Bernoulli polynomials).* Explicitly these are,

$$B_0(x) = 1 \,.$$

$$B_1(x) = x - 1/2 \,,$$

$$B_2(x) = x^2 - x + 1/6 \,,$$

$$B_3(x) = x(x-1)(x-1/2) = x^3 - 3x^2/2 + x/2 \,,$$

$$B_4(x) = x^4 - 2x^3 + x^2 - 1/30 \,,$$

$$B_5(x) = x(x-1)(x-1/2)(x^2-x-1/3) = x^5 - 5x^4/2 + 5x^3/3 - x/6 \,,$$

$$B_6(x) = x^6 - 3x^5 + 5x^4/2 - x^2/2 + 1/42 \,,$$

$$B_7(x) = x^7 - 7x^6/2 + 7x^5/2 - 7x^3/6 + x/6 \,,$$

$$B_8(x) = x^8 - 4x^7 + 14x^6/3 - 7x^4/3 + 2x^2/3 - 1/30 \,,$$

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*There is not uniformity among authors in the definition of either the Bernoulli numbers or the Bernoulli polynomials. We notice that

$$B_0(0) = 1 \,, \quad B_1(0) = -\tfrac{1}{2} \,, \quad B_{2n+1}(0) = 0 \,,$$

$$B_{2n}(0) = (-1)^{n-1} B_n \,, \qquad n > 1.$$

Some authors prefer to call $B_n(0)$ the Bernoulli numbers rather than B_n; others designate the nth Bernoulli number by B_{2n+1} and the nth Euler number (See next part) by B_{2n}. E. T. Whittaker and G. N. Watson define the Bernoulli polynomials to be the series given above when it ends in either x or x^2; N. Nielsen adopts a definition which is equivalent to

$$\phi_n(x) = (-1)^n B_n(-x)/n!$$

The definition employed here coincides with the one used by N. E. Nörlund.

$$B_9(x) = x^9 - 9x^8/2 + 6x^7 - 21x^5/5 + 2x^3 - 3x/10 \ ,$$

$$B_{10}(x) = x^{10} - 5x^9 + 15x^8/2 - 7x^6 + 5x^4 - 3x^2/2 + 5/66 \ ,$$

$$B_{11}(x) = x^{11} - 11x^{10}/2 + 55x^9/6 - 11x^7 + 11x^5 - 11x^3/2 + 5x/6 \ ,$$

$$B_{12}(x) = x^{12} - 6x^{11} + 11x^{10} - 33x^8/2 + 22x^6 - 33x^4/2 + 5x^2 - 691/2730 \ ,$$

$$B_{13}(x) = x^{13} - 13x^{12}/2 + 13x^{11} - 143x^9/6 + 286x^7/7 - 429x^5/10 + 65x^3/3 - 691x/210 \ ,$$

$$B_{14}(x) = x^{14} - 7x^{13} + 91x^{12}/6 - 1001x^{10}/30 + 143x^8/2 - 1001x^6/10 + 455x^4/6 - 691x^2/30 + 7/6 \ ,$$

$$B_{15}(x) = x^{15} - 15x^{14}/2 + 35x^{13}/2 - 91x^{11}/2 + 715x^9/6 - 429x^7/2 + 455x^5/2 - 691x^3/6 + 35x/2 \ ,$$

$$B_n(x) = x^n - \tfrac{1}{2} n\, x^{n-1} + {}_nC_2\, B_1\, x^{n-2} - {}_nC_4\, B_2\, x^{n-4} + {}_nC_6\, B_3\, x^{n-6} - \cdots \text{ (Last term } x \text{ or a constant)} \ ,$$

$$= x^n - \tfrac{1}{2} n\, x^{n-1} + \sum_{m=1}^{\leq n/2} (-1)^{m-1}\, {}_nC_{2m}\, B_m\, x^{n-2m} \ ,$$

where ${}_nC_r = n!/(n-r)!\,r!$, and the coefficients, B_m, form the sequence,

$$B_0 = 1, \quad B_1 = 1/6, \quad B_2 = 1/30, \quad B_3 = 1/42, \quad B_4 = 1/30, \quad B_5 = 5/66 \ ,$$

$$B_6 = 691/2730, \quad B_7 = 7/6, \quad B_8 = 3617/510,$$

These constants are called the Bernoulli numbers and have been extensively tabulated. (See Tables 30, 31 and 32).

If we define the symbolic expression,

$$[x + B(0)]^{(n)} \ ,$$

to mean the development by the binomial theorem in which $B^n(0)$ is replaced in the final result by $B_n(0)$, then we may represent the Bernoulli polynomials very compactly by means of the formula,

$$B_n(x) = [x + B(0)]^{(n)} \ .$$

Setting $x = 1$ and recalling that $B_n(1) = B_n(0)$, except for $n = 1$, we may also write,

$$B_n(0) = [1 + B(0)]^{(n)} \ , \quad n \neq 1 \ .$$

Since $B_{2n+1}(0) = 0$, $B_{2n}(0) = (-1)^{n-1} B_n$, this gives us a simple way to compute B_n in terms of the Bernoulli numbers of lower order. This expansion is seen to be identical with (6.2).

2. Properties of the Bernoulli Polynomials of First Order. These polynomials, generated by the expansion,

$$t\, e^{xt}/\,(e^t-1) = \sum_{n=0}^{\infty} B_n(x)\; t^n/n! \ , \quad |\, t\,| < 2\pi \ ,$$

satisfy the difference equation,

$$B_n(x+1) - B_n(x) = n\, x^{n-1} \ ,$$

which is equivalent to the differential equation of infinite order,

$$B'_n(x) + B''_n(x)/2! + B^{(3)}{}_n(x)/3! + \cdots\cdots$$
$$+ B^{(m)}{}_n(x)/m! + \cdots\cdots = n\, x^{n-1} \ .$$

These polynomials also satisfy the equations,

$$B_n(1-x) = (-1)^n B_n(x) \ ,$$

$$B_n(kx) = k^{n-1} \sum_{r=0}^{k-1} B_n(x+r/k) \ ,$$

$$B_n(x+h) = \sum_{r=0}^{n} {}_nC_r\, B_{n-r}(x)\, h^r \ ,$$

$$\sum_{r=0}^{n-1} {}_nC_r\, B_r(x) = n\, x^{n-1} \ , \qquad n > 0 \ ,$$

$$\sum^{n-1} {}_nC_r\, B_r(0) = 0 \ .$$

The first derivative of the nth polynomial is computed from the polynomial of next lower degree by means of the equation,

$$B_n'(x) = n\, B_{n-1}(x) \ .$$

From the integral,

$$\int_x^{x+1} B_n(t)\ dt = x^n \ ,$$

we attain the celebrated sum,

$$S_n(k-1) = 1^n + 2^n + \cdots + (k-1)^n$$
$$= \int_0^k B_n(t)\ dt = \{B_{n+1}(k) - B_{n+1}(0)\}/(n+1) \ , \quad n \geqq 0 \ .\text{*}$$

The Bernoulli numbers have a semi-orthogonality property over the interval (0,1) as we see from the following formula:

$$\int_0^1 B_m(t)\ B_n(t)\ dt$$
$$= (-1)^{m+1} m!n!B_r/(2r)!,\ m, n > 0 \ ; \ m+n = 2r \ ;$$
$$= 0, \text{ when } m+n \text{ is an odd integer.}$$

We also note the integral,

$$\int_0^{1/2} B_m(t+1/2)\ B_n(t)\ dt = (-1)^{m+1}\ m!\ n!\ D_{2r}/[(2r)!\ 2^{2r+1}],$$
$$m, n > 0;\ m+n = 2r.$$

The numbers, D_n, which enter into this formula are intimately connected with the Bernoulli numbers and are defined to be,

$$D_{2n} = 2(1 - 2^{2n-1})\ (-1)^{n-1} B_n \ ,$$
$$D_{2n+1} = 0 \ .$$

We note the following relations:

$$\sum_{s=0}^{n} {}_nC_s\, D_{n-s} - \sum_{s=0}^{n} (-1)^s\, {}_nC_s\, D_{n-s} = 0 \ , \ n > 1;$$

$$B_n(1/2) = D_n/2^n \ , \quad B_{2n}(1/4) = D_{2n}/4^{2n} \ , B_{2n+1}(1/4)$$
$$= (-1)^{n+1}\ (2n+1)\ E_n/4^{2n+1} \ ,$$

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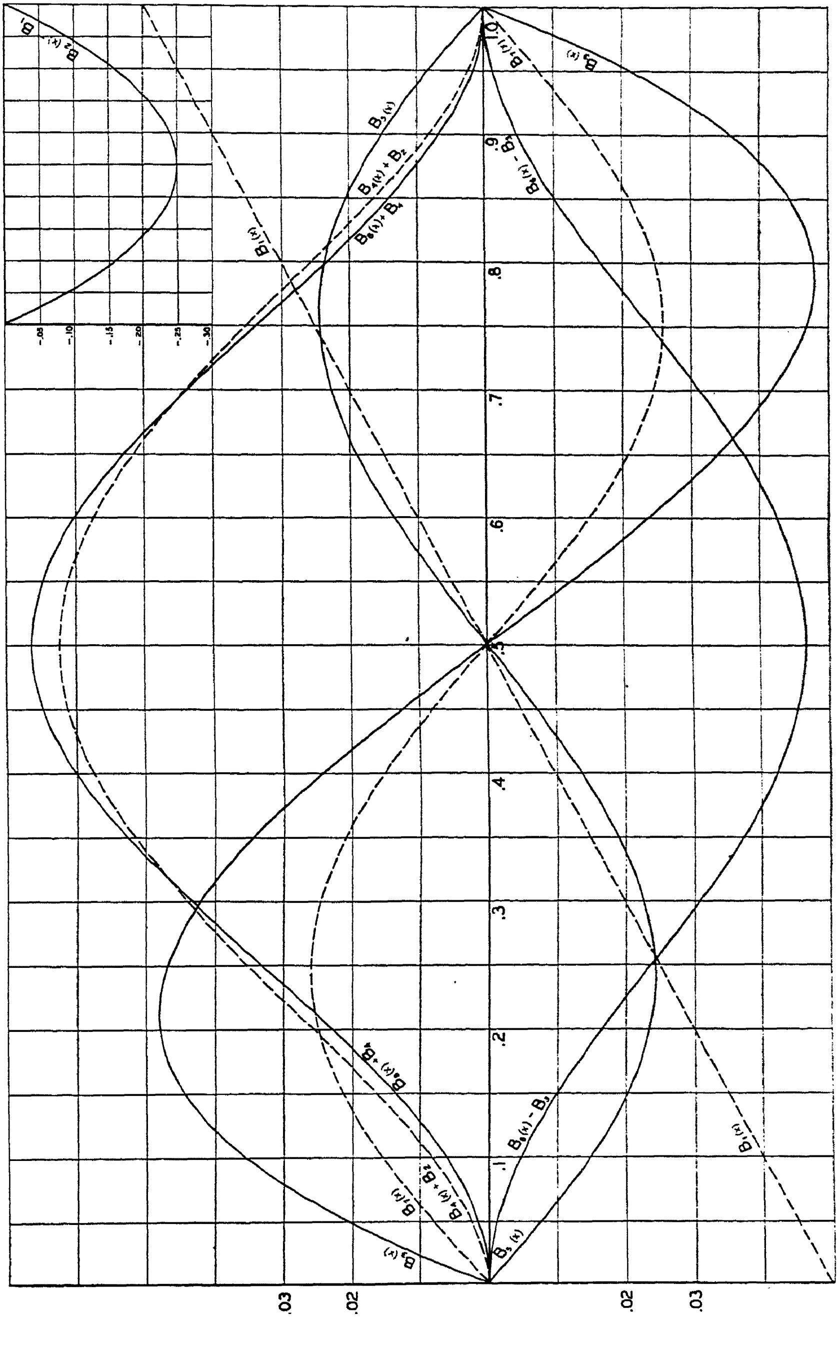
*The Bernoulli numbers were first given by Jakob (James) Bernoulli (1654-1705) in *Ars Conjectandi* (Basel, 1713, p. 97). He exhibited their usefulness in the summation just written down and boasted: "Huius laterculi beneficio intra semi-quadrantem horae reperi, quod potestates decimae sive quadrato-sursolida mille priorum numerorum ab unitate in summum collecta efficiunt

91, 409, 924, 241, 424, 243, 424, 241, 924, 242, 500."



THE B OULLI POLYNOM

where E_n is the nth Euler number (See Table 37);

$$B_n(x) = \sum_{s=0}^{n} {}_nC_s\, D_s(x-\tfrac{1}{2})^{n-s}/2^s \ .$$

Another set of numbers similar to the D_n is defined by the equations:

$$C_0 = 1 \ , \qquad C_{2n} = 0 \ , \qquad n > 0 \ ,$$
$$C_{2n-1} = 2^{2n-1}(1-2^{2n})(-1)^{n-1} B_n/n \ .$$

In terms of these numbers we may evaluate the integral of $B_n(t)$ between 0 and 1/2,

$$\int_0^{\frac{1}{2}} B_n(t)\, dt = C_n/2^{2n+1} \ .$$

The numbers C_n and D_n are found in the coefficients of the power series expansions of tan x and csc x respectively and are sometimes referred to as the *tangent* and *cosecant numbers.* We note the expansions:

$$\tan x = \sum_{n=1}^{\infty} (-1)^n C_{2n-1}\, x^{2n-1}/(2n-1)!$$

$$\csc x = 1/x + \sum_{n=1}^{\infty} (-1)^n D_{2n}\, x^{2n-1}/(2n)! \ .$$

Tables are provided below for both these numbers. A table of the values of C_n to $n = 29$ was computed by $=$ L. Saalschütz [See Supplementary Bibliography, Saalschütz, p. 23] and is reprinted in N. Nielsen: *Traité des Nombres de Bernoulli,* Paris (1923), p. 177. Nörlund [Bibliography, Nörlund, p. 458] gives values of C_n to $n = 19$ and D_n to $n = 20$. The tables given below were computed twice by R. E. Thompson and checked as far as possible against the existing values.

TABLE OF THE CONSTANTS D_n AND C_n

C_n	n
—1	1
2	3
—16	5
272	7
—7 936	9
353 792	11
—22 368 256	13
1 903 757 312	15
—209 865 342 976	17
29 088 885 112 832	19
—4 951 498 053 124 096	21
1 015 423 886 506 852 352	23
—246 921 480 190 207 983 616	25
70 251 601 603 943 959 887 872	27
—23 119 184 187 809 597 841 473 536	29

$C_0 = 1$.

D_n Numerator	Denominator	n
—1	3	2
7	15	4
—31	21	6
127	15	8
—2 555	33	10
1 414 477	1 365	12
—57 337	3	14
118 518 239	255	16
—5 749 691 557	399	18
91 546 277 357	165	20
—1 792 042 792 463	69	22
1 982 765 468 311 237	1 365	24
—286 994 504 449 393	3	26
3 187 598 676 787 461 083	435	28
625 594 554 880 206 790 555	7 161	30

$D_0 = 1$

The Bernoulli polynomials have also the following expansions:

$$B_n(x) = \sum_{r=0}^{n} {}_nC_r\, B_{n-r}(0)\, x^r \ ,$$

$$B_0(x) = 1, \qquad B_n^*(x) =$$

$$-n! \sum_{k=-\infty}^{\infty}{}' e^{2\pi ikx}/(2\pi ik)^n \ , \qquad n > 0 .^*$$

The last expansion can be broken up into the following two Fourier series, valid in the interval $0 \leqq x < 1$:

$$B_{2m}^*(x) = (-1)^{m+1}\, 2\cdot(2m)!\ \sum_{k=1}^{\infty} \cos 2\pi kx/(2\pi k)^{2m} \ ,$$

$$B_{2m+1}^*(x) = (-1)^{m+1}\, 2\cdot(2m+1)!\ \sum_{k=1}^{\infty} \sin 2\pi kx/(2\pi k)^{2m+1} \ .$$

3. Summation of Powers of Integers and Powers of Reciprocals of Integers. In the last section the sum of powers of integers was given in terms of Bernoulli polynomials. This summation may be more conveniently expressed in terms of Bernoulli numbers as follows:

$$\begin{aligned} S_n(p) &= 1^n + 2^n + 3^n + \cdots + p^n \ , \\ &= p^{n+1}/(n+1) + p^n/2 \\ &\quad + \sum_{s=1}^{\leqq n/2} (-1)^{s-1}\, {}_{n+1}C_{2s}\, B_s\, p^{n-2s+1}/(n+1). \end{aligned}$$

Explicitly this formula yields,

$$\begin{aligned} S_0(p) &= p \ , \\ S_1(p) &= p(p+1)/2 \ , \\ S_2(p) &= p(p+1)(2p+1)/6 \ , \\ S_3(p) &= \{p(p+1)/2\}^2 \ , \\ S_4(p) &= p(p+1)(2p+1)(3p^2+3p-1)/30 \ , \\ S_5(p) &= p^2(p+1)^2(2p^2+2p-1)/12 \ , \\ S_6(p) &= p(p+1)(2p+1)(3p^4+6p^3-3p+1)/42 \ , \\ S_7(p) &= p^2(p+1)^2(3p^4+6p^3-p^2-4p+2)/24 \ , \\ S_8(p) &= p(p+1)(2p+1)(5p^6+15p^5+5p^4-15p^3 \\ &\qquad -p^2+9p-3)/90 \ , \end{aligned}$$

*The symbol Σ' means that the term for $k = 0$ is omitted from the summation. $B_n^*(x)$ means the periodic Bernoullian function. (See Section 4).

$$S_9(p) = p^2(p+1)^2(2p^6+6p^5+p^4-8p^3+p^2 \\ +6p-3)/20\ ,$$

$$S_{10}(p) = p(p+1)(2p+1)(3p^8+12p^7+8p^6-18p^5 \\ -10p^4+24p^3+2p^2-15p+5)/66\ ,$$

$$S_{11}(p) = p^2(p+1)^2(2p^8+8p^7+4p^6-16p^5-5p^4 \\ +26p^3-3p^2-20p+10)/24\ ,$$

$$S_{12}(p) = p(p+1)(2p+1)(105p^{10}+525p^9+525p^8 \\ -1050p^7-1190p^6+2310p^5+1420p^4 \\ -3285p^3-287p^2+2073p-691)/2730\ ,$$

$$S_{13}(p) = p^2(p+1)^2(30p^{10}+150p^9+125p^8-400p^7 \\ -326p^6+1052p^5+367p^4-1786p^3+202p^2 \\ +1382p-691)/420\ ,$$

$$S_{14}(p) = p(p+1)(2p+1)(3p^{12}+18p^{11}+24p^{10}-45p^9 \\ -81p^8+144p^7+182p^6-345p^5-217p^4 \\ +498p^3+44p^2-315p+105)/90\ ,$$

$$S_{15}(p) = p^2(p+1)^2(6p^{12}+36p^{11}+42p^{10}-120p^9 \\ -166p^8+452p^7+406p^6-1264p^5-452p^4 \\ +2168p^3-244p^2-1680p+840)/96\ .$$

Tables of the coefficients of various developments of $S_n(p)$ have been computed by S. A. Joffe [See Supplementary Bibliography I Joffe (1)]. The three tables reproduced below refer to the following expansions:

$$S_n(p) = a_1 p^{n+1} + a_2 p^n + a_3 p^{n-1} - a_4 p^{n-3} + \cdots \\ + (-1)^{m-1} a_m p^{n+5-2m} + \cdots\ ;$$

$$S_n(p) = (1/2^{n+1})\{b_1 s^{n+1} - b_2 s^{n-1} + b_3 s^{n-3} - b_4 s^{n-5} + \cdots \\ + (-1)^{m-1} b_m s^{n+3-2m} + \cdots\}\ ,$$

$$S_{2n}(p) = s\{c_1 w^n - c_2 w^{n-1} + c_3 w^{n-2} - \cdots \\ + (-1)^{m-1} c_m w^{n-m+1} + \cdots\}\ ,$$

$$S_{2n+1}(p) = c_1 w^{n+1} - c_2 w^n + c_3 w^{n-1} - \cdots \\ + (-1)^{m-1} c_m w^{n-m+2} \cdots\ ,$$

where we abbreviate, $s = 2p+1$, $w = p^2+p$.

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
1	$\frac{1}{2}$	$\frac{1}{2}$							
2	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$						
3	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$						
4	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{30}$					
5	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{1}{12}$					
6	$\frac{1}{7}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{42}$				
7	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{7}{12}$	$\frac{7}{24}$	$\frac{1}{12}$				
8	$\frac{1}{9}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{7}{15}$	$\frac{2}{9}$	$\frac{1}{30}$			
9	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{1}{2}$	$\frac{3}{20}$			
10	$\frac{1}{11}$	$\frac{1}{2}$	$\frac{5}{6}$	1	1	$\frac{1}{2}$	$\frac{5}{66}$		
11	$\frac{1}{12}$	$\frac{1}{2}$	$\frac{11}{12}$	$\frac{11}{8}$	$\frac{11}{6}$	$\frac{11}{8}$	$\frac{5}{12}$		
12	$\frac{1}{13}$	$\frac{1}{2}$	1	$\frac{11}{6}$	$\frac{22}{7}$	$\frac{33}{10}$	$\frac{5}{3}$	$\frac{691}{2730}$	
13	$\frac{1}{14}$	$\frac{1}{2}$	$\frac{13}{12}$	$\frac{143}{60}$	$\frac{143}{28}$	$\frac{143}{20}$	$\frac{65}{12}$	$\frac{691}{420}$	
14	$\frac{1}{15}$	$\frac{1}{2}$	$\frac{7}{6}$	$\frac{91}{30}$	$\frac{143}{18}$	$\frac{143}{10}$	$\frac{91}{6}$	$\frac{691}{90}$	$\frac{7}{6}$
15	$\frac{1}{16}$	$\frac{1}{2}$	$\frac{5}{4}$	$\frac{91}{24}$	$\frac{143}{12}$	$\frac{429}{16}$	$\frac{455}{12}$	$\frac{691}{24}$	$\frac{35}{4}$
16	$\frac{1}{17}$	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{14}{3}$	$\frac{52}{3}$	$\frac{143}{3}$	$\frac{260}{3}$	$\frac{1382}{15}$	$\frac{140}{3}$
17	$\frac{1}{18}$	$\frac{1}{2}$	$\frac{17}{12}$	$\frac{17}{3}$	$\frac{221}{9}$	$\frac{2431}{30}$	$\frac{1105}{6}$	$\frac{11747}{45}$	$\frac{595}{3}$
18	$\frac{1}{19}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{34}{5}$	34	$\frac{663}{5}$	$\frac{1105}{3}$	$\frac{23494}{35}$	714

		a_2	a_3	a_4					a_9
19	$\frac{1}{20}$	$\frac{1}{2}$	$\frac{19}{12}$	$\frac{323}{40}$	$\frac{323}{7}$	$\frac{4199}{20}$	$\frac{4199}{6}$	$\frac{223193}{140}$	2261
20	$\frac{1}{21}$	$\frac{1}{2}$	$\frac{5}{3}$	$\frac{19}{2}$	$\frac{1292}{21}$	323	$\frac{41990}{33}$	$\frac{223193}{63}$	6460
21	$\frac{1}{22}$	$\frac{1}{2}$	$\frac{7}{4}$	$\frac{133}{12}$	$\frac{323}{4}$	$\frac{969}{2}$	$\frac{146965}{66}$	$\frac{223193}{30}$	$\frac{33915}{2}$
22	$\frac{1}{23}$	$\frac{1}{2}$	$\frac{11}{6}$	$\frac{77}{6}$	$\frac{209}{2}$	$\frac{3553}{5}$	$\frac{11305}{3}$	$\frac{223193}{15}$	$\frac{124355}{3}$
23	$\frac{1}{24}$	$\frac{1}{2}$	$\frac{23}{12}$	$\frac{1771}{120}$	$\frac{4807}{36}$	$\frac{81719}{80}$	$\frac{37145}{6}$	$\frac{5133439}{180}$	$\frac{572033}{6}$
24	$\frac{1}{25}$	$\frac{1}{2}$	2	$\frac{253}{15}$	$\frac{506}{3}$	$\frac{14421}{10}$	$\frac{29716}{3}$	$\frac{10266878}{195}$	208012
25	$\frac{1}{26}$	$\frac{1}{2}$	$\frac{25}{12}$	$\frac{115}{6}$	1265	$\frac{24035}{12}$	$\frac{185725}{12}$	$\frac{25667195}{273}$	

n		a_1			
16	$\frac{3617}{510}$				
17	$\frac{3617}{60}$				
18	$\frac{3617}{10}$	$\frac{43867}{798}$			
19	$\frac{68723}{40}$	$\frac{43867}{84}$			
20	$\frac{68723}{10}$	$\frac{219335}{63}$	$\frac{174611}{330}$		
21	$\frac{481061}{20}$	$\frac{219335}{12}$	$\frac{1222277}{220}$		
22	$\frac{755953}{10}$	$\frac{482537}{6}$	$\frac{1222277}{30}$	$\frac{854513}{138}$	
23	$\frac{17386919}{80}$	$\frac{11098351}{36}$	$\frac{28112371}{120}$	$\frac{854513}{12}$	
24	$\frac{17386919}{30}$	$\frac{22196702}{21}$	$\frac{28112371}{25}$	$\frac{1709026}{3}$	$\frac{236364091}{2730}$
25	$\frac{17386919}{12}$	$\frac{277458775}{84}$	28112371	21362825	$\frac{1181820455}{1092}$

n	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
1	$\frac{1}{2}$	$\frac{1}{2}$							
2	$\frac{1}{3}$	$\frac{1}{3}$							
3	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$						
4	$\frac{1}{5}$	$\frac{2}{3}$	$\frac{7}{15}$						
5	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{7}{6}$	$\frac{1}{2}$					
6	$\frac{1}{7}$	1	$\frac{7}{3}$	$\frac{31}{21}$					
7	$\frac{1}{8}$	$\frac{7}{6}$	$\frac{49}{12}$	$\frac{31}{6}$	$\frac{17}{8}$				
8	$\frac{1}{9}$	$\frac{4}{3}$	$\frac{98}{15}$	$\frac{124}{9}$	$\frac{127}{15}$				
9	$\frac{1}{10}$	$\frac{3}{2}$	$\frac{49}{5}$	31	$\frac{381}{10}$	$\frac{31}{2}$			
10	$\frac{1}{11}$	$\frac{5}{3}$	14	62	127	$\frac{2555}{33}$			
11	$\frac{1}{12}$	$\frac{11}{6}$	$\frac{77}{4}$	$\frac{341}{3}$	$\frac{1397}{4}$	$\frac{2555}{6}$	$\frac{691}{4}$		
12	$\frac{1}{13}$	2	$\frac{77}{3}$	$\frac{1364}{7}$	$\frac{4191}{5}$	$\frac{5110}{3}$	$\frac{1414477}{1365}$		
13	$\frac{1}{14}$	$\frac{13}{6}$	$\frac{1001}{30}$	$\frac{4433}{14}$	$\frac{18161}{10}$	$\frac{33215}{6}$	$\frac{1414477}{210}$	$\frac{5461}{2}$	
14	$\frac{1}{15}$	$\frac{7}{3}$	$\frac{637}{15}$	$\frac{4433}{9}$	$\frac{18161}{5}$	$\frac{46501}{3}$	$\frac{1414477}{45}$	$\frac{57337}{3}$	
15	$\frac{1}{16}$	$\frac{5}{2}$	$\frac{637}{12}$	$\frac{4433}{6}$	$\frac{54483}{8}$	$\frac{232505}{6}$	$\frac{1414477}{12}$	$\frac{286685}{2}$	$\frac{929569}{16}$
16	$\frac{1}{17}$	$\frac{8}{3}$	$\frac{196}{3}$	$\frac{3224}{3}$	$\frac{36322}{3}$	$\frac{265720}{3}$	$\frac{5657908}{15}$	$\frac{2293480}{3}$	$\frac{118518239}{255}$
17	$\frac{1}{18}$	$\frac{17}{6}$	$\frac{238}{3}$	$\frac{13702}{9}$	$\frac{308737}{15}$	$\frac{564655}{3}$	$\frac{48092218}{45}$	$\frac{9747290}{3}$	$\frac{118518239}{30}$
18	$\frac{1}{19}$	3	$\frac{476}{5}$	2108	$\frac{168402}{5}$	$\frac{1129310}{3}$	$\frac{96184436}{35}$	11696748	$\frac{118518239}{5}$

n	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
19	$\frac{1}{20}$	$\frac{19}{6}$	$\frac{2261}{20}$	$\frac{20026}{7}$	$\frac{533273}{10}$	$\frac{2145689}{3}$	$\frac{456876071}{70}$	37039702	$\frac{2251846541}{20}$
20	$\frac{1}{21}$	$\frac{10}{3}$	133	$\frac{80104}{21}$	82042	$\frac{42913780}{33}$	$\frac{913752142}{63}$	105827720	$\frac{2251846541}{5}$
21	$\frac{1}{22}$	$\frac{7}{2}$	$\frac{931}{6}$	$\frac{10013}{2}$	123063	$\frac{75099115}{33}$	$\frac{456876071}{15}$	277797765	$\frac{15762925787}{10}$
22	$\frac{1}{23}$	$\frac{11}{3}$	$\frac{539}{3}$	6479	$\frac{902462}{5}$	$\frac{11553710}{3}$	$\frac{913752142}{15}$	$\frac{2037183610}{3}$	$\frac{24770311951}{5}$
23	$\frac{1}{24}$	$\frac{23}{6}$	$\frac{12397}{60}$	$\frac{149017}{18}$	$\frac{10378313}{40}$	$\frac{18981095}{3}$	$\frac{10508149633}{90}$	$\frac{4685522303}{3}$	$\frac{569717174873}{40}$
24	$\frac{1}{25}$	4	$\frac{3542}{15}$	$\frac{31372}{3}$	$\frac{1831467}{5}$	$\frac{30369752}{3}$	$\frac{42032598532}{195}$	3407652584	$\frac{569717174873}{15}$
25	$\frac{1}{26}$	$\frac{25}{6}$	$\frac{805}{3}$	$\frac{39215}{3}$	$\frac{3052445}{6}$	$\frac{94905475}{6}$	$\frac{105081496330}{273}$	$\frac{21297828650}{3}$	$\frac{569717174873}{6}$

n	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}
17	$\frac{3202291}{2}$				
18	$\frac{5749691557}{399}$				
19	$\frac{5749691557}{42}$	$\frac{221930581}{4}$			
20	$\frac{57496915570}{63}$	$\frac{91546277357}{165}$			
21	$\frac{28748457785}{6}$	$\frac{640823941499}{110}$	$\frac{4722116521}{2}$		
22	$\frac{63246607127}{3}$	$\frac{640823941499}{15}$	$\frac{1792042792463}{69}$		
23	$\frac{1454671963921}{18}$	$\frac{14738950654477}{60}$	$\frac{1792042792463}{6}$	$\frac{968383680827}{8}$	
24	$\frac{5818687855684}{21}$	$\frac{29477901308954}{25}$	$\frac{7168171169852}{3}$	$\frac{1982765468311237}{1365}$	
25	$\frac{36366799098025}{42}$	$\frac{14738950654477}{3}$	$\frac{44801069811575}{3}$	$\frac{9913827341556185}{546}$	$\frac{14717667114151}{2}$

n	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
1	$\frac{1}{2}$								
2	$\frac{1}{6}$								
3	$\frac{1}{4}$								
4	$\frac{1}{10}$	$\frac{1}{30}$							
5	$\frac{1}{6}$	$\frac{1}{12}$							
6	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{42}$						
7	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{12}$						
8	$\frac{1}{18}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{30}$					
9	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{3}{20}$					
10	$\frac{1}{22}$	$\frac{5}{33}$	$\frac{17}{66}$	$\frac{5}{22}$	$\frac{5}{66}$				
11	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{17}{24}$	$\frac{5}{6}$	$\frac{5}{12}$				
12	$\frac{1}{26}$	$\frac{5}{26}$	$\frac{41}{78}$	$\frac{236}{273}$	$\frac{691}{910}$	$\frac{691}{2730}$			
13	$\frac{1}{14}$	$\frac{5}{12}$	$\frac{41}{30}$	$\frac{59}{21}$	$\frac{691}{210}$	$\frac{691}{420}$			
14	$\frac{1}{30}$	$\frac{7}{30}$	$\frac{14}{15}$	$\frac{22}{9}$	$\frac{359}{90}$	$\frac{7}{2}$	$\frac{7}{6}$		
15	$\frac{1}{16}$	$\frac{1}{2}$	$\frac{7}{3}$	$\frac{22}{3}$	$\frac{359}{24}$	$\frac{35}{2}$	$\frac{35}{4}$		
16	$\frac{1}{34}$	$\frac{14}{51}$	$\frac{77}{51}$	$\frac{293}{51}$	$\frac{1519}{102}$	$\frac{1237}{51}$	$\frac{3617}{170}$	$\frac{3617}{510}$	
17	$\frac{1}{18}$	$\frac{7}{12}$	$\frac{11}{3}$	$\frac{293}{18}$	$\frac{1519}{30}$	$\frac{1237}{12}$	$\frac{3617}{30}$	$\frac{3617}{60}$	
18	$\frac{1}{38}$	$\frac{6}{19}$	$\frac{217}{95}$	$\frac{1129}{95}$	$\frac{8487}{190}$	$\frac{6583}{57}$	$\frac{750167}{3990}$	$\frac{43867}{266}$	$\frac{43867}{798}$

n	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}
19	$\frac{1}{20}$	$\frac{2}{3}$	$\frac{217}{40}$	$\frac{1129}{35}$	$\frac{2829}{20}$	$\frac{6583}{15}$	$\frac{750167}{840}$	$\frac{43867}{42}$	$\frac{43867}{84}$		
20	$\frac{1}{42}$	$\frac{5}{14}$	$\frac{23}{7}$	$\frac{470}{21}$	$\frac{689}{6}$	$\frac{28399}{66}$	$\frac{1540967}{1386}$	$\frac{1254146}{693}$	$\frac{174611}{110}$	$\frac{174611}{330}$	
21	$\frac{1}{22}$	$\frac{3}{4}$	$\frac{23}{3}$	$\frac{235}{4}$	$\frac{689}{2}$	$\frac{198793}{132}$	$\frac{1540967}{330}$	$\frac{627073}{66}$	$\frac{1222277}{110}$	$\frac{1222277}{220}$	
22	$\frac{1}{46}$	$\frac{55}{138}$	$\frac{209}{46}$	$\frac{902}{23}$	$\frac{60511}{230}$	$\frac{928151}{690}$	$\frac{1737577}{345}$	$\frac{299264}{23}$	$\frac{4871093}{230}$	$\frac{854513}{46}$	$\frac{854513}{138}$
23	$\frac{1}{24}$	$\frac{5}{6}$	$\frac{209}{20}$	$\frac{902}{9}$	$\frac{60511}{80}$	$\frac{132593}{30}$	$\frac{1737577}{90}$	$\frac{299264}{5}$	$\frac{4871093}{40}$	$\frac{854513}{6}$	$\frac{854513}{12}$
24	$\frac{1}{50}$	$\frac{11}{25}$	$\frac{913}{150}$	$\frac{649}{10}$	$\frac{5511}{10}$	$\frac{276208}{75}$	$\frac{6114166}{325}$	$\frac{22888038}{325}$	$\frac{4730237}{26}$	$\frac{26947575}{91}$	$\frac{236364091}{910}$
25	$\frac{1}{26}$	$\frac{11}{12}$	$\frac{83}{6}$	$\frac{649}{4}$	$\frac{9185}{6}$	$\frac{34526}{3}$	$\frac{6114166}{91}$	$\frac{3814673}{13}$	$\frac{23651185}{26}$	$\frac{673689375}{364}$	$\frac{1181820455}{546}$

For $n = 24$, we have $c_{12} = \dfrac{236364091}{2730}$ and for $n = 25$, $c_{12} = \dfrac{1181820455}{1092}$

Values of $S_n(p)$ will be found in Table 35 from $n = 1$ to $n = 10$ over the range $p = 1$ to $p = 100$, and for $n = 1$, 2 and 3 from $p = 100$ to $p = 1000$. These tables have been computed by R. J. Terkhorn. Tables of $S_n(p)$ for the first 100 integers for n ranging from 1 to 8 are found also in K. Pearson's *Tables for Statisticians and Biometricians*, Part I, pp. 40-41.

For numerous formulas and other information relating to the functions $S_n(p)$, the reader is referred to J. W. L. Glashier:* "On the sums of the series $1^n + 2^n + \cdots + x^n$ and $1^n - 2^n + \ldots \pm x^n$."

*The theory of Bernoulli numbers on the one hand and the science of table computation on the other owe much to the work of James Whitbread Lee Glaisher, born November 5, 1848 and died December 7, 1928, whose picture appears elsewhere in the book. Glaisher was the elder son of James Glaisher, himself an astronomer, a mathematician and a pioneer in meteorology. In 1867 Glaisher took up residence at Trinity College, Cambridge and lived there throughout the remainder of his life. He graduated as Second Wrangler in 1871 and was elected a fellow of Trinity the same year. Glaisher's first original work was an elaborate paper on the sine, cosine, and exponential integrals [See Bibliography: Glaisher: (2)], which contained an extensive table of these functions. This was followed in rapid succession by numerous other researches so that by the age of 25 he had produced more

Quarterly Journal of Math., vol. 30 (1899), pp. 166-204, and to N. Nielson: "*Traité des Nombres de Bernoulli*", Paris (1923), chap. 16.

The Bernoulli numbers are also important in computing the sums to infinity of the reciprocals of powers of the integers. Employing the abbreviations,*

$$S_n = \sum_{r=1}^{\infty} 1/r^n = 1/1^n + 1/2^n + 1/3^n + 1/4^n + \cdots ,$$

$$T_n = \sum_{r=0}^{\infty} (-1)^r 1/(2r+1)^n = 1/1^n - 1/3^n + 1/5^n - 1/7^n + \cdots ,$$

$$s_n = \sum_{r=0}^{\infty} 1/(2r+1)^n = 1/1^n + 1/3^n + 1/5^n + 1/7^n + \cdots ,$$

$$t_n = \sum_{r=1}^{\infty} (-1)^{r-1} 1/r^n = 1/1^n - 1/2^n + 1/3^n - 1/4^n + \cdots$$

we have the formulas,*

$$S_{2n} = 2^{2n-1} \pi^{2n} B_n/(2n)!\ ,$$
$$s_{2n} = (2^{2n} - 1)\, \pi^{2n} B_n/2 \cdot (2n)!\ ,$$
$$t_{2n} = (2^{2n-1} - 1)\, \pi^{2n} B_n/(2n)!\ .$$

Tables have been provided (Table 33) for values of these three sums when n is an odd integer. We also note the formula,†

than 60 articles. In all, the bibliography of his work contains approximately 400 titles. Many of these papers were related either directly or indirectly to the calculation of mathematical tables and Glaisher had a remarkable facility in adapting formulas for this purpose. In addition to his interest in computation, Glaisher found time to explore extensively in the theory of numbers, the subject of differential equations, and the theory of elliptic functions. The latter was his particular hobby, especially as it relates to the Jacobian development, and he gave the first systematic exposition of this subject at Cambridge. Glaisher was also deeply interested in the subject of ceramics and became one of the leading collectors of pottery in England. A more extensive account of his life and work will be found in an article by A. R. Forsyth in the *Journal of the London Mathematical Society*, vol. 4 (1929), pp. 101-112. The picture of Glaisher, which is reproduced elsewhere, is from a portrait made for the Master and Fellows of Trinity College, Cambridge by Francis Dodd.

*For T_{2n} see Euler's numbers (section 4).

†See Nörlund: *loc. cit.*, p. 66.

JAMES WHITBREAD LEE GLAISHER
(1848-1928)
From the Portrait by Francis Dodd made for the
Master and Fellows of Trinity College, Cambridge.

$$S_{2n+1} = (-1)^{n+1}(2\pi)^{2n+1} \int_0^1 B_{2n+1}(t) \cot \pi t \, dt / [2 \cdot (2n+1)!] \ .$$

The values of s_n and t_n are easily constructed from S_n by means of the formulas:

$$s_n = S_n - S_n/2^n \ , \qquad t_n = S_n - S_n/2^{n-1} \ .$$

Closely connected with the sum of the reciprocal powers is the sum of the reciprocal powers of the prime numbers,

$$\Sigma_n = 1/2^n + 1/3^n + 1/5^n + 1/7^n + 1/11^n + \cdots \ ,$$
(summation over primes only)

The following elegant formula is due to J. W. L. Glaisher [See Supplementary Bibliography, Glaisher (15)]:

$$\Sigma_n = \log S_n - \frac{1}{2} \log S_{2n} - \frac{1}{3} \log S_{3n} - \frac{1}{5} \log S_{5n}$$

$$+ \frac{1}{6} \log S_{6n} - \frac{1}{7} \log S_{7n} + \frac{1}{10} \log S_{10} - \cdots \ ,$$

where the numbers which occur in the coefficients include all the integers which do not contain a square factor, and the sign of each term is positive or negative according as the number of prime factors is even or odd

4. The Euler-Maclaurin Sum Formula. In the calculus of finite difference the *Euler-Maclaurin sum formula,* which we have already set forth in volume 1, p. 85, plays an essential rôle. We shall state the formula here with its remainder, which is highly important in many investigations.

$$\frac{1}{d} \int_x^{x+pd} f(t) \, dt = \sum_{m=0}^{p} f(x+md) - \frac{1}{2} [f(x) + f(x+pd)]$$

$$- \frac{d}{12}[f'(x+pd) - f'(x)] + \frac{d^3}{720}[f^{(3)}(x+pd) - f^{(3)}(x)]$$

(4.1)

$$-\frac{d^5}{30240}[f^{(5)}(x+pd)-f^{(5)}(x)]+\frac{d^7}{1209600}[f^{(7)}(x+pd)$$
$$-f^{(7)}(x)]-\cdots$$
$$+(-1)^n\frac{B_n d^{2n-1}}{(2n)!}[f^{(2n-1)}(x+pd)-f^{(2n-1)}(x)]+R_n(x)\ ,$$

where B_n is the nth Bernoulli number, p is an integer, and $R_n(x)$, the remainder term, is the expression,

$$R_n(x)=$$
$$\frac{d^{2n+2}}{(2n+2)!}\int_0^1 \psi_{2n+2}(t)\sum_{m=0}^{p-1} f^{(2n+2)}(x+md+td)\,dt, \qquad (4.2)$$

in which we use the abbreviation:

$$\psi_n(t)=B_n(t)-B_n(0)\ .$$

A useful form has been given to the Euler-Maclaurin series by N. E. Nörlund in terms of what are called the *periodic Bernoullian functions*.* These functions, $B_n^*(x)$, are defined to be periodic functions of unit period, which coincide with the $B_n(x)$ in the interval between 0 and 1.

$$B_n^*(x)=B_n(x)\ , \qquad 0\leqq x<1\ ,$$
$$B_n^*(x+1)=B_n^*(x)\ , \qquad \text{for all values of } x\ .$$

From the fact that $B_1(1)=-B_1(0)=\frac{1}{2}$; $B_{2n+1}(1)=B_{2n+1}(0)=0$, $n>0$; $B_{2n}(1)=B_{2n}(0)$, it is clear that $B_n(x)$ is continuous when n is greater than 1, but possesses discontinuities of magnitude 1 at 0 and all positive and negative integers when $n=1$.

In terms of these functions we may then write the formula,

$$f(x+hd)=\frac{1}{d}\int_x^{x+d} f(t)\,dt+\sum_{m=1}^{n} d^{m-1}B_m(h)\,[f^{(m-1)}(x+d)$$
$$-f^{(m-1)}(x)]/m!+R_n(x)\ , \qquad (4.3)$$

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*See: *Vorlesungen über Differenzenrechnung.* Berlin, (1924), pp. 29-36. Also: Mémoire sur le calcul aux différences finies. *Acta Mathematica*, vol. 44 (1923), pp. 37-212, in particular, pp. 98-108.

where the remainder term is given by the expression

$$R_n(x) = -d^n \int_0^1 B_n^*(h-t)\, f^{(n)}(x+td)\, dt\ n!\ . \qquad (4.4)$$

Writing $x+d, x+2d, \cdots, x+(p-1)d$ for x in this formula and summing, we obtain the following expansions:

$$\sum_{r=0}^{p-1} f(x+rd+hd) = \frac{1}{d} \int_x^{x+pd} f(t)\, dt$$

$$+ \sum_{m=1}^{n} d^{m-1} B_m(h) [f^{(m-1)}(x+pd) - f^{(m-1)}(x)]\ m!$$

$$-d^n \int_0^1 B_n^*(h-t) \sum_{r=0}^{p-1} f^{(n)}(x+rd+td)\, dt/n!\ . \qquad (4.5)$$

Examples: In equation (4.3) we write $f(x) = e^x$, and thus obtain the expansion:

$$d\, e^{hd}/(e^d-1) = \sum_{m=0}^{n} \{d^m B_m(h)/m!\}$$

$$-\{d^{n+1}/(e^d-1)\} \int_0^1 B_n^*(h-t) e^{td}\, dt/n!\ .$$

For $h = 0$, and $n = \infty$, this becomes,

$$d/(e^d-1) = \sum_{m=0}^{\infty} d^m B_m(0)/\mathrm{m}!\ , \qquad |d| < 2\pi\ .$$

Similarly, letting $f(x) = \cos x$, $x = -d/2$, $h = 1$, we get,

$$\tfrac{1}{2} d \cot \tfrac{1}{2} d = \sum_{m=0}^{n} (-1)^m d^{2m} B_{2m}(0)/(2m)! + R_n\ ,$$

where R_n is either

$$(-1)^{n+1} d^{2n+2} \int_0^1 B_{2n+1}(t) \sin[d(t-\tfrac{1}{2})]dt/(2 \sin \tfrac{1}{2} d) \cdot (2n+1)!$$

or

$$(-1)^{n+1}\, d^{2n+1} \int_0^1 B_{2n}(t) \cos [d(t-\tfrac{1}{2})]\,dt/(2 \sin \tfrac{1}{2} d) \cdot (2n)!$$

5. Lubbock's Summation Formula. In the Euler-Maclaurin formula (4.1) let us subdivide the interval d into r parts. We thus obtain the expansion:

$$\frac{r}{d} \int_x^{x+pd} f(t)\,dt = \sum_{m=0}^{pr} f(x+md/r) - \tfrac{1}{2}[f(x) + f(x+pd)]$$
$$- \frac{d}{12\,r}[f'(x+pd) - f'(x)] + \frac{d^3}{720\,r^3}[f^{(3)}(x+pd)$$
$$- f^{(3)}(x)] - \cdots .$$

If we now eliminate the integrals between this equation and (4.1) we obtain what is called *Lubbock's summation formula,* published in 1829 by J. W. Lubbock (1803-1865).*

$$\sum_{m=0}^{pr} f(x+md/r) = r \sum_{m=0}^{p} f(x+md) - \frac{r-1}{2}[f(x+pd) + f(x)]$$
$$- \frac{r^2-1}{12\,r} d[f'(x+pd) - f'(x)] + \frac{r^4-1}{720\,r^3} d^3[f^{(3)}(x+pd)$$
$$- f^{(3)}(x)] - \cdots .$$

If the derivatives in this formula are replaced by their equivalent values in terms of differences, we obtain the following expression:

$$\sum_{m=0}^{pr} f(x+md/r) = r \sum_{m=0}^{p} f(x+md) - A_1(r)[f(x+pd) + f(x)]$$
$$- A_2(r)\{\Delta f[x + (p-1)d] - \Delta f(x)\}$$
$$- A_3(r)\{\Delta^2 f[x + (p-2)d] + \Delta^2 f(x)\}$$
$$- A_4(r)\{\Delta^3 f[x + (p-3)d] - \Delta^3 f(x)\}$$
$$- A_5(r)\{\Delta^4 f[x + (p-4)d] + \Delta^4 f(x)\}$$
$$- A_6(r)\{\Delta^5 f[x + (p-5)d] - \Delta^5 f(x)\} - \cdots ,$$

**Cambridge Philosophical Transactions,* vol. 3 (1829), p. 323.

where the first seven coefficients are the following functions:

$$A_1(r) = (r-1)/2\ , \qquad A_4(r) = (r^2-1)\,(19r^2-1)/720\cdot r^3\ ,$$
$$A_2(r) = (r^2-1)/12\cdot r\ , \quad A_5(r) = (r^2-1)\,(9r^2-1)/480\cdot r^3\ ,$$
$$A_3(r) = (r^2-1)/24\cdot r\ ,$$
$$A_6(r) = (r^2-1)\,(863r^4 - 145r^2 + 2)/60480\cdot r^5\ ,$$
$$A_7(r) = (r^2-1)\,(275r^4 - 61r^2 + 2)/24192\cdot r^5\ .$$

Numerical values of these functions from $r = 2$ to $r = 100$ will be found in Table 34.

It is possible to compute coefficients of higher order from the following formula:

$$ {}_RC_1\,A_n(r) = {}_RC_2\,A_{n-1}(r) - {}_RC_3\,A_{n-2}(r) + {}_RC_4\,A_{n-3}(r)$$
$$- \cdots + (-1)^n\,{}_RC_n\,A_1(r) + (-1)^n\,{}_RC_{n+1}\,r\ .$$

where we abbreviate $R = 1/r$.

6. Computation of the Bernoulli Numbers. The $(n+1)$-*st* Bernoulli number can be computed from those of lower order by means of the following formula:

$$B_{n+1} = \frac{(2n+2)!}{(2n+3)}\left[\frac{B_1\,B_n}{2!(2n)!} + \frac{B_2\,B_{n-1}}{4!(2n-2)!} + \cdots + \frac{B_n\,B_1}{(2n)!\,2!}\right]^{*} \qquad (6.1)$$

The following equation is also useful in the same connection:

$$(-1)^n\,{}_{2n+1}C_{2n}\,B_n + (-1)^{n-1}\,{}_{2n+1}C_{2n-2}\,B_{n-1} + \cdots - {}_{2n+1}C_2\,B_1$$
$$+ n - \tfrac{1}{2} = 0. \qquad (6.2)$$

In the computation and check of his remarkable tables, however, J. C. Adams (1819-1892) used the following theorem due to

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*See R. S. Underwood: *An Expression for the Summation,* $\sum_{m=1}^{n} m^p$. American Math. Monthly, vol. 35 (1928), pp. 424-428.

K. G. C. von Staudt (1798-1867) and T. Clausen (1801-1885) :*

If 1, 2, a, b, c, $\cdots$, $2n$ *be all the divisors of* $2n$ *and if unity be added to each to form the series,* 2, 3, $a+1$, $b+1$, $c+1$, $\cdots$, $2n+1$, *and if from this series only the prime numbers be selected* $(2, 3, p, q, \cdots)$, *then the fractional part of the n-th Bernoulli number will be* $(-1)^n f_n$, *where we abbreviate,* $f_n = (1/2 + 1/3 + 1/p + 1/q + \cdots)$.

Example: Corresponding to the value $n = 10$, we get the two series: (1, 2, 4, 5, 10, 20) and (1, 3, 5, 6, 11, 21). Selecting the primes from the second series we compute,

$$(-1)^{10} f_{10} = (1/2 + 1/3 + 1/5 + 1/11)$$
$$= 371/330 = 1 + 41/330 \ .$$

The fraction 41/330 is the fractional part of the Bernoulli number, $B_{10} = 174611/330 = 529 + 41/330$.

Let us write the nth Bernoulli number in the form,

$$B_n = I_n + (-1)^n (f_n - 1) \ ,$$

where $(-1)^n f_n$ is the fractional part of B_n given by the von Staudt-Clausen theorem, so that I_n is an integer.

Introducing this value into identity (6.2) we get the series,

$$\begin{aligned}(-1)^n C_{2n} I_n + (-1)^{n-1} C_{2n-2} I_{n-1} + \cdots - C_2 I_1 \\ + C_2 f_1 + C_4 f_2 + \cdots + C_{2n} f_n \\ - C_2 - C_4 - \cdots - C_{2n} + n - \tfrac{1}{2} = 0 \ ,\end{aligned}$$

where we employ the abbreviation, $C_{2r} = {}_{2n+1}C_{2r}$.

*K. G. C. von Staudt: Beweis eines Lehrsatzes die Bernoullischen Zahlen betreffend, *Journal für Math.*, vol. 21 (1840), pp. 372-374; Thomas Clausen: *Astronomische Nachrichten*, vol. 17 (1840), cols. 351-352. See also: Schläfli: *Quarterly Journal of Math.*, vol. 6 (1864), pp. 75-77; L. Saalschütz: *Vorlesungen über die Bernoullischen Zahlen*, Berlin (1893), pp. 138-140; E. Lucas: *Théorie des Nombres*, vol. 1, Paris (1891), pp. 433-434; K. Schwering: *Mathematische Annalen*, vol. 52 (1899), pp. 171-173; J. C. Kluyver: *Mathematische Annalen*, vol 53 (1900), pp. 591-592; N. Nielsen: *Nombres de Bernoulli*, Paris (1923), pp. 240-245; N. E. Nörlund: *Vorlesungen über Differenzenrechnung*, Berlin (1924), pp. 32-33; G. Ricci: Sui coefficienti binomiali e polinomiali. *Giornale di Mat.*, vol. 69 (1931), pp. 9-12.

Since ½ occurs in each f_n we get for the value of the part of

$$C_2 f_1 + C_4 f_2 + \cdots + C_{2n} f_n$$

associated with the fraction ½, the following quantity:

$$\tfrac{1}{2}(C_2 + C_4 + \cdots + C_{2n}) = \tfrac{1}{2}(2^{2n} - 1) \ .$$

Moreover, if $2r + 1 = p$ be an odd prime, then the factor $1/p$ will occur in each of the terms $f_r, f_{2r}, f_{3r}, \cdots$. Hence the part of $(C_2 f_1 + C_4 f_2 + \cdots + C_{2n} f_n)$ which contains $1/p$ will be $(C_{2r} + C_{4r} + C_{6r} + \cdots)/p$. Thus, substituting $C_{2n} = 2n + 1$ in the series given above, we shall have,

$$\begin{aligned}(-1)^{n-1}(2n+1) I_n = &-(C_2 I_1 + C_6 I_3 + \cdots) + (C_4 I_2 \\ &+ C_8 I_4 + \cdots) - 2^{2n-1} + n + (1/3)(C_2 + C_4 \\ &+ \cdots + C_{2n}) + (1/5)(C_4 + C_8 + C_{12} + \cdots) \\ &+ (1/7)(C_6 + C_{12} + C_{18} + \cdots) + \cdots \\ &+ (1/p)(C_{2r} + C_{4r} + C_{6r} + \cdots) + \cdots \ .\end{aligned}$$

Since all the expressions on the right are integers, the right member must be an integer divisible by $(2n + 1)$. This supplies a useful check to be applied to the calculations of I_n.

As an example let us compute B_7, given the values $I_1 = I_2 = \cdots = I_6 = 0$. Since $2n = 14$, we form the series (1, 2, 7, 14), (2, 3, 8, 15) and hence compute $f_7 = 1/2 + 1/3 = 5/6$. The binomial coefficients, ${}_{15}C_{2r}$, are then found to be,

${}_{15}C_{2r}$	r
105	1
1365	2
5005	3
6435	4
3003	5
455	6
15	7

$$16383 = 2^{14} - 1$$

Employing the abbreviation, ${}_{15}C_{2r} = C_{2r}$, we apply the formula given above and thus obtain,

$$\begin{aligned} 15\, I_7 = &\ (1/3)\,(C_2 + C_4 + C_6 + C_8 + C_{10} + C_{12} + C_{14}) \\ &+ (1/5)\,(C_4 + C_8 + C_{12}) \\ &+ (1/7)\,(C_6 + C_{12}) \\ &+ (1/11)\,(C_{10}) \\ &+ (1/13)\,(C_{12}) \\ &- 2^{13} + 7 \ , \\ &= 15 \ . \end{aligned}$$

Hence we get, $I_7 = 1$, and the Bernoulli number is,

$$B_7 = 1 - (5/6 - 1) = 7/6 \ .$$

An ingenious method for computing the Bernoulli numbers has been furnished the writer by F. J. Feinler as follows:*

Let us designate by $[a/b]$ the integral part of the fraction a/b. For example, we shall have $[24/7] = [3 + 3/7] = 3$. Designating by p the succession of primes, $p = 2, 3, 5, 7, 11, 13, \cdots$, we compute the table given below (Table 1) for the symbol, $[k/(p-1)]$, $k = 1, 2, 3, 4, 5$, etc.

We next form a table (see Table 2) giving the exponents of the prime factors, p, of $k!$.

If we designate by $k(p)$ the numbers in Table 2 which correspond to k, then we may write $k! = p^{k(p)}$. For example,

$$10! = 2^8\, 3^4\, 5^2\, 7^1 \ .$$

By means of Tables 1 and 2 it is now possible to compute values of the symbol,

$$M_n{}^k = p^{[k/(p-1)]} / \{p^{[n/(p-1)]} \cdot p^{(k-n+1)(p)}\} \ .$$

For example, we get from Tables 1 and 2 the following:

$$M_6{}^{11} = \frac{2^{11} \cdot 3^5 \cdot 5^2 \cdot 7 \cdot 11}{(2^6 \cdot 3^3 \cdot 5 \cdot 7)\ (2^4 \cdot 3^2 \cdot 5)} = 22$$

Some of these values are given below in Table 3.

*See also: A New Method for Calculating the Bernoulli Numbers. *Messenger of Mathematics*, vol. 55 (1926), pp. 40-44.

TABLE 1

k/p	2	3	5	7	11	13	17	19	23	29	31
1	1										
2	2	1									
3	3	1									
4	4	2	1								
5	5	2	1								
6	6	3	1	1							
7	7	3	1	1							
8	8	4	2	1							
9	9	4	2	1							
10	10	5	2	1	1						
11	11	5	2	1	1						
12	12	6	3	2	1	1					
13	13	6	3	2	1	1					
14	14	7	3	2	1	1					
15	15	7	3	2	1	1					
16	16	8	4	2	1	1	1				
17	17	8	4	2	1	1	1				
18	18	9	4	3	1	1	1	1			
19	19	9	4	3	1	1	1	1			
20	20	10	5	3	2	1	1	1			
21	21	10	5	3	2	1	1	1			
22	22	11	5	3	2	1	1	1	1		
23	23	11	5	3	2	1	1	1	1		
24	24	12	6	4	2	2	1	1	1		
25	25	12	6	4	2	2	1	1	1		
26	26	13	6	4	2	2	1	1	1		
27	27	13	6	4	2	2	1	1	1		
28	28	14	7	4	2	2	1	1	1	1	
29	29	14	7	4	2	2	1	1	1	1	
30	30	15	7	5	3	2	1	1	1	1	1
31	31	15	7	5	3	2	1	1	1	1	1

The Bernoulli numbers can now be computed from the following formula:

$$B_k = c_{2k}/d_{2k} \ ,$$

where the numerator and denominator are given by,

$$d_{2k} = p^{[2k/(p-1)]}/p^{2k(p)} \ ,$$

$$c_{2k} = (-1)^k [M_0{}^{2k} c_0 - M_1{}^{2k} c_1 + M_2{}^{2k} c_2 - M_4{}^{2k} c_4 + M_6{}^{2k} c_6 - \cdots] \ ,$$

the series terminating when the subscript is $2k - 2$.

TABLE 2

k/p	2	3	5	7	11	13	17	19	23	29	31
2	1										
3	1	1									
4	3	1									
5	3	1	1								
6	4	2	1								
7	4	2	1	1							
8	7	2	1	1							
9	7	4	1	1							
10	8	4	2	1							
11	8	4	2	1	1						
12	10	5	2	1	1						
13	10	5	2	1	1	1					
14	11	5	2	2	1	1					
15	11	6	3	2	1	1					
16	15	6	3	2	1	1					
17	15	6	3	2	1	1	1				
18	16	8	3	2	1	1	1				
19	16	8	3	2	1	1	1	1			
20	18	8	4	2	1	1	1	1			
21	18	9	4	3	1	1	1	1			
22	19	9	4	3	2	1	1	1			
23	19	9	4	3	2	1	1	1	1		
24	22	10	4	3	2	1	1	1	1		
25	22	10	6	3	2	1	1	1	1		
26	23	10	6	3	2	2	1	1	1		
27	23	13	6	3	2	2	1	1	1		
28	25	13	6	4	2	2	1	1	1		
29	25	13	6	4	2	2	1	1	1	1	
30	26	14	7	4	2	2	1	1	1	1	
31	26	14	7	4	2	2	1	1	1	1	1

TABLE 29

THE BERNOULLI POLYNOMIALS

Description: Values of $B_n(x)$, $n = 2, 3, 4, 5, 6, 7$ and 8, from $x = .00$ to $x = 1.00$.

$$c_{14} = (-1)^7\,(24\cdot 1 - 180\cdot 1 + 420\cdot 1 - 1092\cdot 1 + 1430\cdot 2 - 1716\cdot 3 + 546\cdot 10 - 2\cdot 1382)\ ,$$
$$= 420\ .$$

From these we thus obtain, $B_7 = 420/360 = 7/6$.

TABLE 4

k	c_k	d_k	k	c_k	d_k
0	1	1	16	10851	1530
1	1	2	18	438670	7980
2	1	6	20	7333662	13860
4	1	30	22	51270780	8280
6	2	84	24	7090922730	81900
8	3	90	26	2155381956	1512
10	10	132	28	94997844116	3480
12	1382	5460	30	68926730208040	114576
14	420	360			

7. *The Bernoulli Polynomials of Higher Order.* We have already defined the Bernoulli polynomial of mth order and nth degree, $B_n^{(m)}(x)$, to be the coefficient of $t^n/n!$ in the development of the function $t^m e^{xt}/(e^t - 1)^m$.

These polynomials satisfy the equation,

$$B_m^{(n+1)}(x) = (1 - m/n)\,B_m^{(n)}(x) + (x - n)\,(m/n)\,B_{m-1}^{(n)}(x)\ .$$

They may be calculated for any positive integral value of the upper or lower index by means of the following equation:

$$B_m^{(n+1)}(x) = (m!/n!)\,\frac{d^{n-m}}{dx^{n-m}}\{(x-1)(x-2)\cdots(x-n)\}\ .$$

The following identities should be observed:

$$B_m^{(n+1)}(x) =$$

$$(n+1)\ {}_mC_{n+1} \sum_{s=0}^{n} (-1)^{n-s}\ {}_nC_s\, B_{m-s}(x)\ B_s^{(n-1)}(x)/(m-s)\ .$$

$$B_n^{(n+1)}(x) = (x-1)(x-2)\cdots(x-n) = (x-n)\ B_{n-1}^{(n)}(x)\ ,$$

$$B_n^{(n-1)}(0) = (-1)^n\, n!\ ;$$

$$B_n^{(n)}(x) = \int_x^{x+1} (t-1)(t-2)\cdots(t-n)\ dt\ ;$$

$$B_{n-1}^{(n+1)}(x) = (1/n)(x-1)(x-2)\cdots(x-n)[1/(x-1) + 1/(x-2) + \cdots + 1/(x-n)]\ ,$$

$$B_{n-1}^{(n+1)}(0) = (-1)^{n-1}(n-1)!\ (1 + 1/2 + 1/3 + \cdots + 1/n)\ ;$$

$$(-1)^n B_n^{(n)}(0)/n! = \sum_{s=0}^{n}\ {}_nC_s\, B_s^{(s)}(0)/s!\ ;$$

$$\sum_{s=0}^{n} (-1)^s B_s^{(s)}/(n-s+1)\cdot s! = 1\ .$$

The following expansions are of interest:

$$\{t/(1+t)\cdot\log(1+t)\} = \sum^{\infty} t^m B_m^{(m)}(0)/m!\ , \qquad |t| < 1\ ,$$

$$\{\log(1+t)/t\}^n = n \sum_{m=0}^{\infty} t^m B_m^{(n+m)}(0)/m!\cdot(n+m)\ , \qquad |t| < 1\ .$$

The values of $B_m^{(n)}(0)$ as polynomials in n are given in the following table:

$B_0^{(n)}(0) = 1$,

$B_1^{(n)}(0) = -n/2$,

$B_2^{(n)}(0) = n(3n-1)/12$,

$$B_3^{(n)}(0) = -n^2(n-1)/8 \ ,$$

$$B_4^{(n)}(0) = n(15n^3 - 30n^2 + 5n + 2)/240 \ ,$$

$$B_5^{(n)}(0) = -n^2(n-1)(3n^2 - 7n - 2)/96 \ ,$$

$$B_6^{(n)}(0) = n(63n^5 - 315n^4 + 315n^3 + 91n^2 - 42n - 16)/4032 \ ,$$

$$B_7^{(n)}(0) = -n^2(n-1)(9n^4 - 54n^3 + 51n^2 + 58n + 16)/1152 \ ,$$

$$B_8^{(n)}(0) = n(135n^7 - 1260n^6 + 3150n^5 - 840n^4 - 2345n^3 - 540n^2 + 404n + 144)/34560 \ ,$$

$$B_9^{(n)}(0) = -n^2(n-1)(15n^6 - 165n^5 + 465n^4 + 17n^3 - 648n^2 - 548n - 144)/7680 \ ,$$

$$B_{10}^{(n)}(0) = n(99n^9 - 1485n^8 + 6930n^7 - 8778n^6 - 8085n^5 + 8195n^4 + 11792n^3 + 2068n^2 - 2288n - 768)/101376 \ ,$$

$$B_{11}^{(n)}(0) = -n^2(n-1)(9n^8 - 156n^7 + 834n^6 - 1080n^5 - 1927n^4 + 1252n^3 + 4156n^2 + 3056n + 768)/18432 \ ,$$

$$B_{12}^{(n)}(0) = n(12285n^{11} - 270270n^{10} + 2027025n^9 - 5495490n^8 + 315315n^7 + 12882870n^6 + 5760755n^5 - 14444430n^4 - 15875860n^3 - 2037672n^2 + 3327584n + 1061376)/50319360 \ .$$

8. *Differences of Zero.* The Bernoulli numbers are connected with what are somewhat picturesquely called the differences of zero, $\Delta^r 0^n$, by means of the following formula:

$$(\Delta - \Delta^2/2 + \Delta^3/2^2 - \Delta^4/2^3 - \cdots)\, 0^{2n-1} = (-1)^{n-1}\, 2(2^{2n} - 1)\, B_n/n \ .$$

For example, referring to the accompanying tables, we find,

$$\begin{aligned} B_5\, 2(2^{10} - 1)/5 &= \Delta\, 0^9 - \Delta^2\, 0^9/2 + \Delta^3\, 0^9/4 - \Delta^4\, 0^9/8 + \cdots + \Delta^9\, 0^9/256, \\ &= 1 - 255 + 4537.5 - 23310 + 52132.5 - 59535 + 36382.5 - 11340 + 1417.5 \ , \\ &= 31 \ . \end{aligned}$$

Hence we get,

$$B_5 = (5 \cdot 31)/(2 \cdot 1023) = 5/66 \ .$$

This formula is well known being found in Sir J. F. W. Herschel's: *Examples of the Calculus of Finite Differences,* (1820), p. 85. J. W. L. Glaisher has generalized it by considering the function:*

$$\begin{aligned} F_n(\vartheta) = \{1/(4\cos^2\vartheta)\}\{&\Delta\, 0^{2n-1} - \Delta^2\, 0^{2n-1}(\cos\vartheta/2\cos\vartheta) \\ &+ \Delta^3\, 0^{2n-1}(\cos 2\vartheta/4\cos^2\vartheta) - \Delta^4\, 0^{2n-1}(\cos 3\vartheta/8\cos^3\vartheta) \\ &+ \cdots + \Delta^{2n-1}\, 0^{2n-1}[\cos(2n-2)\vartheta/2^{2n-2}\cos^{2n-2}\vartheta]\} \ . \end{aligned}$$

Glaisher then showed that

$$\begin{aligned} (-1)^{n-1}(2^{2n}-1)\, B_n/(2n) &= F_n(0) \ , \\ (-1)^{n-1}\, 2^{2n}(2^{2n}-1)\, B_n/(4n) &= F_n(\pi/4) \ , \\ (-1)^{n-1}(3^{2n}-1)\, B_n/(4n) &= F_n(\pi/6) \ , \\ (-1)^{n-1}(2^{2n}-1)(3^{2n}-1)\, B_n/(4n) &= F_n(\pi/3) \ . \end{aligned}$$

Differences of zero find one of their principal applications in effecting the expansion,

$$x^n = \sum_{r=0}^{n} x^{(r)}\, \Delta^r\, 0^n/r! \ ,$$

where we employ the familiar definition:

$$x^{(r)} = x(x-1)(x-2)\cdots(x-r+1) \ .$$

By definition,

$$\Delta^r\, 0^n = r^n - {}_rC_1(r-1)^n + {}_rC_2(r-2)^n - {}_rC_3(r-3)^n + \cdots \ ,$$

from which we may derive the iteration formula,

$$\Delta^r\, 0^n = r(\Delta^r\, 0^{n-1} + \Delta^{r-1}\, 0^{n-1}) \ , \quad \Delta^r\, 0^r = r! \ , \quad \Delta\, 0^n = 1 \ .$$

The following table gives the values of $\Delta^r\, 0^n$ and $[\Delta^r\, 0^n]/r!$ for r and n from 1 to 12:

Messenger of Mathematics, vol. 39 (1909-10), pp. 154-173; also *Quarterly Journal of Math.*, vol. 41 (1910), pp. 265-301.

$$D^r\,0^{(-n-1)} = n\,D^r\,0^{(-n)} + r\,D^{r-1}\,0^{(-n)}\ ,$$
$$D^r\,0^{(-r)} = r!\ ,\quad D\,0^{(-r)} = (r-1)!\ .$$

These coefficients are used in effecting the expansions:

$$x^{(-n)} = \sum^{n} x^r\,D^r\,0^{(-n)}/r!\ ,$$

$$x^{(n)} = \sum_{r=0} x^r\,D^r\,0^{(n)}/r!\ ,$$

where we employ the abbreviations:

$$x^{(-n)} = x(x+1)(x+2)\cdots(x+n-1)\ ,$$
$$D^r\,0^{(n)} = (-1)^{n+r}\,D^r\,0^{(-n)}\ .$$

Analogous to central differences of zero we may define central differential coefficients of zero, $D^r\,0^{[n]}$, by means of the following set of recurrence formulas:

$$D^{2r}\,0^{[2n+2]} = (2r)(2r-1)\,D^{2r-2}\,0^{[2n]} - n^2\,D^{2r}\,0^{[2n]}\ ,$$
$$D^{2r}\,0^{[2r]} = (2r)!\ ,\quad D^2\,0^{[2n]} = (-1)^{n-1}\,2\cdot[(n-1)!]^2\ ;$$
$$D^{2r+1}\,0^{[2n+1]} = (2r+1)(2r)\,D^{2r-1}\,0^{[2n-1]}$$
$$- \tfrac{1}{4}(2n-1)^2\,D^{2r+1}\,0^{[2n-1]}\ ,$$
$$D^{2r-1}\,0^{[2r-1]} = (2r-1)!\ ,$$
$$D\,0^{[2n-1]} = (-1)^{n-1}[1\cdot 3\cdot 5\cdots(2n-3)]^2/2^{2n-2}\ .$$

These differences satisfy the following identities:

$$\sum_{r=1}^{n} D^{2r}\,0^{[2n]}/(2r)! = 0\ ,\qquad n > 1.$$

$$\sum_{r=0}^{n} D^{2r+1}\,0^{[2n+1]}/(2r+1)!$$
$$= (-1)^{n-1}(3/2^{2n})(9-4)(25-4)(49-4)\cdots[(2n-1)^2-4]\ ,$$
$$n > 1\ .$$

These differences are useful in effecting the following expansions:

$$x^{[2n]} = \sum_{r=1}^{n} x^{2r} D^{2r} 0^{[2n]} / (2r)\,! \ ,$$

$$x^{[2n+1]} = \sum_{r=0}^{n} x^{2r+1} D^{2r+1} 0^{[2n+1]} / (2r+1)\,! \ .$$

Values of $[D^r\, 0^{(-n)}]/r!$ and $[D^r\, 0^{[n]}]/r!$ for r and n from 1 to 12 are given in the following table:

$[D^r\, 0^{(-n)}]/r!$

	$0^{(-1)}$	$0^{(-2)}$	$0^{(-3)}$	$0^{(-4)}$	$0^{(-5)}$	$0^{(-6)}$	$0^{(-7)}$	$0^{(-8)}$	$0^{(-9)}$	$0^{(-10)}$	$0^{(-11)}$	$0^{(-12)}$
$D/1!$	1	1	2	6	24	120	720	5040	40320	362880	3628800	39916800
$D^2/2!$		1	3	11	50	274	1764	13068	109584	1026576	10628640	120543840
$D^3/3!$			1	6	35	225	1624	13132	118124	1172700	12753576	150917976
$D^4/4!$				1	10	85	735	6769	67284	723680	8409500	105258076
$D^5/5!$					1	15	175	1960	22449	269325	3416930	45995730
$D^6/6!$						1	21	322	4536	63273	902055	13339535
$D^7/7!$							1	28	546	9450	157773	2637558
$D^8/8!$								1	36	870	18150	357423
$D^9/9!$									1	45	1320	32670
$D^{10}/10!$										1	55	1925
$D^{11}/11!$											1	66
$D^{12}/12!$												1

$[D^r\, 0^{[n]}]/r!$

	$0^{[1]}$	$0^{[3]}$	$0^{[5]}$	$0^{[7]}$	$0^{[9]}$	$0^{[11]}$
$D/1!$	1	–1/4	9/16	–225/64	11025/256	–893025/1024
$D^3/3!$		1	–5/2	259/16	–3229/16	1057221/256
$D^5/5!$			1	–35/4	987/8	–86405/32
$D^7/7!$				1	–21	4389/8
$D^9/9!$					1	–165/4
$D^{11}/11!$						1

	$0^{[2]}$	$0^{[4]}$	$0^{[6]}$	$0^{[8]}$	$0^{[10]}$	$0^{[12]}$
$D^2/2!$	1	–1	4	–36	576	–14400
$D^4/4!$		1	–5	49	–820	21076
$D^6/6!$			1	–14	273	–7645
$D^8/8!$				1	–30	1023
$D^{10}/10!$					1	–55
$D^{12}/12!$						1

9. The Origin and Calculation of the Tables. A brief account of the origin and calculation of the tables which accompany this part of the work is given below.

The Bernoulli Polynomials. Values of $B_n(x)$, $n = 2, 3, 4, 5, 6, 7$ and 8 over the range from $x = .00$ to $x = 1.00$ at intervals of .01 were computed by Ralph E. McClain. For the polynomials of even order the constant term, a Bernoulli number, was omitted from the computation. Employing the identity,

$$B_n(1-x) = (-1)^n B_n(x) \quad ,$$

one found it necessary only to evaluate the polynomials over half the range. The values were finally rounded off to 10 decimal places. They are recorded in Table 29.

The Bernoulli Numbers. An account has already been given in section 6 of the method employed in computing the first 62 values of B_n which are recorded in Table 30. This table has been augmented by 28 addition values computed by S. Z. Serebrennikoff in 1905. J. C. Adams [See Bibliography: Adams (2) and (3)] to whom the original computations are due also reduced the Bernoulli numbers to repeating decimals. He makes only the following remark about the calculation of this table (Table 31):

"It readily follows from Staudt's theorem that if the fractional part of the nth number of Bernoulli be converted into a repeating decimal, then the number of figures in the repeating part will be either $2n$ or a divisor of $2n$, and the first figure of the repeating part will occupy the second place of decimals."

Table 32, which gives the logarithms to 10 decimal places and the values to 9 significant figures of the first 250 Bernoulli numbers, was the work of J. W. L. Glaisher [See Bibliography: Glaisher (11)]. The first seven were computed directly from their values and the remainder from the formula (See expression for S_{2n}):

$$\log B_n = \log 2 + \log 1 + \log 2 + \cdots + \log 2n - 2n \log (2\pi) + M(1/2^{2n} + 1/3^{2n} + \cdots\cdots) \quad ,$$

where M is the modulus, $M = .43429\ 44819\ 03251\ 82765\ 11289\ 18917 \cdots$.

Replacing $(2n)!$ by its Stirling approximation to three terms, one obtains the following formula:

$$\log B_n = \log 4 + (2n+1/2) \log n$$
$$- (2n-1/2) \log \pi - 2Mn + M/24n$$

which is accurate to 10 decimal places when n exceeds 250.

The first table of Bernoulli numbers was due to L. Euler* [See Bibliography: Euler (2)], who gave the first 15. This was extended to 31 by Rothe and published in 1840 by M. Ohm (See Bibliography). Adams as we have already indicated, computed the numbers to B_{62} and this heroic calculation has been exceeded by S. Z. Serebrennikoff who has calculated the B_n to B_{90}.†

Prior to Glaisher's work, apparently the only table of the logarithms of the Bernoulli numbers was found in Grunert's *Supplement to Klügel's Wörterbuch.* This table, which contains the logarithms of the first 18 numbers, was found to be inaccurate. It is attributed to Eytelwein's *Grundlehren der höhern Analysis,* Berlin (1824), vol. 1, p. 488.

Tables of S_n, s_n, and t_n. The first table of the Series S_n was computed by L. Euler [See Bibliography, Euler (2), p. 349] to 16 decimal places from $n = 2$ to $n = 16$. Several errors detected in this table led A. M. Legendre [See Bibliography, Legendre (1), p. 432] to compute the values of S_n to 16 decimal places from $n = 2$ to $n = 35$. In 1887 T. J. Stieltjes (See Bibliography) published a new table of S_n to 32 decimal places for the range $n = 2$ to $n = 70$. In this computation he detected the following errors in the table of Legendre, the correction applying to the last decimal place:

	S_5	S_7	S_{10}	S_{11}	S_{16}	S_{35}
Correction:	—1	1	1	1	1	1

The table of Legendre was reprinted by A. De Morgan in his *Differential and Integral Calculus* (1842), p. 554 and by J. Edwards in vol. 2 of his *Integral Calculus* (1922), p. 144 without the corrections noted by Stieltjes. Twenty of the corrected Legendrian values were printed in volume 1 of this work, p. 280.

It is probable that the values given by Euler, Legendre and Stieltjes were computed by means of the Euler-Maclaurin sum formula.

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*Bernoulli computed only the first five numbers.

†Mémoires de L'Académie de Saint-Pétersbourg (8th series), vol. 16, (1905).

Th. Clausen in vol. 5 (1830), pp. 380-382 of the Journal für Mathematik, gave S_3 to 22 places, employing the following formula:

$$S_3 = \pi\left[\frac{1}{2} + \frac{1}{2}\frac{1}{3^2\,2^3} + \frac{1\cdot 3}{2\cdot 4}\frac{1}{5^2\cdot 2^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{1}{7^2\cdot 2^7} + \cdots\right]$$

$$-\frac{3}{4}\left[\frac{1}{2} + \frac{1}{3}\frac{1}{2^2\,2^2} + \frac{1\cdot 2}{3\cdot 5}\frac{1}{2^3\,3^2} + \frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7}\frac{1}{2^4\cdot 4^2} + \cdots\right]$$

J. W. L. Glaisher in 1877 [See Bibliography, Glaisher (10)] computed S_2 to 24 places, S_4 to 23 places, and S_n, $n = 6, 8, 10, 12$ to 22 places using the formula,

$$S_{2n} = 2^{2n-1}\,\pi^{2n}\,B_n/(2n)\,! \ ,$$

for the application of which he found it necessary to compute the values of the first twelve powers of π .

In 1914 Glaisher published tables of S_n to 32 decimal places from $n = 2$ to $n = 107$ [See Supplementary Bibliography I, Glaisher (21)]. From $n = 34$ to the end of the table the values were computed by means of the direct series to 34 places and compared with these given by Stieltjes. The following changes in the last place in the table of Stieltjes were made by Glaisher:

	S_{39}	S_{42}	S_{43}	S_{46}	S_{47}	S_{56}	S_{57}	S_{61}	S_{65}	S_{67}
Correction:	—1	—1	—1	—1	—1	—1	—1	1	—1	1

The values before $n = 34$ were reprinted from the table of Stieltjes.

In the present work the table of Stieltjes as corrected by Glaisher with the additions made by the latter to $n = 100$ has been reprinted. The last significant figures in the seven values omitted, i. e. from $n = 101$ to $n = 107$, are the following: 39, 20, 10, 05, 02, 01, and 01.

Tables for the series s_n and t_n are easily constructed from the tables for S_n by means of the formulas:

$$s_n = S_n - S_n/2^n \ , \quad t_n = S_n - S_n/2^{n-1} \ .$$

Glaisher published a table of these values from $n = 1$ to $n = 35$ to 16 places in 1872. [See Bibliography, Glaisher (6)]. These were obtained from the Legendrian table of S_n . Five years

later, [Bibliography, Glaisher (10)], he gave values of s_2 and t_2 to 24 places, s_4 and t_4 to 23 places and s_n and t_n, $n = 6, 8, 10$, and 12 to 22 places. The values of s_n and t_n which appear in table 33 are taken from Glaisher. [Supplementary Bibliography I, Glaisher (17)].

Tables of Σ_n and $\log_e S_n$. The first table of the values of the series Σ_n was computed by L. Euler and appeared in the first volume of his *Introductio in Analysin Infinitorum*, sections 281 and 282, in 1748.* In this table the values of the first 18 even numbers, Σ_n, were given to 15 places. Euler used in his computation the series:

$$\Sigma_n = (S_n - 1)(1 - 1/2^n)(1 - 1/3^n) + 1/6^n - 1/25^n - 1/35^n - 1/45^n - \cdots ,$$

which converges with sufficient rapidity for large values of n. The sums for smaller subscripts were then deduced by means of the series:

$$\Sigma_2 + \frac{1}{2}\Sigma_4 + \frac{1}{3}\Sigma_6 + \frac{1}{4}\Sigma_8 + \cdots = \log(\pi^2/6) ,$$

$$\Sigma_4 + \frac{1}{2}\Sigma_8 + \frac{1}{3}\Sigma_{12} + \frac{1}{4}\Sigma_{16} + \cdots = \log(\pi^4/90) ,$$

$$\Sigma_2 + \frac{1}{3}\Sigma_5 + \frac{1}{5}\Sigma_{10} \cdot \cdot \frac{1}{7}\Sigma_{14} + \cdot \cdot = \frac{1}{2}\log(5/2)$$

In 1881 C. W. Merrifield [See Supplementary Bibliography I] completed Euler's table by evaluating Σ_n to 15 decimal places for the odd numbers up to and including $n = 35$. In his calculation he made use of the formula deduced by Glaisher and discussed below.

In a paper published in vol. 25 (1891) of the Quarterly Journal of Mathematics [See Supplementary Bibliography I, Glaisher (15)], Glaisher computed Σ_n to 24 decimal places (the last being doubtful) over the even numbers from $n = 2$ to $n = 80$.

For this computation he used the series:

*See his *Opera Omnia*, vol. 8 (1922), pp. 298-299. Errors in the last five figures of Euler's original table have been corrected by the editors.

$$\Sigma_n = \log S_n - \frac{1}{2}\log S_{2n} - \frac{1}{3}\log S_{3n} - \frac{1}{5}\log S_{5n}$$

$$+ \frac{1}{6}\log S_{6n} - \frac{1}{7}\log S_{7n} + \frac{1}{10}\log S_{10n} - \cdots \qquad (9.1)$$

where the numbers which occur in the coefficients include all the integers which do not contain a square factor, and the sign of each term is positive or negative according as the number of prime factors is even or odd.

This series Glaisher had previously derived by means of the following theorem due to A. F. Möbius (1790-1868) :*

If we define the function $F(x)$ by means of the series,

$$F(x) = f(x) + \frac{1}{2}f(x^2) + \frac{1}{3}f(x^3) + \frac{1}{4}f(x^4) + \cdots ,$$

then $f(x)$ may be written in the form,

$$f(x) = F(x) - \frac{1}{2}F(x^2) - \frac{1}{3}F(x^3) - \frac{1}{5}F(x^5)$$
$$+ \frac{1}{6}F(x^6) - \cdots ,$$

where the law for the coefficients is that stated above in the expansion of Σ_n.

Applying this theorem to the expansion of $-\log(1-x)$, we obtain,

$$x = -\log(1-x) + \frac{1}{2}\log(1-x^2) + \frac{1}{3}\log(1-x^3)$$
$$+ \frac{1}{5}\log(1-x^5) - \cdots .$$

Setting x successively equal to $1/2^n$, $1/3^n$, $1/5^n$, $1/7^n$, etc. and recalling Euler's famous equation:

$$S_n = 1/[(1-1/2^n)(1-1/3^n)(1-1/5^n)(1-1/7^n)\cdots]$$

we readily deduce equation (9.1).

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*Über eine besondere Art von Umkehrung der Reihen. *Journal für Math.*, vol. 9 (1832), pp. 105-123. *Werke*, vol. 4, pp. 591-612.

For the determination of Σ_n by this formula, Glaisher computed a table to 24 decimal places for log S_n, n ranging over the even numbers from 2 to 80. For small values of n he used the formula,

$$\log S_{2n} = \log B_n + 2n \log (2\pi) - \log(2n)\,! - \log 2 \ ,$$

and for values beyond $n = 22$ the direct expansion of the logarithm.

Tables of $\log_e S_n$ and Σ_n for the odd numbers to $n = 79$ have been computed by Lucy C. Kantz and E. W. Scotten, the approximation being carried to 24 decimal places to conform with the calculations of Glaisher.

Values of $\log_e S_n$ for small values of n were readily obtained by the following device. The number (a) formed by the first seven or eight digits of S_n was factored into prime factors by means of the *Factor Tables* of D. N. Lehmer. In every case these factors were less than 10,000 and hence their logarithms were available through Wolfram's table (see Vega's *Thesaurus*) to 48 decimal places. The number S_n/a is then unity followed by six or seven zeros before the first significant number. Hence $\log_e(S_n/a)$ was readily computed by means of the ordinary power series which converges with great rapidity and the desired logarithm was determined from the formula: $\log_e S_n = \log_e(S_n/a) + \log_e a$. For example, from $S_3 = 1.2020569031\ 5959428539\ 9738161511$, we find $a = 1.202056 = 8 \cdot 31 \cdot 4847 \cdot 10^{-6}$ and $S_3/1.202056 = 1.000000751\ 3456896229\ 45801328$. The series expansion for the logarithm of this last number converges to sufficient accuracy when four terms of it are used.

Lubbock's Coefficients. Table 34, mainly the work of J. S. Mentzer and Kathryn E. Withers, gives the values of the coefficients, $A_n(r)$, $n = 2$ to $n = 7$, computed to 10 decimal places from $r = 2$ to $r = 100$.

The values of $A_2(r)$, $A_4(r)$, and $A_6(r)$ were computed directly from tables of reciprocal powers. The coefficients of odd subscript were more conveniently evaluated by the following formulas:

$$A_3(r) = A_2(r)/2 \ ,$$
$$A_5(r) = 3A_4(r)/2 - A_3(r)/2 \ ,$$
$$A_7(r) = 5A_6(r)/2 - 10A_5(r)/3 + 5A_4(r)/2 - 2A_3(r)/3 \ .$$

The tables were checked in part by duplicate calculation and

in part by comparing the sum of the values of each coefficient with the sum directly computed from the tables of the polygamma functions. For example, the following formula:

$$\sum_{r=2}^{n} A_7(r) = 275(n-1)(n+2)/48384$$
$$-(1/72)[\Psi(n+1)+C-1]+(1/384)[\tfrac{1}{2}\Psi^{(2)}(n+1)$$
$$+S_3-1]-(1/12096)[(1/24)\Psi^{(4)}(n+1)+S_5-1] ,$$

where C is Euler's constant, was conveniently employed in the check.

Values of these coefficients to 10 decimal places for r from 2 to 11 inclusive are found in J. F. Steffensen's *Interpolation,* Baltimore (1927), pp. 140-141 and to eight decimal places for r from 2 to 12 inclusive in Glover's *Tables* (See Bibliography), p. 430. The *Text Book of the Institute of Actuaries,* Part II, (2nd edition) London (1902), by George King, p. 471, gives the values of the coefficients and their logarithms to six places for r from 2 to 11 inclusive. This table originates with T. B. Sprague in the Journal of the Institute of Actuaries, vol. 18 (1874). The computation is attributed to McLauchlan.

The Sums of Powers. K. Pearson published a short table of $S_n(p)$ for n from 1 to 7 and p from 1 to 20 in Biometrika, vol. 2 (1902-1903), p. 10. This table was extended by W. P. Elderton for p from 1 to 100 and appeared in Biometrika, vol. 2, pp. 474-480. It was later reproduced in Pearson's *Tables for Statisticians and Biometricians,* Part I, (1st edition), pp. 40-41.

Table 35 in this work, computed by R. J. Terkhorn, includes $S_n(p)$, for n from 1 to 10 and p from 1 to 100; also $S_1(p)$, $S_2(p)$, and $S_3(p)$ for the first 1000 integers.

10. Bibliography. Principal sources of information concerning the Bernoulli numbers and Bernoulli polynomials are the following:

N. Nielsen: Traité Élémentaire des Nombres de Bernoulli. Paris, (1923), x + 398 p.

N. E. Nörlund: Vorlesungen über Differenzenrechnung. Berlin, (1924), 551 p.

An extensive bibliography of the literature prior to 1882 is found in the *Bibliography of Bernoulli's Numbers* by G. S. Ely. American Journal of Mathematics, vol. 5 (1882) pp. 228-235.

TABLE 29

THE BERNOULLI POLYNOMIALS

Description: Values of $B_n(x)$, $n = 2, 3, 4, 5, 6, 7$ and 8, from $x = .00$ to $x = 1.00$.

x	$B_2(x) - B_1$	$B_3(x)$	$B_4(x) + B_2$	$B_5(x)$	x
.00	.0000	.000000	.0000 0000	00000 00000	.00
.01	–.0099	.004851	.0000 9801	–.00166 50249	.01
.02	–.0196	.009408	.0003 8416	–.00332 03968	.02
.03	–.0291	.013677	.0008 4681	–.00495 70007	.03
.04	–.0384	.017664	.0014 7456	–.00656 62976	.04
.05	–.0475	.021375	.0022 5625	–.00814 03125	.05
.06	–.0564	.024816	.0031 8096	–.00967 16224	.06
.07	–.0651	.027993	.0042 3801	–.01115 33443	.07
.08	–.0736	.030912	.0054 1696	–.01257 91232	.08
.09	–.0819	.033579	.0067 0761	–.01394 31201	.09
.10	–.0900	.036000	.0081 0000	–.01524 00000	.10
.11	–.0979	.038181	.0095 8441	–.01646 49199	.11
.12	–.1056	.040128	.0111 5136	–.01761 35168	.12
.13	–.1131	.041847	.0127 9161	–.01868 18957	.13
.14	–.1204	.043344	.0144 9616	–.01966 66176	.14
.15	–.1275	.044625	.0162 5625	–.02056 46875	.15
.16	–.1344	.045696	.0180 6336	–.02137 35424	.16
.17	–.1411	.046563	.0199 0921	–.02209 10393	.17
.18	–.1476	.047232	.0217 8576	–.02271 54432	.18
.19	–.1539	.047709	.0236 8521	–.02324 54151	.19
.20	–.1600	.048000	.0256 0000	–.02368 00000	.20
.21	–.1659	.048111	.0275 2281	–.02401 86149	.21
.22	–.1716	.048048	.0294 4656	–.02426 10368	.22
.23	–.1771	.047817	.0313 6441	–.02440 73907	.23
.24	–.1824	.047424	.0332 6976	–.02445 81376	.24
.25	–.1875	.046875	.0351 5625	–.02441 40625	.25
.26	–.1924	.046176	.0370 1776	–.02427 62624	.26
.27	–.1971	.045333	.0388 4841	–.02404 61343	.27
.28	–.2016	.044352	.0406 4256	–.02372 53632	.28
.29	–.2059	.043239	.0423 9481	–.02331 59101	.29
.30	–.2100	.042000	.0441 0000	–.02282 00000	.30
.31	–.2139	.040641	.0457 5321	–.02224 01099	.31
.32	–.2176	.039168	.0473 4976	–.02157 89568	.32
.33	–.2211	.037587	.0488 8521	–.02083 94857	.33
.34	–.2244	.035904	.0503 5536	–.02002 48576	.34
.35	–.2275	.034125	.0517 5625	–.01913 84375	.35
.36	–.2304	.032256	.0530 8416	–.01818 37824	.36
.37	–.2331	.030303	.0543 3561	–.01716 46293	.37
.38	–.2356	.028272	.0555 0736	–.01608 48832	.38
.39	–.2379	.026169	.0565 9641	–.01494 86051	.39
.40	–.2400	.024000	.0576 0000	–.01376 00000	.40
.41	–.2419	.021771	.0585 1561	–.01252 34049	.41
.42	–.2436	.019488	.0593 4096	–.01124 32768	.42
.43	–.2451	.017157	.0600 7401	–.00992 41807	.43
.44	–.2464	.014784	.0607 1296	–.00857 07776	.44
.45	–.2475	.012375	.0612 5625	–.00718 78125	.45
.46	–.2484	.009936	.0617 0256	–.00578 01024	.46
.47	–.2491	.007473	.0620 5081	–.00435 25243	.47
.48	–.2496	.004992	.0623 0016	–.00291 00032	.48
.49	–.2499	.002499	.0624 5001	–.00145 75001	.49
.50	–.2500	.000000	.0625 0000	00000 00000	.50

x	$B_2(x) - B_1$	$B_3(x)$	$B_4(x) + B_2$	$B_5(x)$	x
.50	–.2500	.000000	.0625 0000	00000 00000	.50
.51	–.2499	–.002499	.0624 5001	.00145 75001	.51
.52	–.2496	–.004992	.0623 0016	.00291 00032	.52
.53	–.2491	–.007473	.0620 5081	.00435 25243	.53
.54	–.2484	–.009936	.0617 0256	.00578 01024	.54
.55	–.2475	–.012375	.0612 5625	.00718 78125	.55
.56	–.2464	–.014784	.0607 1296	.00857 07776	.56
.57	–.2451	–.017157	.0600 7401	.00992 41807	.57
.58	–.2436	–.019488	.0593 4096	.01124 32768	.58
.59	–.2419	–.021771	.0585 1561	.01252 34049	.59
.60	–.2400	–.024000	.0576 0000	.01376 00000	.60
.61	–.2379	–.026169	.0565 9641	.01494 86051	.61
.62	–.2356	–.028272	.0555 0736	.01608 48832	.62
.63	–.2331	–.030303	.0543 3561	.01716 46293	.63
.64	–.2304	–.032256	.0530 8416	.01818 37824	.64
.65	–.2275	–.034125	.0517 5625	.01913 84375	.65
.66	–.2244	–.035904	.0503 5536	.02002 48576	.66
.67	–.2211	–.037587	.0488 8521	.02083 94857	.67
.68	–.2176	–.039168	.0473 4976	.02157 89568	.68
.69	–.2139	–.040641	.0457 5321	.02224 01099	.69
.70	–.2100	–.042000	.0441 0000	.02282 00000	.70
.71	–.2059	–.043239	.0423 9481	.02331 59101	.71
.72	–.2016	–.044352	.0406 4256	.02372 53632	.72
.73	–.1971	–.045333	.0388 4841	.02404 61343	.73
.74	–.1924	–.046176	.0370 1776	.02427 62624	.74
.75	–.1875	–.046875	.0351 5625	.02441 40625	.75
.76	–.1824	–.047424	.0332 6976	.02445 81376	.76
.77	–.1771	–.047817	.0313 6441	.02440 73907	.77
.78	–.1716	–.048048	.0294 4656	.02426 10368	.78
.79	–.1659	–.048111	.0275 2281	.02401 86149	.79
.80	–.1600	–.048000	.0256 0000	.02368 00000	.80
.81	–.1539	–.047709	.0236 8521	.02324 54151	.81
.82	–.1476	–.047232	.0217 8576	.02271 54432	.82
.83	–.1411	–.046563	.0199 0921	.02209 10393	.83
.84	–.1344	–.045696	.0180 6336	.02137 35424	.84
.85	–.1275	–.044625	.0162 5625	.02056 46875	.85
.86	–.1204	–.043344	.0144 9616	.01966 66176	.86
.87	–.1131	–.041847	.0127 9161	.01868 18957	.87
.88	–.1056	–.040128	.0111 5136	.01761 35168	.88
.89	–.0979	–.038181	.0095 8441	.01646 49199	.89
.90	–.0900	–.036000	.0081 0000	.01524 00000	.90
.91	–.0819	–.033579	.0067 0761	.01394 31201	.91
.92	–.0736	–.030912	.0054 1696	.01257 91232	.92
.93	–.0651	–.027993	.0042 3801	.01115 33443	.93
.94	–.0564	–.024816	.0031 8096	.00967 16224	.94
.95	–.0475	–.021375	.0022 5625	.00814 03125	.95
.96	–.0384	–.017664	.0014 7456	.00656 62976	.96
.97	–.0291	–.013677	.0008 4681	.00495 70057	.97
.98	–.0196	–.009408	.0003 8416	.00332 03968	.98
.99	–.0099	–.004851	.0000 9801	.00166 50249	.99
1.00	.0000	.000000	.0000 0000	.00000 00000	1.00

x	$B_6(x) - B_3$	$B_7(x)$	$B_8(x) + B_4$	x
.00	.00000 00000	.00000 00000	.00000 00000	.00
.01	–.00004 99753	.00166 55003	.00006 66433	.01
.02	–.00019 96095	.00332 40110	.00026 62936	.02
.03	–.00044 80472	.00496 85825	.00059 81133	.03
.04	–.00079 39031	.00659 23442	.00106 07118	.04
.05	–.00123 52969	.00818 85398	.00165 21532	.05
.06	–.00176 98861	.00975 05611	.00236 99667	.06
.07	–.00239 48995	.01127 19789	.00321 11600	.07
.08	–.00310 71683	.01274 65723	.00417 22345	.08
.09	–.00390 31583	.01416 84549	.00524 92030	.09
.10	–.00477 90000	.01553 16000	.00643 76100	.10
.11	–.00573 05187	.01683 08623	.00773 25526	.11
.12	–.00675 32636	.01806 09986	.00912 87043	.12
.13	–.00784 25361	.01921 71862	.01062 03401	.13
.14	–.00899 34177	.02029 49392	.01220 13622	.14
.15	–.01020 07969	.02129 01227	.01386 53282	.15
.16	–.01145 93956	.02219 89657	.01560 54791	.16
.17	–.01276 37945	.02301 80718	.01741 47694	.17
.18	–.01410 84582	.02374 44282	.01928 58969	.18
.19	–.01548 77588	.02437 54128	.02121 13343	.19
.20	–.01689 60000	.02490 88000	.02318 33600	.20
.21	–.01832 74392	.02534 27648	.02519 40906	.21
.22	–.01977 63097	.02567 58851	.02723 55128	.22
.23	–.02123 68420	.02590 71427	.02929 95156	.23
.24	–.02270 32842	.02603 59227	.03137 79226	.24
.25	–.02416 99219	.02606 20117	.03346 25244	.25
.26	–.02563 10970	.02598 55945	.03554 51106	.26
.27	–.02708 12266	.02580 72493	.03761 75011	.27
.28	–.02851 48201	.02552 79420	.03967 15778	.28
.29	–.02992 64964	.02514 90193	.04169 93151	.29
.30	–.03131 10000	.02467 22000	.04369 28100	.30
.31	–.03266 32166	.02409 95661	.04564 43117	.31
.32	–.03397 81878	.02343 35522	.04754 62501	.32
.33	–.03525 11249	.02267 69341	.04939 12637	.33
.34	–.03647 74228	.02183 28164	.05117 22260	.34
.35	–.03765 26719	.02090 46195	.05288 22719	.35
.36	–.03877 26705	.01989 60651	.05451 48220	.36
.37	–.03983 34357	.01881 11613	.05606 36061	.37
.38	–.04083 12140	.01765 41873	.05752 26857	.38
.39	–.04176 24909	.01642 96766	.05888 64749	.39
.40	–.04262 40000	.01514 24000	.06014 97600	.40
.41	–.04341 27311	.01379 73482	.06130 77180	.41
.42	–.04412 59379	.01239 97134	.06235 59333	.42
.43	–.04476 11449	.01095 48705	.06369 04131	.43
.44	–.04531 61533	.00946 83580	.06410 76013	.44
.45	–.04578 90469	.00794 58586	.06480 43907	.45
.46	–.04617 81959	.00639 31790	.06537 81338	.46
.47	–.04648 22618	.00481 62300	.06582 66520	.47
.48	–.04670 01999	.00322 10056	.06614 82432	.48
.49	–.04683 12625	.00161 35627	.06634 16871	.49
.50	–.04687 50000	.00000 00000	.06640 62500	.50

x	$B_6(x)-B_3$	$B_7(x)$	$B_8(x)+B_4$	x
.50	−.04687 50000	.00000 00000	.06640 62500	.50
.51	−.04683 12625	−.00161 35627	.06634 16871	.51
.52	−.04670 01999	−.00322 10056	.06614 82432	.52
.53	−.04648 22618	−.00481 62300	.06582 66520	.53
.54	−.04617 81959	−.00639 31790	.06537 81338	.54
.55	−.04578 90469	−.00794 58586	.06480 43907	.55
.56	−.04531 61533	−.00946 83580	.06410 76013	.56
.57	−.04476 11449	−.01095 48705	.06369 04131	.57
.58	−.04412 59379	−.01239 97134	.06235 59333	.58
.59	−.04341 27311	−.01379 73482	.06130 77180	.59
.60	−.04262 40000	−.01514 24000	.06014 97600	.60
.61	−.04176 24909	−.01642 96766	.05888 64749	.61
.62	−.04083 12140	−.01765 41873	.05752 26857	.62
.63	−.03983 34357	−.01881 11613	.05606 36061	.63
.64	−.03877 26705	−.01989 60651	.05451 48220	.64
.65	−.03765 26719	−.02090 46195	.05288 22719	.65
.66	−.03647 74228	−.02183 28164	.05117 22260	.66
.67	−.03525 11249	−.02267 69341	.04939 12637	.67
.68	−.03397 81878	−.02343 35522	.04754 62501	.68
.69	−.03266 32166	−.02409 95661	.04564 43117	.69
.70	−.03131 10000	−.02467 22000	.04369 28100	.70
.71	−.02992 64964	−.02514 90193	.04169 93151	.71
.72	−.02851 48201	−.02552 79420	.03967 15778	.72
.73	−.02708 12266	−.02580 72493	.03761 75011	.73
.74	−.02563 10970	−.02598 55945	.03554 51106	.74
.75	−.02416 99219	−.02606 20117	.03346 25244	.75
.76	−.02270 32842	−.02603 59227	.03137 79226	.76
.77	−.02123 68420	−.02590 71427	.02929 95156	.77
.78	−.01977 63097	−.02567 58851	.02723 55128	.78
.79	−.01832 74392	−.02534 27648	.02519 40906	.79
.80	−.01689 60000	−.02490 88000	.02318 33600	.80
.81	−.01548 77588	−.02437 54128	.02121 13343	.81
.82	−.01410 84582	−.02374 44282	.01928 58969	.82
.83	−.01276 37945	−.02301 80718	.01741 47694	.83
.84	−.01145 93956	−.02219 89657	.01560 54791	.84
.85	−.01020 07969	−.02129 01227	.01386 53282	.85
.86	−.00899 34177	−.02029 49392	.01220 13622	.86
.87	−.00784 25361	−.01921 71862	.01062 03401	.87
.88	−.00675 32636	−.01806 09986	.00912 87043	.88
.89	−.00573 05187	−.01683 08623	.00773 25526	.89
.90	−.00477 90000	−.01553 16000	.00643 76100	.90
.91	−.00390 31583	−.01416 83549	.00524 92030	.91
.92	−.00310 71683	−.01274 65723	.00417 22345	.92
.93	−.00239 48995	−.01127 19789	.00321 11600	.93
.94	−.00176 98861	−.00975 05611	.00236 99667	.94
.95	−.00123 52969	−.00818 85398	.00165 21532	.95
.96	−.00079 39031	−.00659 23442	.00106 07118	.96
.97	−.00044 80472	−.00496 85825	.00059 81133	.97
.98	−.00019 96095	−.00332 40110	.00026 62936	.98
.99	−.00004 99753	−.00166 55003	.00006 66433	.99
1.00	.00000 00000	.00000 00000	.00000 00000	1.00

TABLE 30

BERNOULLI NUMBERS

Description: Numerators and denominators of the first 90 Bernoulli numbers.

n	Numerator	Denominator	n
1	1	6	1
2	1	30	2
3	1	42	3
4	1	30	4
5	5	66	5
6	691	2730	6
7	7	6	7
8	3617	510	8
9	43867	798	9
10	1 74611	330	10
11	8 54513	138	11
12	2363 64091	2730	12
13	85 53103	6	13
14	2 37494 61029	870	14
15	861 58412 76005	14322	15
16	770 93210 41217	510	16
17	257 76878 58367	6	17
18	26315 27155 30534 77373	1919190	18
19	2 92999 39138 41559	6	19
20	2 61082 71849 64491 22051	13530	20
21	15 20097 64391 80708 02691	1806	21
22	278 33269 57930 10242 35023	690	22
23	5964 51111 59391 21632 77961	282	23
24	560 94033 68997 81768 62491 27547	46410	24
25	49 50572 05241 07964 82124 77525	66	25
26	80116 57181 35489 95734 79249 91853	1590	26
27	29 14996 36348 84862 42141 81238 12691	798	27
28	2479 39292 93132 26753 68541 57396 63229	870	28
29	84483 61334 88800 41862 04677 59940 36021	354	29
30	121 52331 40483 75557 20403 04994 07982 02460 41491	56786730	30
31	123 00585 43408 68585 41953 03985 74033 86151	6	31
32	10 67838 30147 86652 98863 85444 97914 26479 42017	510	32
33	1 47260 00221 26335 65405 16194 28551 93234 22418 99101	64722	33
34	7877 31308 58718 72814 19091 49208 47460 62443 47001	30	34
35	1505 38134 73333 67003 80307 65673 77857 20851 14381 60235	4686	35
36	58279 54961 66994 41104 38277 24464 10673 65282 48830 18442 60429	140100870	36
37	34152 41728 92211 68014 33007 37314 72635 18668 83077 83087	6	37
38	246 55088 82593 53727 07687 19604 05851 99904 36526 78288 65801	30	38
39	41 48463 65575 40082 82951 79035 54954 20734 92199 37537 24004 83487	3318	39
40	4 60378 42994 79457 64693 55749 69019 04684 97942 57872 75128 89196 56867	230010	40
41	1 67701 41491 85145 83682 31545 09786 26990 02077 36027 57025 34148 81613	498	41

	Numerator	Denominator	
42	20 24576 19593 52903 60231 13116 01117 31009 98991 73911 98090 87728 10839 32477	3404310	42
43	660 71461 94176 78653 57384 78474 26261 49627 78306 86653 38893 17619 96983	6	43
44	13114 26488 67401 75079 95511 42401 93118 43345 75027 55720 28644 29691 98905 74047	61410	44
45	117 90572 79021 08279 98841 23351 24921 50837 75254 94966 96471 16231 54521 57279 22535	272118	45
46	129 55859 48207 53752 79894 27828 53857 67496 59341 48371 94351 43023 31632 68299 46247	1410	46
47	122 08138 06579 74446 96073 01679 41320 12039 58508 41520 26966 21436 21510 52846 49447	6	47
48	2 11600 44959 72665 13097 59772 81098 24233 67304 39543 89060 23415 06387 33420 05066 83499 87259 ...	4501770	48
49	67 90826 06729 05495 62405 11175 46403 60560 73421 95728 50448 75090 73961 24999 29470 58239	6	49
50	945 98037 81912 21252 95227 43306 94937 21872 70284 15330 66936 13338 56962 04311 39541 51972 47711 ...	33330	50
51	32040 19410 86090 70782 43020 78211 62417 75491 81719 71527 17450 67900 25010 86861 53083 66781 58791 ...	4326	51
52	31 95336 31363 83001 12871 03352 79617 42746 71189 60607 82727 38327 10347 01628 49568 36554 97212 24053	1590	52
53	3637 39031 72617 41440 81518 20151 59342 71692 31298 64058 16900 38930 81637 82818 79873 38620 23465 72901	642	53
54	34 69342 24784 78287 89552 08865 93238 52541 39976 67857 60491 14687 00058 91371 50126 63197 24897 59230 65973 38057	209191710	54
55	7645 99294 04847 42892 24813 42467 24347 50052 87524 13412 30790 66835 93870 75979 76062 69585 77997 79302 17515	1518	55
56	26508 79602 15509 97133 52597 21468 51620 14443 15149 91925 09896 45178 84276 80966 75651 48755 15366 78120 35526 00109	1671270	56
57	217 37832 31936 91633 33310 76108 66529 91475 72115 66790 90831 36080 61101 14933 60548 42345 93650 90418 86185 62649	42	57
58	30 95539 16571 84297 69125 13458 03384 14168 69004 12806 43298 44245 50404 57210 08957 52457 19682 71388 19959 57547 52259	1770	58
59	36 69631 19969 71311 15349 47151 58558 50066 84606 36108 06992 04301 05944 06764 14485 04580 64618 89371 77635 45170 95799	6	59
60	515 07486 53507 91090 61843 99685 78499 83274 09517 03532 62675 21309 28691 67199 29747 49229 85358 81132 93670 77682 67780 32820 70131	2328255930	60
61	49 63366 60792 62581 91253 26374 75990 75743 87227 90311 06013 97703 09311 79315 06832 14100 43132 90331 13678 09803 79685 64431	6	61
62	95876 77533 42471 28750 77490 31075 42444 62057 88300 13297 33681 95535 12729 35859 33544 35944 41363 19436 10268 47268 90946 09001	30	62

	Numerator	Denominator	
63	555 63302 81949 27485 06163 24408 91895 13805 25567	4357878	63
	30712 67472 46796 78230 43335 94286 40050 89812 87241		
	41993 45296 38692 08151 38026 96639		
64	26 77547 07742 54808 28869 54405 58528 23947 79291	510	64
	45959 25517 40629 97868 60633 57792 73486 35301 45362		
	66309 35198 62048 49590 84537 18017		
65	1 92821 51751 36130 91564 52995 22271 59643 53076	8646	65
	11010 16472 84587 83733 02052 85486 22403 50407 85951		
	74411 69389 38827 39334 73514 25624 18015		
66	4 10951 94584 69933 78209 02048 65235 71938 12325	4206930	66
	80778 70477 50243 34697 47962 65007 07547 04863 81264		
	63928 01863 68669 41068 05747 33537 03129 46831 ...		
67	264 59017 18707 17725 63363 57372 48879 01515 12545		67
	25593 16868 84119 18554 84066 77655 91690 54072 79873		
	16391 25243 43486 64694 63934 94841 90167		
68	842 90226 34336 74051 31287 57806 03661 93649 33661	4110	68
	23975 47435 76718 92069 12230 44224 26282 12786 55823		
	54558 17749 73769 15176 85781 16483 70366 49737 ...		
69	269 48665 48990 88093 60438 51683 72411 30408 49078	274386	69
	49466 42824 83862 15089 30604 78501 55954 62434 23633		
	37569 33257 57795 70943 83259 07154 97359 02881 36429		
70	3 28949 09864 35898 80393 06995 48851 88400 68805	679470	70
	37476 93113 09813 07467 08516 25048 02973 61809 66938		
	59598 12527 47416 04181 46782 66511 44393 87469 66019		
	46049		
71	1473 18532 80888 58956 58700 80442 45321 42398 04217		71
	02399 06426 76194 87899 74075 46061 58164 31065 69966		
	18921 17482 70209 48349 45544 02556 60807 33851 49191		
72	30502 44698 37360 75650 35155 83690 17263 57405 00710	2381714790	72
	42565 66761 88419 18524 34851 03374 47612 76392 69566		
	93296 26855 96518 35032 95793 51741 15260 56244 43102		
	46126 40493		
73	4120 57002 62801 14871 52611 33159 07864 02616 55456		73
	08808 54115 39738 17680 03479 02626 83524 28485 58100		
	08621 90523 82902 40143 48140 30229 87037 27168 39898		
	24863		
74	16917 37145 61401 89798 65561 09511 21661 89607 68285	4470	74
	21473 01400 81648 06759 16957 87117 86484 33284 82149		
	36063 61235 97334 65846 67336 18179 39379 50344 82855		
	78983 47149		
75	46336 55793 89162 74144 32844 25811 80626 49822 33725	2162622	75
	42529 57998 52299 80732 53793 15501 57230 57600 30594		
	76968 82963 08375 19391 37877 03707 69301 02241 01613		
	90422 79790 66275		
76	373 70181 41155 10850 21058 92888 49128 21658 37489	30	76
	53148 89329 51768 50712 71824 09731 32847 20844 56653		
	63981 25301 40212 35537 46189 17309 55282 49258 58430		
	88631 37958 05601		
77	10 25971 86820 38021 05102 77942 38379 18446 10257	138	77
	38652 46056 92339 92776 48975 08813 37506 86380 84486		
	85054 32262 77082 45455 88824 90067 15516 69012 42288		
	01409 69785 04082 84121		
78	8 17180 86083 26262 85107 56459 75367 34523 13595	1794590070	78
	71039 61164 67582 15209 05960 92548 69913 83469 42995		
	50948 82846 50803 97683 63371 64670 49473 38665 59829		
	76884 83635 06624 33481 89614 19869		

n	Numerator	Denominator	n
79	1 71672 67690 11532 10072 18308 35061 03395 13751 39222 74029 56415 05001 35265 30814 81973 58551 99920 58678 70374 01328 97282 60984 26962 35798 80772 40852 23969 75250 68277 35580 18919	6	79
80	424 08607 94203 31037 60655 63492 36115 69499 89398 08708 63732 14710 62577 84584 41940 47783 99818 50928 83042 00292 85687 06670 18046 45453 15976 74029 61229 30594 27657 84122 42119 77361 80867	230010	80
81	1 58445 14951 44416 42839 09342 43279 42614 08365 96476 08078 63169 60222 38078 42393 80974 79988 03643 63647 97816 86345 90418 21585 44197 93716 54938 88659 05348 53437 56299 28732 00878 62335 07729	130074	81
82	20 53806 46091 43216 26557 19795 86692 64683 78053 31023 14864 50681 33372 38393 03449 48316 60059 12039 26388 54094 08148 33173 32279 38043 25084 94509 48285 24860 62609 20135 47281 33535 62000 73083	2490	82
83	5734 03296 93708 60921 63109 53113 92645 73150 52223 58555 20849 85730 88911 30300 17846 52122 96470 32057 52709 19419 30952 46308 61126 41216 78834 25070 44680 82648 31378 81247 54168 67181 58158 21441	1002	83
84	1 38448 28515 17639 60812 38346 58506 35172 28531 10915 69843 45249 26045 39343 17772 75483 67912 58987 51654 03249 83611 56975 86495 25983 34740 85890 45734 17658 92701 43058 50902 63922 46407 57657 82810 97477	3404310	84
85	1953 34207 62663 75304 14976 77923 84622 34481 41033 73509 88427 21513 99957 07346 97912 46869 18267 68817 15363 52650 57253 53303 69818 17697 99519 31477 42759 48727 83018 74989 46991 57917 78246 00358 94085 ...	66	85
86	1144 37022 11333 32844 71871 79942 99184 66130 08046 50603 24217 31755 25814 86652 87832 26493 10247 81365 96263 33017 01773 08847 08416 21804 32820 10080 20129 99695 55494 67573 21765 95876 09679 40553 77395 09973	5190	86
87	4 16616 15546 62042 83188 49595 93250 71729 73956 14318 18256 14120 48180 68407 74078 03317 59127 08311 94619 29383 21074 82426 94565 51433 57909 80725 18528 59279 48317 63734 35697 60763 98830 85093 24649 93471 28331	2478	87
88	13 69347 91048 67057 07645 62136 25128 24332 22036 07744 76594 34835 69387 15366 60804 45886 14657 55743 61317 06543 94846 41599 47970 46434 60702 53278 29198 96963 90096 80079 96146 17317 65551 01187 10460 07607 76388 83999	1043970	88
89	11 24251 81661 79412 90026 48485 12062 99982 77472 04677 12867 27529 20437 01618 82982 67083 95745 45965 41707 18363 18214 34183 14514 08542 66928 57018 42861 49354 12736 06394 68530 33094 32896 80696 56979 23244 62571 01741	1074	89
90	6173 13645 40162 48924 64052 22722 63470 96019 95593 28290 65533 75302 02055 85339 77917 47341 31234 70301 41906 50099 37527 00612 23369 59545 32816 01820 77217 31818 22529 00766 70213 48110 28346 47254 68591 19172 65818 95593 23830 93313	72257138 85390	90

TABLE 31

BERNOULLI NUMBERS

Description: The first 62 Bernoulli numbers expressed as periodic decimals. The period is the part enclosed in parentheses.

n		
	.1(6)	1
	.0(3)	2
	.0(2380 95)	3
	.0(3)	4
	.0(75)	5
	.2(5311 35)	6
	1 .1(6)	7
	7.0(9215 68627 45098 03)	8
	54 .9(7117 79448 62155 3884)	9
	529 .1(24)	10
	6192 .1(2318 84057 97101 44927 536)	11
	86580 .2(5311 35)	12
	14 25517 .1(6)	13
	272 98231 .0(6781 60919 54022 98850 57471 2643)	14
	6015 80873 .9(0064 23683 84303 86817 48359 16771 4)	15
	1 51163 15767 .0(9215 68627 45098 03)	16
	42 96146 43061 .1(6)	17
	1371 16552 05088 .3(3277 21590 87948 5616)	18
	48833 23189 73593 .1(6)	19
	19 29657 93419 40068 .1(4863 26681 4)	20
21	841 69304 75736 82615 .0(0055 37098 56035 43743 07862 67995 57032 11517 165)	21
	40338 07185 40594 55413 .0(7681 15942 02898 55072 463)	22
	21 15074 86380 81991 60560 .1(4539 00709 21985 81560 28368 79432 62411 34751 77304 96)	23
24	1208 66265 22296 52593 46027 .3(1193 70825 25317 81943 54664 94290 02370 17884 07670 7606)	24
	75008 66746 07696 43668 55720 .0(75)	25
	50 38778 10148 10689 14137 89303 .0(5220 12578 6163)	26
	3652 87764 84818 12333 51104 30842 .9(7117 79448 62155 3884)	27
	2 84987 69302 45088 22262 69146 43291 .0(6781 60919 54022 98850 57471 2643)	28
	238 65427 49968 36276 44645 98191 92192 .1(4971 75141 24293 78531 07344 63276 83615 81920 90395 48022 59887 0056)	29
	21399 94925 72253 33665 81074 47651 91097 .3(9267 41511 61723 87457 42183 07692 65988 72659 15822 23522 99560 12610 6)	30
	20 50097 57234 78097 56992 17330 95672 31025 .1(6)	31
	2093 80059 11346 37840 90951 85290 02797 01847 .0(9215 68627 45098 03)	32
	2 27526 96488 46351 55596 49260 35276 92645 81469 .9(6540 58898 05630 23392 35499 52102 83983 80766 97259 04638 29918 72933 46929 94)	33
	262 57710 28623 95760 47303 04973 61582 02081 44900 .0(3)	34
	32125 08210 27180 32518 20479 23042 64985 24352 19411 .0(6167 30687 15322 23644 89970 12377 29406 74349 12505 33504 05463 08151 94195 47588 5)	35
36	41 59827 81667 94710 91391 70744 95262 35893 66896 03011 .3(4647 07892 24934 86300 26351 72786 57869 86190 73528 95096 22602 62909 14538 93184 246)	36

B_n

37	5692 06954 82035 28002 38834 56219 12105 86444 80512 97181 .1(6)	37
38	8 21836 29419 78457 56922 90653 46861 73330 14550 89276 28860 .0(3)	38
39	1250 29043 27166 99301 67323 39829 70289 55241 77196 36444 84775 .0(1115 12959 61422 54370 10247 13682 94153 10427 96865 58167 57082 57986 73899 93972 27245 3285)	39
40	2 00155 83233 24837 02749 25329 19881 32987 68724 22013 28259 15915 .2(0745 61975 56627 97269 68392 67857 91922 09034 38980 91387 33098 56093 21333 85504 97804 44328 5)	40
41	336 74982 91536 43742 33396 67690 33387 53016 21959 89471 93843 67232 .1(5461 84738 95582 32931 72690 76305 22088 35341 36)	41
42	59470 97050 31354 47718 66049 68440 51540 84057 90715 65106 90499 04704 .3(1085 21256 87731 14081 85506 02030 95487 77872 75541 88660 84463 51830 47372 30158 24058 32606 31376)	42
43	110 11910 32362 79775 59564 13079 04376 91604 63051 14442 23148 86269 99497 .1(6)	43
44	21355 25954 52535 01188 65838 50190 41065 67897 32987 39163 46921 18045 90304 .0(8804 75492 59078 32600 55365 57563 91467 18775 44373)	44
45	43 32889 69866 41192 41961 66130 59379 20621 84513 68511 80910 91449 86557 88032 .8(4801 07894 36935 44712 22043 37824 03222 13157 52724 92080 64148 64139 82169 49999 63251 23659 58885 48350 3)	45
46	9188 55282 41669 32822 62005 55215 50189 71389 60388 91627 19959 59100 44871 13437 .0(5460 99290 78014 18439 71631 20567 37588 65248 22695 03)	46
47	20 34689 67763 29074 49345 50279 90220 02006 59751 40253 37827 70239 36918 42141 08241 .1(6)	47
48	4700 38339 58035 73107 85752 55535 00606 06545 96737 36975 90579 15139 76356 41204 83354 .3(2224 63608 75833 28335 29922 67485 89999 04482 01485 19360 16278 04174 80235 55179 40721 09414 74131 28613 85632 76)	48
49	11 31804 34454 84249 27067 51862 57733 93426 78903 65954 75074 79181 78993 54166 54911 76373 .1(6)	49
50	2838 22495 70693 70695 92641 56336 48176 47382 84680 92801 28821 28228 53171 44648 65111 07028 .1(3414)	50
51	7 40642 48979 67885 06297 50827 14092 09841 76879 73178 80887 06673 11610 03487 48532 84412 10855 .0(1410 07859 45446 13962 08969 02450 30050 85529 35737 40175 68192 32547 38788 71937 12436 43088 30328 24780 39759 59315 765)	51
52	2009 64548 02756 60448 34656 19672 71536 31868 67270 82253 28766 24346 13019 89213 56500 97796 98883 .0(5220 12578 6163)	52
53	5 66571 70050 80594 14457 19346 03051 93569 61419 46828 75104 20621 38756 44521 52460 86197 22777 98400 .1(5732 08722 74143 30218 06853 58255 45171 33956 38629 28348 9096)	53
54	1658 45111 54136 21691 58237 13374 31991 23014 94962 61472 54647 27402 46681 55898 78137 71265 07431 49939 .3(4194 64710 14554 06621 99281 22390 70085 52107 53810 46409 53506 23597 84716 13430 57045 61619 57851 96268 05479 05077 11801 7726)	54
55	5 03688 59950 49237 74192 89421 91518 01548 12442 37426 49032 14141 52565 13225 28310 97674 29893 27917 85387 .0(3227 93148 88010 54018 445)	55

n	B_n	n
56	1586 14682 37658 18636 93634 01572 96643 87827 40978 41277 89638 80472 86451 42973 11365 09885 00683 12009 45121 .1(3548 91788 87911 58819 34098 02126 52653 37138 82257 20559 81379 43001 43005 02013 43888 18084 45074 70366 84676 92234 04955 51287 344)	56
57	5 17567 43617 54562 69840 73240 68250 71225 61240 84923 59305 50859 06216 69403 18108 29579 66515 49771 87766 32444 .0(2380 95)	57
58	1748 89218 40217 11733 96900 25877 61815 91451 41476 16182 65448 72627 34721 58762 12289 52384 00153 32666 64382 79521 .0(5028 24858 75706 21468 92655 36723 16384 18079 09604 51977 40112 9943)	58
59	6 11605 19994 95218 52558 24525 26426 41677 80767 72684 67832 00716 84324 01127 35747 50763 44103 14895 29605 90861 82633 .1(6)	59
60	2212 27769 12707 83494 22883 23456 71293 24455 73185 05498 77801 50566 55269 30277 36635 00257 26591 02528 03139 11549 56836 .4(1706 43950 64162 89896 44622 10131 68427 75098 18261 25962 01999 15049 7)	60
61	8 27227 76798 77096 98542 21062 45998 45957 31204 65051 84335 66283 84885 29885 84472 02350 07188 81721 85613 01633 96614 27405 .1(6)	61
62	3195 89251 11415 70958 35916 34369 18081 48735 26276 67109 91122 73184 50424 31195 31118 14531 48045 43981 20342 28242 29698	62

TABLE 32

LOGARITHMS OF THE BERNOULLI NUMBERS

Description: Logarithms of the first 250 Bernoulli numbers computed to ten decimal places.

Also the first nine significant figures in the first 250 Bernoulli numbers.

(The numbers in parentheses denote the number of additional figures before the decimal point.)

n	$\log_{10}B_n$	B_n	n	$\log_{10}B_n$	B_n
1	9.22184 87496	.16666 6667	51	80.86960 86234	74064 2490 (72)
2	8.52287 87453	.03333 3333	52	83.30311 94507	20096 4548 (75)
3	8.37675 07096	.02380 9524	53	85.75325 48783	56657 1701 (77)
4	8.52287 87453	.03333 3333	54	88.21970 26748	16584 5112 (80)
5	8.87942 60688	.07575 7576	55	90.70216 21211	50368 8599 (82)
6	9.40331 54003	.25311 3553	56	93.20034 33859	15861 4682 (85)
7	0.06694 67896	1.1666 6667	57	95.71396 69440	51756 7436 (87)
8	0.85077 83327	7.0921 5686	58	98.24276 30368	17488 9218 (90)
9	1.74013 50433	54.971 1779	59	100.78647 11692	61160 5200 (92)
10	2.72355 76597	529.12 4242	60	103.34483 96399	22122 7769 (95)
11	3.79183 95878	6192.1 2319	61	105.91762 51042	82722 7768 (97)
12	4.93741 88511	86580. 2531	62	108.50459 21641	31958 9251 (100)
13	6.15397 24516	14255 17.17	63	111.10551 29855	12750 0822 (103)
14	7.43613 45056	27298 231.1	64	113.72016 69394	52500 9231 (105)
15	8.77929 40203	60158 0874 (0)	65	116.34834 02653	22301 8179 (108)
16	10.17944 59554	15116 3158 (2)	66	118.98982 57554	97684 5219 (110)
17	11.63307 90755	42961 4643 (3)	67	121.64442 24580	44098 3620 (113)
18	13.13708 98839	13711 6552 (5)	68	124.31193 53982	20508 5709 (116)
19	14.68871 54679	48833 2319 (6)	69	126.99217 53150	98214 4333 (118)
20	16.28548 03295	19296 5793 (8)	70	129.68495 84142	48412 6008 (121)
21	17.92515 37399	84169 3048 (9)	71	132.39010 61345	24553 0888 (124)
22	19.60571 51352	40338 0719 (11)	72	135.10744 49274	12806 9268 (127)
23	21.32532 57440	21150 7486 (13)	73	137.83680 60487	68676 1671 (129)
24	23.08230 51026	12086 6265 (15)	74	140.57802 53621	37846 4686 (132)
25	24.87511 14502	75008 6675 (16)	75	143.33094 31529	21426 1013 (135)
26	26.70232 52332	50387 7810 (18)	76	146.09540 39514	12456 7271 (138)
27	28.56263 51260	36528 7765 (20)	77	148.87125 63663	74345 7875 (140)
28	30.45482 61057	28498 7693 (22)	78	151.65835 29261	45535 7953 (143)
29	32.37776 92183	23865 4275 (24)	79	154.45654 99288	28612 1128 (146)
30	34.33041 27436	21399 9493 (26)	80	157.26570 72990	18437 7236 (149)
31	36.31177 45314	20500 9757 (28)	81	160.08568 84529	12181 1545 (152)
32	38.32093 53181	20938 0059 (30)	82	162.91636 01686	82482 1872 (154)
33	40.35703 28735	22752 6965 (32)	83	165.75759 24642	57225 8779 (157)
34	42.41925 68522	26257 7103 (34)	84	168.60925 84803	40668 5305 (160)
35	44.50684 42463	32125 0821 (36)	85	171.47123 43696	29596 0921 (163)
36	46.61907 53547	41598 2782 (38)	86	174.34339 91902	22049 5226 (166)
37	48.75527 01978	56920 6955 (40)	87	177.22563 48049	16812 5971 (169)
38	50.91478 53168	82183 6294 (42)	88	180.11782 57847	13116 7362 (172)
39	53.09701 09079	12502 9043 (45)	89	183.01985 93166	10467 8940 (175)
40	55.30136 82495	20015 5832 (47)	90	185.93162 51160	85432 8936 (177)
41	57.52730 73841	33674 9829 (49)	91	188.85301 53421	71287 8213 (180)
42	59.77430 50258	59470 9705 (51)	92	191.78392 45182	60802 9315 (183)
43	62.04186 26660	11011 9103 (54)	93	194.72424 94541	52996 7764 (186)
44	64.32950 48541	21355 2595 (56)	94	197.67388 91731	47194 2592 (189)
45	66.63677 76334	43328 8970 (58)	95	200.63274 48416	42928 4138 (192)
46	68.96324 71164	91885 5282 (60)	96	203.60071 97008	39876 7450 (195)
47	71.30849 81818	20346 8968 (63)	97	206.57771 90030	37819 7804 (198)
48	73.67213 32834	47003 8340 (65)	98	209.56364 99490	36614 2337 (201)
49	76.05377 13567	11318 0434 (68)	99	212.55842 16287	36176 0903 (204)
50	78.45304 68146	28382 2496 (70)	100	215.56194 49641	36470 7727 (207)

n	$\log_{10}B_n$	B_n	n	$\log_{10}B_n$	B_n
101	218.57413 26542	37508 7554(210)	151	378.69508 62338	49554 8577(370)
102	221.59489 91229	39345 8673(213)	152	382.06304 27092	11562 2594(374)
103	224.62416 04676	42088 2112(216)	153	385.43670 42383	27334 0660(377)
104	227.66183 44113	45902 2962(219)	154	388.81603 35936	65468 6814(380)
105	230.70784 02554	51031 7258(222)	155	392.20099 40301	15885 2491(384)
106	233.76209 88349	57822 7623(225)	156	395.59154 92765	39043 5480(387)
107	236.82453 24750	66762 4822(228)	157	398.98766 35254	97199 3869(390)
108	239.89506 49493	78535 3076(231)	158	402.38930 14251	24507 6362(394)
109	242.97362 14401	94106 8941(234)	159	405.79642 80706	62578 9210(397)
110	246.06012 84990	11484 9339(238)	160	409.20900 89952	16181 1355(401)
111	249.15451 40104	14272 9587(241)	161	412.62701 01626	42365 2880(404)
112	252.25670 71551	18059 5596(244)	162	416.05039 79584	11230 4707(408)
113	255.36663 83756	23261 5353(247)	163	419.47913 91828	30139 7179(411)
114	258.48423 93431	30495 7517(250)	164	422.91320 10424	81884 3757(414)
115	261.60944 29248	40685 8061(253)	165	426.35255 11435	22519 1059(418)
116	264.74218 31528	55231 0313(256)	166	429.79715 74843	62684 1129(421)
117	267.88239 51945	76277 2794(259)	167	433.24698 84479	17659 9085(425)
118	271.03001 53231	10715 5711(263)	168	436.70201 27956	50351 5444(428)
119	274.18498 08894	15310 2009(266)	169	440.16219 96600	14527 8103(432)
120	277.34723 02954	22244 8917(269)	170	443.62751 85386	42414 9089(435)
121	280.51670 29672	32862 6792(272)	171	447.09793 92869	12529 6600(439)
122	283.69333 93304	49355 9289(275)	172	450.57343 21128	37448 3005(442)
123	286.87708 07852	75349 5712(278)	173	454.05396 75699	11323 1581(446)
124	290.06786 96825	11691 4852(282)	174	457.53951 65520	34635 1085(449)
125	293.26564 93016	18435 2615(285)	175	461.03005 02866	10716 4338(453)
126	296.47036 38271	29536 8262(288)	176	464.52554 03298	33538 2448(456)
127	299.68195 83282	48079 3213(291)	177	468.02595 85605	10615 9426(460)
128	302.90037 87373	79502 1251(294)	178	471.53127 71748	33984 2097(463)
129	306.12557 18298	13352 7842(298)	179	475.04146 86808	11001 9250(467)
130	309.35748 52052	22776 4065(301)	180	478.55650 58935	36016 8638(470)
131	312.59606 72671	39451 8404(304)	181	482.07636 19292	11922 3517(474)
132	315.84126 72058	69385 2577(307)	182	485.60101 02012	39903 4275(477)
133	319.09303 49796	12388 9637(311)	183	489.13042 44143	13502 8180(481)
134	322.35132 12983	22455 4260(314)	184	492.66457 85605	46193 2544(484)
135	325.61607 76057	41312 1318(317)	185	496.20344 69140	15975 2224(488)
136	328.88725 60639	77135 8135(320)	186	499.74700 40268	55847 5373(491)
137	332.16480 95371	14615 3607(324)	187	503.29522 47241	19734 4362(495)
138	335.44869 15762	28099 0461(327)	188	506.84808 41000	70482 9544(498)
139	338.73885 64045	54809 5712(330)	189	510.40555 75133	25442 3670(502)
140	342.03525 89024	10845 7328(334)	190	513.96762 05832	92815 5160(505)
141	345.33785 45939	21769 8078(337)	191	517.53424 91851	34217 5716(509)
142	348.64659 96328	44319 9879(340)	192	521.10541 94467	12747 3364(513)
143	351.96145 07892	91506 2566(343)	193	524.68110 77442	47985 2481(516)
144	355.28236 54370	19158 6735(347)	194	528.26129 06981	18251 1695(520)
145	358.60930 15409	40672 5630(350)	195	531.84594 51697	70136 6744(523)
146	361.94221 76446	87542 2379(353)	196	535.43504 82574	27230 0386(527)
147	365.28107 28587	19101 7369(357)	197	539.02857 72929	10680 1485(531)
148	368.62582 68490	42250 0132(360)	198	542.62650 98377	42316 5095(534)
149	371.97643 98257	94719 5935(363)	199	546.22882 36798	16936 5005(538)
150	375.33287 25320	21521 4997(367)	200	549.83549 68301	68469 4485(541)

n	$\log_{10}B_n$	B_n	n	$\log_{10}B_n$	B_n
201	553.44650 75191	27958 0913(545)	226	645.07193 54329	11801 4517(637)
202	557.06183 41937	11530 1297(549)	227	648.78872 97510	61479 4185(640)
203	560.68145 55137	48023 6885(552)	228	652.50934 62536	32310 6916(644)
204	564.30535 03493	20199 9526(556)	229	656.23376 81950	17130 4273(648)
205	567.93349 77774	85802 0724(559)	230	659.96197 89755	91617 6136(651)
206	571.56587 70785	36802 4794(563)	231	663.69396 21397	49426 7597(655)
207	575.20246 77346	15939 2446(567)	232	667.42970 13746	26896 8471(659)
208	578.84324 94252	69702 6718(570)	233	671.16918 05075	14763 1899(663)
209	582.48820 20253	30775 2809(574)	234	674.91238 35044	81730 3774(666)
210	586.13730 56019	13718 4676(578)	235	678.65929 44683	45634 6231(670)
211	589.79054 04120	61736 2736(581)	236	682.40989 76374	25697 9002(674)
212	593.44788 68992	28047 0313(585)	237	686.56417 73831	14594 1022(678)
213	597.10932 56917	12862 5090(589)	238	689.92211 82087	83583 0488(681)
214	600.77483 75990	59543 9442(592)	239	693.68370 47476	48273 0509(685)
215	604.44440 36100	27822 9779(596)	240	697.44892 17617	28113 9431(689)
216	608.11800 48903	13122 1468(600)	241	701.21775 44396	16510 2686(693)
217	611.79562 27795	62462 9915(603)	242	704.99018 68953	97765 7858(696)
218	615.47723 87890	30008 1201(607)	243	708.76620 51664	58372 0796(700)
219	619.16283 45997	14549 0488(611)	244	712.54579 42129	35139 3896(704)
220	622.85239 20597	71185 5852(614)	245	716.32893 94154	21327 4737(708)
221	626.54589 31818	35147 3982(618)	246	720.11562 62735	13050 4736(712)
222	630.24332 01415	17511 3707(622)	247	723.90584 04050	80508 2534(715)
223	633.94465 52744	88034 9809(625)	248	727.69956 75437	50068 8416(719)
224	637.64988 10748	44656 1291(629)	249	731.49679 35385	31390 1607(723)
225	641.35898 01929	22854 9457(633)	250	735.29750 43517	19838 2954(727)

TABLE 33

THE SUMS S_n, s_n, t_n AND Σ_n, WITH VALUES OF $\log_e S_n$

Description: Values of S_n to 32 decimals from $n=2$ to $n=100$ (table of Stieltjes) with a 32 decimal approximation of C (Euler's constant); values of s_n to 32 decimal places from $n = 2$ to $n = 67$ and of t_n to 32 decimal places from $n = 1$ to $n = 100$ (table of Glaisher); values of Σ_n to 24 decimal places and $\log_e S_n$ to 24 decimal places from $n = 2$ to $n = 80$.

n	S_n	n	S_n
C =	0.5772156649 015 3286060 6512090082 40	51	1.0000000000 000 0044408 9210314381 34
2	1.6449340668 482 2643647 2415166646 03	52	1.0000000000 000 0022204 4605079804 20
3	1.2020569031 595 9428539 9738161511 46	53	1.0000000000 000 0011102 2302514106 61
4	1.0823232337 111 3819151 6003696541 18	54	1.0000000000 000 0005551 1151248454 81
5	1.0369277551 433 6992633 1365486457 03	55	1.0000000000 000 0002775 5575621361 24
6	1.0173430619 844 4913971 4517929790 93	56	1.0000000000 000 0001387 7787809725 23
7	1.0083492773 819 2282683 9797549849 82	57	1.0000000000 000 0000693 8893904544 15
8	1.0040773561 979 4433937 8685238508 65	58	1.0000000000 000 0000346 9446952165 92
9	1.0020083928 260 8221441 7852769232 40	59	1.0000000000 000 0000173 4723476047 58
10	1.0009945751 278 1808533 7145958900 34	60	1.0000000000 000 0000086 7361738011 99
11	1.0004941886 041 1946455 8702282526 46	61	1.0000000000 000 0000043 3680869002 07
12	1.0002460865 533 0804829 8637998047 72	62	1.0000000000 000 0000021 6840434499 72
13	1.0001227133 475 7848914 6751836526 37	63	1.0000000000 000 0000010 8420217249 42
14	1.0000612481 350 5870482 9258545105 14	64	1.0000000000 000 0000005 4210108624 57
15	1.0000305882 363 0702049 3551728510 66	65	1.0000000000 000 0000002 7105054312 23
16	1.0000152822 594 0865187 1732571487 66	66	1.0000000000 000 0000001 3552527156 10
17	1.0000076371 976 3789976 2273600293 54	67	1.0000000000 000 0000000 6776263578 05
18	1.0000038172 932 6499983 9856461644 61	68	1.0000000000 000 0000000 3388131789 02
19	1.0000019082 127 1655393 8925656957 80	69	1.0000000000 000 0000000 1694065894 51
20	1.0000009539 620 3387279 6113152038 70	70	1.0000000000 000 0000000 0847032947 25
21	1.0000004769 329 8678780 6463116719 62	71	1.0000000000 000 0000000 0423516473 63
22	1.0000002384 505 0272773 2990003648 18	72	1.0000000000 000 0000000 0211758236 81
23	1.0000001192 199 2596531 1073067788 73	73	1.0000000000 000 0000000 0105879118 41
24	1.0000000596 081 8905125 9479612440 20	74	1.0000000000 000 0000000 0052939559 20
25	1.0000000298 035 0351465 2280186063 69	75	1.0000000000 000 0000000 0026469779 60
26	1.0000000149 015 5482836 5041234658 50	76	1.0000000000 000 0000000 0013234889 80
27	1.0000000074 507 1178983 5429491981 01	77	1.0000000000 000 0000000 0006617444 90
28	1.0000000037 253 3402478 8457054819 20	78	1.0000000000 000 0000000 0003308722 45
29	1.0000000018 626 5972351 3049006403 90	79	1.0000000000 000 0000000 0001654361 23
30	1.0000000009 313 2743241 9668182871 76	80	1.0000000000 000 0000000 0000827180 61
31	1.0000000004 656 6290650 3378407298 92	81	1.0000000000 000 0000000 0000413590 31
32	1.0000000002 328 3118336 7650549200 16	82	1.0000000000 000 0000000 0000206795 15
33	1.0000000001 164 1550172 7005197759 30	83	1.0000000000 000 0000000 0000103397 58
34	1.0000000000 582 0772087 9027008892 44	84	1.0000000000 000 0000000 0000051698 79
35	1.0000000000 291 0385044 4970996869 29	85	1.0000000000 000 0000000 0000025849 39
36	1.0000000000 145 5192189 1041984235 93	86	1.0000000000 000 0000000 0000012924 70
37	1.0000000000 072 7595983 5057481014 52	87	1.0000000000 000 0000000 0000006462 35
38	1.0000000000 036 3797954 7378651190 24	88	1.0000000000 000 0000000 0000003231 17
39	1.0000000000 018 1898965 0307065947 58	89	1.0000000000 000 0000000 0000001615 59
40	1.0000000000 009 0949478 4026388928 25	90	1.0000000000 000 0000000 0000000807 79
41	1.0000000000 004 5474737 8304215402 68	91	1.0000000000 000 0000000 0000000403 90
42	1.0000000000 002 2737368 4582465251 52	92	1.0000000000 000 0000000 0000000201 95
43	1.0000000000 001 1368684 0768022784 93	93	1.0000000000 000 0000000 0000000100 97
44	1.0000000000 000 5684341 9876275856 09	94	1.0000000000 000 0000000 0000000050 49
45	1.0000000000 000 2842170 9768893018 55	95	1.0000000000 000 0000000 0000000025 24
46	1.0000000000 000 1421085 4828031606 77	96	1.0000000000 000 0000000 0000000012 62
47	1.0000000000 000 0710542 7395210852 71	97	1.0000000000 000 0000000 0000000006 31
48	1.0000000000 000 0355271 3691337113 67	98	1.0000000000 000 0000000 0000000003 16
49	1.0000000000 000 0177635 6843579120 33	99	1.0000000000 000 0000000 0000000001 58
50	1.0000000000 000 0088817 8421093081 59	100	1.0000000000 000 0000000 0000000000 79

n	s_n
1	∞
2	1.23370 05501 36169 82735 43113 74984 52
3	1.05179 97902 64644 99972 47708 91322 53
4	1.01467 80316 04192 05454 62534 65507 36
5	1.00452 37627 95139 61613 35103 15005 25
6	1.00144 70766 40942 12190 64785 87137 95
7	1.00047 15486 52376 55475 51116 31491 62
8	1.00015 51790 25296 11930 29872 49295 73
9	1.00005 13451 83843 77259 28179 00542 49
10	1.00001 70413 63044 82548 81839 02299 85
11	1.00000 56660 51090 10935 13982 28677 57
12	1.00000 18858 48583 11957 59088 38380 23
13	1.00000 06280 55421 80231 94634 14671 33
14	1.00000 02092 40519 21150 01063 68680 27
15	1.00000 00697 24703 12928 80923 30569 73
16	1.00000 00232 37157 37915 67076 73224 55
17	1.00000 00077 44839 45586 96057 36267 66
18	1.00000 00025 81437 55665 97728 44028 10
19	1.00000 00008 60444 11452 28910 74961 54
20	1.00000 00002 86807 69745 55819 97298 22
21	1.00000 00000 95601 16531 13898 90523 75
22	1.00000 00000 31866 77514 04436 04295 63
23	1.00000 00000 10622 20240 71484 46317 20
24	1.00000 00000 03540 72294 39205 07790 20
25	1.00000 00000 01180 23874 33476 59586 11
26	1.00000 00000 00393 41246 69144 48232 18
27	1.00000 00000 00131 13739 94726 72361 55
28	1.00000 00000 00043 71244 85922 91244 69
29	1.00000 00000 00014 57081 26178 76484 44
30	1.00000 00000 00004 85693 68234 05075 65
31	1.00000 00000 00001 61897 87979 60992 59
32	1.00000 00000 00000 53965 95706 86007 73
33	1.00000 00000 00000 17988 65178 35206 98
34	1.00000 00000 00000 05996 21714 66386 40

n	s_n
35	1.00000 00000 00000 01998 73902 59727 02
36	1.00000 00000 00000 00666 24633 74095 52
37	1.00000 00000 00000 00222 08211 15535 84
38	1.00000 00000 00000 00074 02737 03346 08
39	1.00000 00000 00000 00024 67579 00748 86
40	1.00000 00000 00000 00008 22526 33509 65
41	1.00000 00000 00000 00002 74175 44488 56
42	1.00000 00000 00000 00000 91391 81493 25
43	1.00000 00000 00000 00000 30463 93830 50
44	1.00000 00000 00000 00000 10154 64610 05
45	1.00000 00000 00000 00000 03384 88203 33
46	1.00000 00000 00000 00000 01128 29401 10
47	1.00000 00000 00000 00000 00376 09800 37
48	1.00000 00000 00000 00000 00125 36600 12
49	1.00000 00000 00000 00000 00041 78866 71
50	1.00000 00000 00000 00000 00013 92955 57
51	1.00000 00000 00000 00000 00004 64318 52
52	1.00000 00000 00000 00000 00001 54772 84
53	1.00000 00000 00000 00000 00000 51590 95
54	1.00000 00000 00000 00000 00000 17196 98
55	1.00000 00000 00000 00000 00000 05732 33
56	1.00000 00000 00000 00000 00000 01910 78
57	1.00000 00000 00000 00000 00000 00636 93
58	1.00000 00000 00000 00000 00000 00212 31
59	1.00000 00000 00000 00000 00000 00070 77
60	1.00000 00000 00000 00000 00000 00023 59
61	1.00000 00000 00000 00000 00000 00007 86
62	1.00000 00000 00000 00000 00000 00002 62
63	1.00000 00000 00000 00000 00000 00000 87
64	1.00000 00000 00000 00000 00000 00000 29
65	1.00000 00000 00000 00000 00000 00000 10
66	1.00000 00000 00000 00000 00000 00000 03
67	1.00000 00000 00000 00000 00000 00000 01

n	t_n
1	.69314 71805 59945 30941 72321 21458 18
2	.82246 70334 24113 21823 62075 83323 01
3	.90154 26773 69695 71404 98036 21133 59
4	.94703 28294 97245 91757 65032 34473 53
5	.97211 97704 46909 30593 56551 43553 47
6	.98555 10912 97435 10409 84392 44484 96
7	.99259 38199 22830 28267 04257 13133 42
8	.99623 30018 52647 89922 72892 60082 80
9	.99809 42975 41605 33076 77830 31852 59
10	.99903 95075 98271 56563 92218 45699 36
11	.99951 71434 98060 75414 40941 74828 68
12	.99975 76851 43858 19085 31796 78712 74
13	.99987 85427 63265 11549 21749 92816 28
14	.99993 91703 45979 71817 09541 92255 40
15	.99996 95512 13099 23808 26329 32628 79
16	.99998 47642 14906 10644 16827 74961 43
17	99999 23782 92041 01197 69378 72241 78
18	.99999 61878 69610 11347 96892 26411 60
19	.99999 80935 08171 67510 68564 92965 27
20	.99999 90466 11581 52211 50508 42557 74
21	.99999 95232 58215 54281 63166 64327 88
22	.99999 97616 13230 82254 78972 04943 08
23	.99999 98808 01318 43950 32238 24845 67
24	.99999 99403 98892 39462 83614 03140 21
25	.99999 99701 98856 96283 44151 33108 53
26	.99999 99850 99231 99656 87876 61805 85
27	.99999 99925 49550 48496 35158 52742 09
28	.99999 99962 74753 40010 87275 27670 17
29	.99999 99981 37369 41811 21867 46564 99
30	.99999 99990 68682 28145 39786 27279 54
31	.99999 99995 34340 33145 42175 14686 27
32	.99999 99997 67169 89595 14908 22815 30
33	.99999 99998 83584 85804 60304 72654 66
34	.99999 99999 41792 39904 53159 23880 37
35	.99999 99999 70896 18952 98095 22584 75
36	.99999 99999 85448 09143 38847 63955 11
37	.99999 99999 92724 04460 65847 50057 16
38	.99999 99999 96362 02193 31687 55501 93
39	.99999 99999 98181 01084 32087 35550 13
40	.99999 99999 99090 50538 04788 78091 05
41	.99999 99999 99545 25267 65308 73574 43
42	.99999 99999 99772 62633 36958 97734 98
43	.99999 99999 99886 31316 53247 64876 06
44	.99999 99999 99943 15658 21546 53364 00
45	.99999 99999 99971 57829 09080 83388 10
46	.99999 99999 99985 78914 53976 27195 44
47	.99999 99999 99992 89457 26800 08748 02
48	.99999 99999 99996 44728 63337 36086 57
49	.99999 99999 99998 22364 31647 78613 09
50	.99999 99999 99999 11182 15816 92829 55

n	t_n
51	.99999 99999 99999 55591 07906 14255 71
52	.99999 99999 99999 77795 53952 29741 48
53	.99999 99999 99999 88897 76975 89075 28
54	.99999 99999 99999 94448 88487 85939 15
55	.99999 99999 99999 97224 44243 90103 41
56	.99999 99999 99999 98612 22121 94096 32
57	.99999 99999 99999 99306 11060 96729 70
58	.99999 99999 99999 99653 05530 48258 69
59	.99999 99999 99999 99826 52765 24093 96
60	.99999 99999 99999 99913 26382 62035 19
61	.99999 99999 99999 99956 63191 31013 66
62	.99999 99999 99999 99978 31595 65505 52
63	.99999 99999 99999 99989 15797 82752 32
64	.99999 99999 99999 99994 57898 91376 02
65	.99999 99999 99999 99997 28949 45687 96
66	.99999 99999 99999 99998 64474 72843 96
67	.99999 99999 99999 99999 32237 36421 98
68	.99999 99999 99999 99999 66118 68210 99
69	.99999 99999 99999 99999 83059 34105 49
70	.99999 99999 99999 99999 91529 67052 75
71	.99999 99999 99999 99999 95764 83526 37
72	.99999 99999 99999 99999 97882 41763 19
73	.99999 99999 99999 99999 98941 20881 59
74	.99999 99999 99999 99999 99470 60440 80
75	.99999 99999 99999 99999 99735 30220 40
76	.99999 99999 99999 99999 99867 65110 20
77	.99999 99999 99999 99999 99933 82555 10
78	.99999 99999 99999 99999 99966 91277 55
79	.99999 99999 99999 99999 99983 45638 77
80	.99999 99999 99999 99999 99991 72819 39
81	.99999 99999 99999 99999 99995 86409 69
82	.99999 99999 99999 99999 99997 93204 85
83	.99999 99999 99999 99999 99998 96602 42
84	.99999 99999 99999 99999 99999 48301 21
85	.99999 99999 99999 99999 99999 74150 61
86	.99999 99999 99999 99999 99999 87075 30
87	.99999 99999 99999 99999 99999 93537 65
88	.99999 99999 99999 99999 99999 96768 83
89	.99999 99999 99999 99999 99999 98384 41
90	.99999 99999 99999 99999 99999 99192 21
91	99999 99999 99999 99999 99999 99596 10
92	.99999 99999 99999 99999 99999 99798 05
93	.99999 99999 99999 99999 99999 99899 03
94	.99999 99999 99999 99999 99999 99949 51
95	.99999 99999 99999 99999 99999 99974 76
96	.99999 99999 99999 99999 99999 99987 38
97	.99999 99999 99999 99999 99999 99993 69
98	.99999 99999 99999 99999 99999 99996 84
99	.99999 99999 99999 99999 99999 99998 42
100	.99999 99999 99999 99999 99999 99999 21

n	$\log S_n$	Σ_n	
2	0.4977 0030 2470 7453 4747 4378	0.4522 4742 0041 0654 9850 6544	2
3	1840 3417 5391 4914 2150 4959	1747 6263 9299 4435 3642 3111	3
4	791 0987 3067 3356 2976 5227	769 9313 9764 2468 4494 2619	4
5	362 6225 9649 2279 2286 3397	357 5501 7483 9242 5713 2818	5
6	171 9438 7602 6582 9096 8717	170 7008 6850 6365 1295 4135	6
7	83 1461 4969 2752 0485 9397	82 8383 2856 1335 9253 5124	7
8	40 6906 6307 4129 5523 7580	40 6140 5366 5178 3056 0524	8
9	20 0637 8701 5282 9035 1191	20 0446 7574 9624 5066 3073	9
10	9 9408 0865 6690 6068 9992	9 9360 3574 4369 8021 7856	10
11	4 9406 6533 1469 7697 2817	4 9394 7269 1046 5497 5692	11
12	2 4605 6278 9788 2382 9000	2 4602 6470 0345 4567 9526	12
13	1 2270 5818 9115 5783 7198	1 2269 8367 5278 6927 9906	13
14	6124 6259 4682 6463 1441	6124 4396 7254 6447 8378	14
15	3058 7768 4964 5994 8821	3058 7302 8232 7005 2568	15
16	1528 2142 6361 1525 2350	1528 2026 2193 3934 4180	16
17	763 7168 4746 5436 5654	763 7139 3706 4589 7250	17
18	381 7285 9791 5444 5816	381 7278 7031 7499 6631	18
19	190 8210 8959 1836 9223	190 8209 0769 2628 2571	19
20	95 3961 5788 5130 4460	95 3961 1241 0362 3326	20
21	47 6932 8730 5530 5682	47 6932 7593 6842 7251	21
22	23 8450 4742 9841 6384	23 8450 4458 7670 1928	22
23	11 9219 9188 5861 6264	11 9219 9117 5318 8285	23
24	5 9608 1872 7469 1450	5 9608 1854 9833 4533	24
25	2 9803 5030 7052 7878	2 9803 5026 2643 8658	25
26	1 4901 5547 1733 6874	1 4901 5546 0631 4571	26
27	7450 7117 6207 8877	7450 7117 3432 3301	27
28	3725 3340 1784 9400	3725 3340 1091 0506	28
29	1862 6597 2177 8298	1862 6597 2004 3574	29
30	931 3274 3198 5984	931 3274 3155 2303	30
31	465 6629 0639 4957	465 6629 0628 6537	31
32	232 8311 8334 0546	232 8311 8331 3441	32
33	116 4155 0172 0229	116 4155 0171 3452	33
34	58 2077 2087 7333	58 2077 2087 5639	34
35	29 1038 5044 4547	29 1038 5044 4123	35
36	14 5519 2189 0936	14 5519 2189 0830	36
37	7 2759 5983 5031	7 2759 5983 5004	37
38	3 6379 7954 7373	3 6379 7954 7366	38
39	1 8189 8965 0305	1 8189 8965 0304	39
40	9094 9478 4026	9094 9478 4026	40

$\log S_n$

41	0.0000 0000 0000 4547 4737 8304	0.0000 0000 0000 4547 4737 8304	41
42	2273 7368 4582	2273 7368 4582	42
43	1136 8684 0768	1136 8684 0768	43
44	568 4341 9876	568 4341 9876	44
45	284 2170 9769	284 2170 9769	45
46	142 1085 4828	142 1085 4828	46
47	71 0542 7395	71 0542 7395	47
48	35 5271 3692	35 5271 3692	48
49	17 7635 6844	17 7635 6844	49
50	8 8817 8421	8 8817 8421	50
51	4 4408 9210	4 4408 9210	51
52	2 2204 4605	2 2204 4605	52
53	1 1102 2303	1 1102 2303	53
54	5551 1151	5551 1151	54
55	2775 5576	2775 5576	55
56	1387 7788	1387 7788	56
57	693 8894	693 8894	57
58	346 9447	346 9447	58
58	173 4723	173 4723	59
60	86 7362	86 7362	60
61	43 3681	43 3681	61
62	21 6840	21 6840	62
63	10 8420	10 8420	63
64	5 4210	5 4210	64
65	2 7105	2 7105	65
66	1 3553	1 3553	66
67	6776	6776	67
68	3388	3388	68
69	1694	1694	69
70	847	847	70
71	424	424	71
72	212	212	72
73	106	106	73
74	53	53	74
75	26	26	75
76	13	13	76
77	7		77
78	3		78
79	2		79
80	1		80

TABLE 34

COEFFICIENTS IN LUBBOCK'S SUMMATION FORMULA

Description: Values of $A_n(r)$, $n = 2, 3, 4, 5, 6$ and 7, to 12 decimal places from $r = 2$ to $r = 100$.

r	$A_2(r)$	$A_3(r)$	$A_4(r)$	r
2	0.125000 000000	0.062500 000000	0.039062 500000	2
3	0.222222 222222	0.111111 111111	0.069958 847737	3
4	0.312500 000000	0.156250 000000	0.098632 812500	4
5	0.400000 000000	0.200000 000000	0.126400 000000	5
6	0.486111 111111	0.243055 555556	0.153710 133745	6
7	0.571428 571428	0.285714 285714	0.180758 017492	7
8	0.656250 000000	0.328125 000000	0.207641 601562	8
9	0.740740 740741	0.370370 370370	0.234415 485444	9
10	0.825000 000000	0.412500 000000	0.261112 500000	10
11	0.909090 909091	0.454545 454545	0.287753 568745	11
12	0.993055 555556	0.496527 777778	0.314352 655608	12
13	1.076923 076923	0.538461 538462	0.340919 435595	13
14	1.160714 285714	0.580357 142857	0.367460 823615	14
15	1.244444 444444	0.622222 222222	0.393981 893005	15
16	1.328125 000000	0.664062 500000	0.420486 450195	16
17	1.411764 705882	0.705882 352941	0.446977 406880	17
18	1.495370 370370	0.747685 185185	0.473457 028274	18
19	1.578947 368421	0.789473 684210	0.499927 103076	19
20	1.662500 000000	0.831250 000000	0.526389 062500	20
21	1.746031 746032	0.873015 873016	0.552844 065317	21
22	1.829545 454545	0.914772 727272	0.579293 059730	22
23	1.913043 478260	0.956521 739130	0.605736 829128	23
24	1.996527 777778	0.998263 888889	0.632176 026395	24
25	2.080000 000000	1.040000 000000	0.658611 200000	25
26	2.163461 538462	1.081730 769230	0.685042 814065	26
27	2.246913 580247	1.123456 790123	0.711471 263979	27
28	2.330357 142857	1.165178 571428	0.737896 888666	28
29	2.413793 103448	1.206896 551724	0.764319 980319	29
30	2.497222 222222	1.248611 111111	0.790740 792182	30
31	2.580645 161290	1.290322 580645	0.817159 544829	31
32	2.664062 500000	1.332031 250000	0.843576 431275	32
33	2.747474 747478	1.373737 373737	0.869991 621139	33
34	2.830882 352941	1.415441 176470	0.896405 264095	34
35	2.914285 714286	1.457142 857143	0.922817 492711	35
36	2.997685 185185	1.498842 592593	0.949228 424831	36
37	3.081081 081081	1.540540 540541	0.975638 165559	37
38	3.164473 684210	1.582236 842105	1.002046 808937	38
39	3.247863 247863	1.623931 623932	1.028454 439369	39
40	3.331250 000000	1.665625 000000	1.054861 132813	40
41	3.414634 146341	1.707317 073171	1.081266 957821	41
42	3.498015 873016	1.749007 936507	1.107671 976418	42
43	3.581395 348837	1.790697 674418	1.134076 244859	43
44	3.664772 727273	1.832386 363636	1.160479 814285	44
45	3.748148 148148	1.874074 074074	1.186882 731292	45
46	3.831521 739130	1.915760 869565	1.213285 038424	46
47	3.914893 617021	1.957446 808510	1.239686 774607	47
48	3.998263 888889	1.999131 944444	1.266087 975522	48
49	4.081632 653061	2.040816 326530	1.292488 673937	49
50	4.165000 000000	2.082500 000000	1.318888 900000	50

r	$A_2(r)$	$A_3(r)$	$A_4(r)$	r
51	4.248366 013071	2.124183 006535	1.345288 681494	51
52	4.331730 769230	2.165865 384615	1.371688 044066	52
53	4.415094 339622	2.207547 169811	1.398087 011426	53
54	4.498456 790123	2.249228 395061	1.424485 605528	54
55	4.581818 181818	2.290909 090909	1.450883 846734	55
56	4.665178 571428	2.332589 285714	1.477281 753941	56
57	4.748538 011695	2.374269 005847	1.503679 344732	57
58	4.831896 551724	2.415948 275862	1.530076 635471	58
59	4.915254 237288	2.457627 118644	1.556473 641417	59
60	4.998611 111111	2.499305 555556	1.582870 376800	60
61	5.081967 213114	2.540983 606557	1.609266 854935	61
62	5.165322 580645	2.582661 290322	1.635663 088265	62
63	5.248677 248677	2.624338 624339	1.662059 088447	63
64	5.332031 250000	2.666015 625000	1.688454 866409	64
65	5.415384 615385	2.707692 307692	1.714850 432408	65
66	5.498737 373737	2.749368 686869	1.741245 796078	66
67	5.582089 552238	2.791044 776119	1.767640 966476	67
68	5.665441 176470	2.832720 588235	1.794035 952129	68
69	5.748792 270531	2.874396 135265	1.820430 761071	69
70	5.832142 857143	2.916071 428571	1.846825 400874	70
71	5.915492 957746	2.957746 478873	1.873219 878686	71
72	5.998842 592593	2.999421 296296	1.899614 201252	72
73	6.082191 780821	3.041095 890410	1.926008 374956	73
74	6.165540 540541	3.082770 270270	1.952402 405830	74
75	6.248888 888889	3.124444 444444	1.978796 299589	75
76	6.332236 842105	3.166118 421052	2.005190 061643	76
77	6.415584 415584	3.207792 207792	2.031583 697125	77
78	6.498931 623932	3.249465 811965	2.057977 210904	78
79	6.582278 481012	3.291139 240506	2.084370 607599	79
80	6.665625 000000	3.332812 500000	2.110763 891602	80
81	6.748971 193415	3.374485 596707	2.137157 067085	81
82	6.832317 073171	3.416158 536585	2.163550 138020	82
83	6.915662 650602	3.457831 325301	2.189943 108186	83
84	6.999007 936507	3.499503 968253	2.216335 981179	84
85	7.082352 941176	3.541176 470588	2.242728 760432	85
86	7.165697 674418	3.582848 837209	2.269121 449212	86
87	7.249042 145593	3.624521 072796	2.295514 050640	87
88	7.332386 363636	3.666193 181818	2.321906 567694	88
89	7.415730 337078	3.707865 168539	2.348299 003217	89
90	7.499074 074074	3.749537 037037	2.374691 001905	90
91	7.582417 582418	3.791208 791209	2.401083 640427	91
92	7.665760 869565	3.832880 434782	2.427475 847195	92
93	7.749103 942652	3.874551 971326	2.453867 982611	93
94	7.832446 808510	3.916223 404255	2.480260 048954	94
95	7.915789 473684	3.957894 736842	2.506652 048403	95
96	7.999131 944444	3.999565 972222	2.533043 983049	96
97	8.082474 226804	4.041237 113402	2.559435 854901	97
98	8.165816 326530	4.082908 163265	2.585827 665875	98
99	8.249158 249158	4.124579 124579	2.612219 417817	99
100	8.332500 000000	4.166250 000000	2.638611 112500	100

r	$A_5(r)$	$A_6(r)$	$A_7(r)$	r
2	.027343 750000	.020507 812500	.016113 281250	2
3	.049382 716051	.037341 868617	.029568 663308	3
4	.069824 218750	.052947 998048	.042037 963870	4
5	.089600 000000	.068032 000000	.054080 000000	5
6	.109037 422841	.082848 549908	.065901 595960	6
7	.128279 883381	.097510 391078	.077595 219679	7
8	.147399 902343	.112074 851991	.089208 126073	8
9	.166438 042981	.126574 100731	.100766 908588	9
10	.185418 750000	.141027 562500	.112287 656250	10
11	.204357 625846	.155447 653228	.123780 665749	11
12	.223265 094524	.169842 676279	.135252 829453	12
13	.242148 384163	.184218 393561	.146708 933373	13
14	.261012 663996	.198578 928038	.158152 403908	14
15	.279861 728397	.212927 307423	.169585 758266	15
16	.298698 425293	.227265 805006	.181010 887026	16
17	.317524 933850	.241596 160747	.192429 240719	17
15	.279861 728397	.212927 307423	.169585 758266	15
19	.355153 812509	.270237 579353	.215249 874903	19
20	.373958 593750	.284550 568360	.226653 764650	20
21	.392758 161469	.298859 389421	.238054 183272	21
22	.411553 225959	.313164 610180	.249451 603397	22
23	.430344 374132	.327466 699644	.260846 415403	23
24	.449132 095149	.341766 048828	.272238 944969	24
25	.467916 800000	.356062 986240	.283629 465600	25
26	.486698 836483	.370357 789901	.295018 208818	26
27	.505478 500908	.384650 696676	.306405 371862	27
28	.524256 047285	.398941 909657	.317791 123906	28
29	.543031 694617	.413231 603938	.329175 610770	29
30	.561805 632718	.427519 931295	.340558 958892	30
31	.580578 026922	.441807 023892	.351941 278299	31
32	.599349 021913	.456092 997455	.363322 665448	32
33	.618118 744841	.470377 953641	.374703 204989	33
34	.636887 307908	.484661 982159	.386082 971628	34
35	.655654 810496	.498945 162502	.397462 031618	35
36	.674421 340951	.513227 565319	.408840 443810	36
37	.693186 978068	.527509 253619	.420218 260692	37
38	.711951 792354	.541790 283791	.431595 529237	38
39	.730715 847088	.556070 706414	.442972 291544	39
40	.749479 199219	.570350 567078	.454348 585664	40
41	.768241 900147	.584629 906861	.465724 445768	41
42	.787003 996374	.598908 762962	.477099 902865	42
43	.805765 530079	.613187 169121	.488474 985075	43
44	.824526 539609	.627465 155994	.499849 717910	44
45	.843287 059901	.641742 751545	.511224 124707	45
46	.862047 122854	.656019 981280	.522598 226703	46
47	.880806 757656	.670296 868504	.533972 043251	47
48	.899565 991061	.684573 434682	.545345 592344	48
49	.918324 847640	.698849 699425	.556718 890252	49
50	.937083 350000	.713125 680821	.568091 952053	50

r	$A_5(r)$	$A_6(r)$	$A_7(r)$	r
51	0.955841 518974	0.727401 395544	0.579464 791658	51
52	0.974599 373792	0.741676 858968	0.590837 421868	52
53	0.993356 932234	0.755952 085302	0.602209 854499	53
54	1.012114 210762	0.770227 087739	0.613582 100587	54
55	1.030871 224647	0.784501 878490	0.624954 170297	55
56	1.049627 988055	0.798776 468865	0.636326 073022	56
57	1.068384 514175	0.813050 869427	0.647697 817583	57
58	1.087140 815275	0.827325 090004	0.659069 412196	58
59	1.105896 902804	0.841599 139709	0.670440 864372	59
60	1.124652 787423	0.855873 027121	0.681812 181256	60
61	1.143408 479124	0.870146 760204	0.693183 369730	61
62	1.162163 987237	0.884420 346439	0.704554 435755	62
63	1.180919 320502	0.898693 792788	0.715925 385189	63
64	1.199674 487113	0.912967 105834	0.727296 223564	64
65	1.218429 494766	0.927240 291723	0.738666 954960	65
66	1.237184 350683	0.941513 356221	0.750037 587226	66
67	1.255939 061655	0.955786 304776	0.761408 121867	67
68	1.274693 634077	0.970059 142502	0.772778 564164	68
69	1.293448 073975	0.984331 874198	0.784148 918079	69
70	1.312202 387026	0.998604 504447	0.795519 187502	70
71	1.330956 578593	1.012877 037496	0.806889 375896	71
72	1.349710 653730	1.027149 477411	0.818259 486694	72
73	1.368464 617229	1.041421 828024	0.829629 523080	73
74	1.387218 473610	1.055694 092935	0.840999 488033	74
75	1.405972 227162	1.069966 275610	0.852369 384498	75
76	1.424725 881939	1.084238 379246	0.863739 215058	76
77	1.443479 441791	1.098510 406961	0.875108 982384	77
78	1.462232 910374	1.112782 361653	0.886478 688836	78
79	1.480986 291146	1.127054 246105	0.897848 336769	79
80	1.499739 587403	1.141326 062950	0.909217 928370	80
81	1.518492 802275	1.155597 814687	0.920587 465709	81
82	1.537245 938738	1.169869 503699	0.931956 950781	82
83	1.555998 999603	1.184141 132253	0.943326 385554	83
84	1.574751 987643	1.198412 702514	0.954695 771587	84
85	1.593504 905354	1.212684 216531	0.966065 110836	85
86	1.612257 755214	1.226955 676272	0.977434 404857	86
87	1.631010 539562	1.241227 083603	0.988803 655204	87
88	1.649763 260632	1.255498 440323	1.000172 863391	88
89	1.668515 920556	1.269769 748115	1.011542 030784	89
90	1.687268 521377	1.284041 008629	1.022910 263720	90
91	1.706021 065037	1.298312 223414	1.034280 248674	91
92	1.724773 553401	1.312583 393971	1.045649 301724	92
93	1.743525 988253	1.326854 521716	1.057018 319090	93
94	1.762278 371304	1.341125 608014	1.068387 301903	94
95	1.781030 704184	1.355396 654184	1.079756 251293	95
96	1.799782 988463	1.369667 661474	1.091125 168283	96
97	1.818535 225651	1.383938 631100	1.102494 053898	97
98	1.837287 417180	1.398209 564181	1.113862 909030	98
99	1.856039 564437	1.412480 461864	1.125231 734693	99
100	1.874791 668750	1.426751 325184	1.136600 531710	100

TABLE 35

Sums of Powers of Integers

Description: Values of $S_n(p)$ from $n = 1$ to $n = 10$ for the range $p = 1$ to $p = 100$. Also values of $S_1(p)$, $S_2(p)$ and $S_3(p)$ from $p = 101$ to $p = 1000$.

p	$S_1(p)$	$S_2(p)$	$S_3(p)$	$S_4(p)$	$S_5(p)$	p
1	1	1	1	1	1	1
2	3	5	9	17	33	2
3	6	14	36	98	276	3
4	10	30	100	354	1300	4
5	15	55	225	979	4425	5
6	21	91	441	2275	12201	6
7	28	140	784	4676	29008	7
8	36	204	1296	8772	61776	8
9	45	285	2025	15333	120825	9
10	55	385	3025	25333	220825	10
11	66	506	4356	39974	381876	11
12	78	650	6084	60710	630708	12
13	91	819	8281	89271	1002001	13
14	105	1015	11025	127687	1539825	14
15	120	1240	14400	178312	2299200	15
16	136	1496	18496	243848	3347776	16
17	153	1785	23409	327369	4767633	17
18	171	2109	29241	432345	6657201	18
19	190	2470	36100	562666	9133300	19
20	210	2870	44100	722666	12333300	20
21	231	3311	53361	917147	16417401	21
22	253	3795	64009	1151403	21571033	22
23	276	4324	76176	1431244	28007376	23
24	300	4900	90000	1763020	35970000	24
25	325	5525	105625	2153645	45735625	25
26	351	6201	123201	2610621	57617001	26
27	378	6930	142884	3142062	71965908	27
28	406	7714	164836	3756718	89176276	28
29	435	8555	189225	4463999	109687425	29
30	465	9455	216225	5273999	133987425	30
31	496	10416	246016	6197520	162616576	31
32	528	11440	278784	7246096	196171008	32
33	561	12529	314721	8432017	235306401	33
34	595	13685	354025	9768353	280741825	34
35	630	14910	396900	11268978	333263700	35
36	666	16206	443556	12948594	393729876	36
37	703	17575	494209	14822755	463073833	37
38	741	19019	549081	16907891	542309001	38
39	780	20540	608400	19221332	632533200	39
40	820	22140	672400	21781332	734933200	40
41	861	23821	741321	24607093	850789401	41
42	903	25585	815409	27718789	981480633	42
43	946	27434	894916	31137590	1128489076	43
44	990	29370	980100	34885686	1293405300	44
45	1035	31395	1071225	38986311	1477933425	45
46	1081	33511	1168561	43463767	1683896401	46
47	1128	35720	1272384	48343448	1913241408	47
48	1176	38024	1382976	53651864	2168045376	48
49	1225	40425	1500625	59416665	2450520625	49
50	1275	42925	1625625	65666665	2763020625	50

p	$S_1(p)$	$S_2(p)$	$S_3(p)$	$S_4(p)$	$S_5(p)$	p
51	1326	45526	1758276	72431866	3108045876	51
52	1378	48230	1898884	79743482	3488249908	52
53	1431	51039	2047761	87633963	3906445401	53
54	1485	53955	2205225	96137019	4365610425	54
55	1540	56980	2371600	105287644	4868894800	55
56	1596	60116	2547216	115122140	5419626576	56
57	1653	63365	2732409	125678141	6021318633	57
58	1711	66729	2927521	136994637	6677675401	58
59	1770	70210	3132900	149111998	7392599700	59
60	1830	73810	3348900	162071998	8170199700	60
61	1891	77531	3575881	175917839	9014796001	61
62	1953	81375	3814209	190694175	9930928833	62
63	2016	85344	4064256	206447136	10923365376	63
64	2080	89440	4326400	223224352	11997107200	64
65	2145	93665	4601025	241074977	13157397825	65
66	2211	98021	4888521	260049713	14409730401	66
67	2278	102510	5189284	280200834	15759855508	67
68	2346	107134	5503716	301582210	17213789076	68
69	2415	111895	5832225	324249331	18777820425	69
70	2485	116795	6175225	348259331	20458520425	70
71	2556	121836	6533136	373671012	22262749776	71
72	2628	127020	6906384	400544868	24197667408	72
73	2701	132349	7295401	428943109	26270739001	73
74	2775	137825	7700625	458929685	28489745625	74
75	2850	143450	8122500	490570310	30862792500	75
76	2926	149226	8561476	523932486	33398317876	76
77	3003	155155	9018009	559085527	36105102033	77
78	3081	161239	9492561	596100583	38992276401	78
79	3160	167480	9985600	635050664	42069332800	79
80	3240	173880	10497600	676010664	45346132800	80
81	3321	180441	11029041	719057385	48832917201	81
82	3403	187165	11580409	764269561	52540315633	82
83	3486	194054	12152196	811727882	56479356276	83
84	3570	201110	12744900	861515018	60661475700	84
85	3655	208335	13359025	913715643	65098528825	85
86	3741	215731	13995081	968416459	69802799001	86
87	3828	223300	14653584	1025706220	74787008208	87
88	3916	231044	15335056	1085675756	80064327376	88
89	4005	238965	16040025	1148417997	85648386825	89
90	4095	247065	16769025	1214027997	91553286825	90
91	4186	255346	17522596	1282602958	97793608276	91
92	4278	263810	18301284	1354242254	104384423508	92
93	4371	272459	19105641	1429047455	111341307201	93
94	4465	281295	19936225	1507122351	118680347425	94
95	4560	290320	20793600	1588572976	126418156800	95
96	4656	299536	21678336	1673507632	134571883776	96
97	4753	308945	22591009	1762036913	143159224033	97
98	4851	318549	23532201	1854273729	152198432001	98
99	4950	328350	24502500	1950333330	161708332500	99
100	5050	338350	25502500	2050333330	171708332500	100

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2	65	129	257	2
3	794	2316	6818	3
4	4890	18700	72354	4
5	20515	96825	462979	5
6	67171	376761	2142595	6
7	184820	1200304	7907396	7
8	446964	3297456	24684612	8
9	978405	8080425	67731333	9
10	1978405	18080425	167731333	10
11	3749966	37567596	382090214	11
12	6735950	73399404	812071910	12
13	11562759	136147921	1627802631	13
14	19092295	241561425	3103591687	14
15	30482920	412420800	5666482312	15
16	47260136	680856256	9961449608	16
17	71397705	1091194929	16937207049	17
18	105409929	1703414961	27957167625	18
19	152455810	2597286700	44940730666	19
20	216455810	3877286700	70540730666	20
21	302221931	5678375241	108363590027	21
22	415601835	8172733129	163239463563	22
23	563637724	11577558576	241550448844	23
24	754740700	16164030000	351625763020	24
25	998881325	22267545625	504213653645	25
26	1307797101	30299355801	713040718221	26
27	1695217590	40759709004	995470254702	27
28	2177107894	54252637516	1373272253038	28
29	2771931215	71502513825	1873518665999	29
30	3500931215	93372513825	2529618665999	30
31	4388434896	120885127936	3382509703440	31
32	5462176720	155244866304	4482021331216	32
33	6753644689	197863309281	5888429949457	33
34	8298449105	250386659425	7674223854353	34
35	10136714730	314725956300	9926099244978	35
36	12313497066	393090120396	12747209152434	36
37	14879223475	488021997529	16259688606355	37
38	17890159859	602437580121	20607480744851	38
39	21408903620	739668586800	25959490005332	39
40	25504903620	903508586800	32513090005332	40
41	30255007861	1098262860681	40498015234453	41
42	35744039605	1328802193929	50180667230869	42
43	42065402654	1600620805036	61868867508470	43
44	49321716510	1919898614700	75917091133686	44
45	57625482135	2293568067825	92732216524311	45
46	67099779031	2729385725041	112779828756247	46
47	77878994360	3236008845504	136591115418008	47
48	90109584824	3823077187776	164770395847064	48
49	103950872025	4501300260625	198003326416665	49
50	119575872025	5282550260625	237065826416665	50

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51	137172159826	6179960938476	282833770987066	51
52	156942769490	7208032641004	336293499518522	52
53	179107130619	8382743780841	398553189929883	53
54	203902041915	9721668990825	470855151269019	54
55	231582682540	11244104225200	554589089159644	55
56	262423661996	12971199074736	651306400733660	56
57	296720109245	14926096567929	762735557845661	57
58	334788801789	17134080735481	890798639563677	58
59	376969335430	19622732220300	1037629077167998	59
60	423625335430	22422092220300	1205590677167998	60
61	475145709791	25564835056321	1397297990165279	61
62	531945945375	29086449662529	1615638095750175	62
63	594469447584	33025430301696	1863793876017696	63
64	663188924320	37423476812800	2145268852728352	64
65	738607814945	42325704703425	2463913665618977	65
66	821261764961	47780865404481	2823954271888673	66
67	911720147130	53841577009804	3230021949445314	67
68	1010587629754	60564565828236	3687185189098690	68
69	1118505792835	68010919080825	4200983563527331	69
70	1236154792835	76246349080825	4777463663527331	70
71	1364255076756	85341469239216	5423217194773092	71
72	1503569146260	95372082243504	6145421331081828	72
73	1654903372549	106419480762601	6951881422975909	73
74	1819109862725	118570761035625	7851076163179685	74
75	1997088378350	131919149707500	8852205313570310	75
76	2189788306926	146564344279276	9965240101025286	76
77	2398210687015	162612867546129	11200976392572967	77
78	2623410287719	180178436401041	12571090763256103	78
79	2866497743240	199382345387200	14088199573162664	79
80	3128641743240	220353865387200	15765921173162664	80
81	3411071279721	243230657842161	17618941362014505	81
82	3715077951145	268159204898929	19663082220669481	82
83	4042018324514	295295255888556	21915374452808522	83
84	4393316356130	324804290544300	24394133363891018	84
85	4770465871755	356861999372425	27119038614281643	85
86	5175033106891	391654781594121	30111217885347499	86
87	5608659307900	429380261081904	33393334600784620	87
88	6073063394684	470247820718896	36989679848839916	88
89	6570044685645	514479155614425	40926268654541997	89
90	7101485685645	562308845614425	45230940754541997	90
91	7669354937686	613984947550156	49933466030693518	91
92	8275709939030	669769607673804	55065654762069134	92
93	8922700122479	729939694734561	60661472858719535	93
94	9612569903535	794787454153825	66757162244130351	94
95	10347661794160	864621183763200	73391366557020976	95
96	11130419583856	939765931574016	80605262346859312	96
97	11963391588785	1020564216052129	88442695941236273	97
98	12849233969649	1107376769376801	96950326167054129	98
99	13790714119050	1200583304167500	106177773111333330	99
100	14790714119050	1300583304167500	116177773111333330	100

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2	513	1025	2
3	20196	60074	3
4	282340	1 108650	4
5	2 235465	10 874275	5
6	12 313161	71 340451	6
7	52 666768	353 815700	7
8	186 884496	1427 557524	8
9	574 304985	4914 341925	9
10	1574 304985	14914 341925	10
11	3932 252676	40851 766526	11
12	9092 033028	102769 130750	12
13	19696 532401	240627 622599	13
14	40357 579185	529882 277575	14
15	78800 938560	1 106532 668200	15
16	147520 415296	2 206044 295976	16
17	266108 291793	4 222038 196425	17
18	464467 582161	7 792505 423049	18
19	787155 279940	13 923571 680850	19
20	1 299155 279940	24 163571 680850	20
21	2 093435 326521	40 843452 659051	21
22	3 300704 544313	67 403375 450475	22
23	5 101857 205776	108 829886 664124	23
24	7 743664 746000	172 233267 629500	24
25	11 558362 011625	267 600699 270125	25
26	16 987865 690601	408 767794 923501	26
27	24 613463 175588	614 658927 018150	27
28	35 191919 128996	910 855693 713574	28
29	49 699065 104865	1331 562927 013775	29
30	69 382065 104865	1922 052927 013775	30
31	95 821687 265536	2741 681213 994576	31
32	131 006059 354368	3867 581120 837200	32
33	177 417543 756321	5399 160106 101649	33
34	238 134536 522785	7463 537860 161425	34
35	316 950175 194660	10222 085213 677050	35
36	418 510131 863076	13878 243653 740026	36
37	548 471871 658153	18686 828026 157875	37
38	713 687972 921001	24965 039874 146099	38
39	922 416334 079760	33105 445959 337700	39
40	1184 560334 079760	43591 205959 337700	40
41	1511 942268 473721	57013 865269 490101	41
42	1918 613652 323193	74094 063391 167925	42
43	2421 206264 260036	95705 545704 452174	43
44	3039 328103 769540	122902 906642 870350	44
45	3796 008746 347665	156953 535558 885975	45
46	4718 198909 016721	199374 283041 662551	46
47	5837 329382 119488	251973 415277 492600	47
48	7189 934842 714176	316898 477386 037624	48
49	8818 348440 624625	396690 743683 649625	49
50	10771 473440 624625	494346 993683 649625	50

p	$S_9(p)$	$S_{10}(p)$	p
51	13 10563 86137 15076	613 38941 75112 62626	51
52	15 88554 44973 50788	757 94452 34603 19650	52
53	19 18530 80891 52921	932 83199 38258 32699	53
54	23 08961 40014 66265	1143 66451 30907 53275	54
55	27 69498 05854 50640	1396 95967 52098 93900	55
56	33 11115 00335 95536	1700 26516 43060 08076	56
57	39 46261 19889 79593	2062 29849 57628 99325	57
58	46 89027 07286 24521	2493 10270 26623 05149	58
59	55 55326 65472 79460	3004 21945 59629 46550	59
60	65 63096 25472 79460	3608 88121 59629 46550	60
61	77 32510 86401 13601	4322 22412 76258 29151	61
62	90 86219 51863 77153	5161 52349 34941 69375	62
63	106 49600 93432 30976	6146 45378 53759 60224	63
64	124 51040 78527 12960	7299 37528 99828 07200	64
65	145 22232 06906 03585	8645 64962 44456 97825	65
66	168 98500 07044 03521	10213 98650 53564 93601	66
67	196 19153 51006 98468	12036 82430 99082 55050	67
68	227 27863 53971 28036	14150 74713 00654 65674	68
69	262 73072 32327 04265	16596 94119 07202 25475	69
70	303 08433 02327 04265	19421 69368 07202 25475	70
71	348 93283 09511 53296	22676 93723 17301 06676	71
72	400 93152 87653 82288	26420 84347 43545 94100	72
73	459 80311 54736 50201	30718 46930 40581 51749	73
74	526 34352 62487 29625	35642 45970 14140 29125	74
75	601 42821 25280 26500	41273 81117 23612 94750	75
76	686 01885 63746 04676	47702 70010 47012 36126	76
77	781 17055 08237 76113	55029 38057 72874 36775	77
78	888 03947 17370 60721	63365 15640 35236 36199	78
79	1007 89106 77196 79040	72833 43249 11504 83400	79
80	1142 10879 57196 79040	83570 85073 11504 83400	80
81	1292 20343 10166 78161	95728 51619 02074 12201	81
82	1459 82298 14263 86193	109473 31932 38034 70825	82
83	1646 76323 66939 26596	124989 36051 10093 24274	83
84	1854 97898 52248 56260	142479 48338 76074 16050	84
85	2086 59593 15080 59385	162166 92382 16796 81675	85
86	2343 92334 88197 23001	184297 08171 04827 52651	86
87	2629 46750 30627 52528	209139 42312 96263 21500	87
88	2945 94588 48916 18576	236989 52073 05665 33724	88
89	3296 30228 85991 03785	268171 24066 05327 17325	89
90	3683 72277 75991 03785	303039 08467 05327 17325	90
91	4111 65257 77288 92196	341980 69648 23434 62726	91
92	4583 81394 10154 48868	385419 54190 47066 76550	92
93	5104 22502 40039 36161	433817 77262 26359 94799	93
94	5677 21982 62325 52865	487679 28403 21259 64975	94
95	6307 46923 59571 62240	547552 97795 59638 55600	95
96	7000 00323 17816 42496	614036 24155 51139 60176	96
97	7760 23429 04362 07713	687778 65424 46067 86225	97
98	8593 98205 25663 57601	769485 93493 33614 75249	98
99	9507 49930 00499 98500	859924 14243 42419 24250	99
100	10507 49930 00499 98500	959924 14243 42419 24250	100

p	$S_1(p)$	$S_2(p)$	$S_3(p)$	p	$S_1(p)$	$S_2(p)$	$S_3(p)$
101	5151	348551	26532801	151	11476	1159076	131698576
102	5253	358955	27594009	152	11628	1182180	135210384
103	5356	369564	28686736	153	11781	1205589	138791961
104	5460	380380	29811600	154	11935	1229305	142444225
105	5565	391405	30969225	155	12090	1253330	146168100
106	5671	402641	32160241	156	12246	1277666	149964516
107	5778	414090	33385284	157	12403	1302315	153834409
108	5886	425754	34644996	158	12561	1327279	157778721
109	5995	437635	35940025	159	12720	1352560	161798400
110	6105	449735	37271025	160	12880	1378160	165894400
111	6216	462056	38638656	161	13041	1404081	170067681
112	6328	474600	40043584	162	13203	1430325	174319209
113	6441	487369	41486481	163	13366	1456894	178649956
114	6555	500365	42968025	164	13530	1483790	183060900
115	6670	513590	44488900	165	13695	1511015	187553025
116	6786	527046	46049796	166	13861	1538571	192127321
117	6903	540735	47651409	167	14028	1566460	196784784
118	7021	554659	49294441	168	14196	1594684	201526416
119	7140	568820	50979600	169	14365	1623245	206353225
120	7260	583220	52707600	170	14535	1652145	211266225
121	7381	597861	54479161	171	14706	1681386	216266436
122	7503	612745	56295009	172	14878	1710970	221354884
123	7626	627874	58155876	173	15051	1740899	226532601
124	7750	643250	60062500	174	15225	1771175	231800625
125	7875	658875	62015625	175	15400	1801800	237160000
126	8001	674751	64016001	176	15576	1832776	242611776
127	8128	690880	66064384	177	15753	1864105	248157009
128	8256	707264	68161536	178	15931	1895789	253796761
129	8385	723905	70308225	179	16110	1927830	259532100
130	8515	740805	72505225	180	16290	1960230	265364100
131	8646	757966	74753316	181	16471	1992991	271293841
132	8778	775390	77053284	182	16653	2026115	277322409
133	8911	793079	79405921	183	16836	2059604	283450896
134	9045	811035	81812025	184	17020	2093460	289680400
135	9180	829260	84272400	185	17205	2127685	296012025
136	9316	847756	86787856	186	17391	2162281	302446881
137	9453	866525	89359209	187	17578	2197250	308986084
138	9591	885569	91987281	188	17766	2232594	315630756
139	9730	904890	94672900	189	17955	2268315	322382025
140	9870	924490	97416900	190	18145	2304415	329241025
141	10011	944371	100220121	191	18336	2340896	336208896
142	10153	964535	103083409	192	18528	2377760	343286784
143	10296	984984	106007616	193	18721	2415009	350475841
144	10440	1005720	108993600	194	18915	2452645	357777225
145	10585	1026745	112042225	195	19110	2490670	365192100
146	10731	1048061	115154361	196	19306	2529086	372721636
147	10878	1069670	118330884	197	19503	2567895	380367009
148	11026	1091574	121572676	198	19701	2607099	388129401
149	11175	1113775	124880625	199	19900	2646700	396010000
150	11325	1136275	128255625	200	20100	2686700	404010000

p	$S_1(p)$	$S_2(p)$	$S_3(p)$	p	$S_1(p)$	$S_2(p)$	$S_3(p)$
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202	20503	2767905	420373009	252	31878	5366130	1016206884
203	20706	2809114	428738436	253	32131	5430139	1032401161
204	20910	2850730	437228100	254	32385	5494655	1048788225
205	21115	2892755	445843225	255	32640	5559680	1065369600
206	21321	2935191	454585041	256	32896	5625216	1082146816
207	21528	2978040	463454784	257	33153	5691265	1099121409
208	21736	3021304	472453696	258	33411	5757829	1116294921
209	21945	3064985	481583025	259	33670	5824910	1133668900
210	22155	3109085	490844025	260	33930	5892510	1151244900
211	22366	3153606	500237956	261	34191	5960631	1169024481
212	22578	3198550	509766084	262	34453	6029275	1187009209
213	22791	3243919	519429681	263	34716	6098444	1205200656
214	23005	3289715	529230025	264	34980	6168140	1223600400
215	23220	3335940	539168400	265	35245	6238365	1242210025
216	23436	3382596	549246096	266	35511	6309121	1261031121
217	23653	3429685	559464409	267	35778	6380410	1280065284
218	23871	3477209	569824641	268	36046	6452234	1299314116
219	24090	3525170	580328100	269	36315	6524595	1318779225
220	24310	3573570	590976100	270	36585	6597495	1338462225
221	24531	3622411	601769961	271	36856	6670936	1358364736
222	24753	3671695	612711009	272	37128	6744920	1378488384
223	24976	3721424	623800576	273	37401	6819449	1398834801
224	25200	3771600	635040000	274	37675	6894525	1419405625
225	25425	3822225	646430625	275	37950	6970150	1440202500
226	25651	3873301	657973801	276	38226	7046326	1461227076
227	25878	3924830	669670884	277	38503	7123055	1482481009
228	26106	3976814	681523236	278	38781	7200339	1503965961
229	26335	4029255	693532225	279	39060	7278180	1525683600
230	26565	4082155	705699225	280	39340	7356580	1547635600
231	26796	4135516	718025616	281	39621	7435541	1569823641
232	27028	4189340	730512784	282	39903	7515065	1592249409
233	27261	4243629	743162121	283	40186	7595154	1614914596
234	27495	4298385	755975025	284	40470	7675810	1637820900
235	27730	4353610	768952900	285	40755	7757035	1660970025
236	27966	4409306	782097156	286	41041	7838831	1684363681
237	28203	4465475	795409209	287	41328	7921200	1708003584
238	28441	4522119	808890481	288	41616	8004144	1731891456
239	28680	4579240	822542400	289	41905	8087665	1756029025
240	28920	4636840	836366400	290	42195	8171765	1780418025
241	29161	4694921	850363921	291	42486	8256446	1805060196
242	29403	4753485	864536409	292	42778	8341710	1829957284
243	29646	4812534	878885316	293	43071	8427559	1855111041
244	29890	4872070	893412100	294	43365	8513995	1880523225
245	30135	4932095	908118225	295	43660	8601020	1906195600
246	30381	4992611	923005161	296	43956	8688636	1932129936
247	30628	5053620	938074384	297	44253	8776845	1958328009
248	30876	5115124	953327376	298	44551	8865649	1984791601
249	31125	5177125	968765625	299	44850	8955050	2011522500
250	31375	5239625	984390625	300	45150	9045050	2038522500

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302	45753	9226855	2093337009	352	62128	14600080	3859888384
303	46056	9318664	2121155136	353	62481	14724689	3903875361
304	46360	9411080	2149249600	354	62835	14850005	3948237225
305	46665	9504105	2177622225	355	63190	14976030	3992976100
306	46971	9597741	2206274841	356	63546	15102766	4038094116
307	47278	9691990	2235209284	357	63903	15230215	4083593409
308	47586	9786854	2264427396	358	64281	15358379	4129476121
309	47895	9882335	2293931025	359	64620	15487260	4175744400
310	48205	9978435	2323722025	360	64980	15616860	4222400400
311	48516	10075156	2353802256	361	65341	15747181	4269446281
312	48828	10172500	2384173584	362	65703	15878225	4316884209
313	49141	10270469	2414837881	363	66066	16009994	4364716356
314	49455	10369065	2445797025	364	66430	16142490	4412944900
315	49770	10468290	2477052900	365	66795	16275715	4461572025
316	50086	10568146	2508607396	366	67161	16409671	4510599921
317	50403	10668635	2540462409	367	67528	16544360	4560030784
318	50721	10769759	2572619841	368	67896	16679784	4609866816
319	51040	10871520	2605081600	369	68265	16815945	4660110225
320	51360	10973920	2637849600	370	68635	16952845	4710763225
321	51681	11076961	2670925761	371	69006	17090486	4761828036
322	52003	11180645	2704312009	372	69378	17228870	4813306884
323	52326	11284974	2738010276	373	69751	17367999	4865202001
324	52650	11389950	2772022500	374	70125	17507875	4917515625
325	52975	11495575	2806350625	375	70500	17648500	4970250000
326	53301	11601851	2840996601	376	70876	17789876	5023407376
327	53628	11708780	2875962384	377	71253	17932005	5076990009
328	53956	11816364	2911249936	378	71631	18074889	5131000161
329	54285	11924605	2946861225	379	72010	18218530	5185440100
330	54651	12033505	2982798225	380	72390	18362930	5240312100
331	54946	12143066	3019062916	381	72771	18508091	5295618441
332	55278	12253290	3055657284	382	73153	18654015	5351361409
333	55611	12364179	3092583321	383	73536	18800704	5407543296
334	55945	12475735	3129843025	384	73920	18948160	5464166400
335	56280	12587960	3167438400	385	74305	19096385	5521233025
336	56616	12700856	3205371456	386	74691	19245381	5578745481
337	56953	12814425	3243644209	387	75078	19395150	5636706084
338	57291	12928669	3282258681	388	75466	19545694	5695117156
339	57630	13043590	3321216900	389	75855	19697015	5753981025
340	57970	13159190	3360520900	390	76245	19849115	5813300025
341	58311	13275471	3400172721	391	76636	20001996	5873076496
342	58653	13392435	3440174409	392	77028	20155660	5933312784
343	58996	13510084	3480528016	393	77421	20310109	5994011241
344	59340	13628420	3521235600	394	77815	20465345	6055174225
345	59685	13747445	3562299225	395	78210	20621370	6116804100
346	60031	13867161	3603720961	396	78606	20778186	6178903236
347	60378	13987570	3645502884	397	79003	20935795	6241474009
348	60726	14108674	3687647076	398	79401	21094199	6304518801
349	61075	14230475	3730155625	399	79800	21253400	6368040000
350	61425	14352975	3773030625	400	80200	21413400	6432040000

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402	81003	21735805	6561486009	452	102378	30884030	10481254884
403	81406	21898214	6626936836	453	102831	31089239	10574214561
404	81810	22061430	6692876100	454	103285	31295355	10667791225
405	82215	22225455	6759306225	455	103740	31502380	10761987600
406	82621	22390291	6826229641	456	104196	31710316	10856806416
407	83028	22555940	6893648784	457	104653	31919165	10952250409
408	83436	22722404	6961566096	458	105111	32128929	11048322321
409	83845	22889685	7029984025	459	105570	32339610	11145024900
410	84255	23057785	7098905025	460	106030	32551210	11242360900
411	84666	23226706	7168331556	461	106491	32763731	11340333081
412	85078	23396450	7238266084	462	106953	32977175	11438944209
413	85491	23567019	7308711081	463	107416	33191544	11538197056
414	85905	23738415	7379669025	464	107880	33406840	11638094400
415	86320	23910640	7451142400	465	108345	33623065	11738639025
416	86736	24083696	7523133696	466	108811	33840221	11839833721
417	87153	24257585	7595645409	467	109278	34058310	11941681284
418	87571	24432309	7668680041	468	109746	34277334	12044184516
419	87990	24607870	7742240100	469	110215	34497295	12147346225
420	88410	24784270	7816328100	470	110685	34718195	12251169225
421	88831	24961511	7890946561	471	111156	34940036	12355656336
422	89253	25139595	7966098009	472	111628	35162820	12460810384
423	89676	25318524	8041784976	473	112101	35386549	12566634201
424	90100	25498300	8118010000	474	112575	35611225	12673130625
425	90525	25678925	8194775625	475	113050	35836850	12780302500
426	90951	25860401	8272084401	476	113526	36063426	12888152676
427	91378	26042730	8349938884	477	114003	36290955	12996684009
428	91806	26225914	8428341636	478	114481	36519439	13105899361
429	92235	26409955	8507295225	479	114960	36748880	13215801600
430	92665	26594855	8586802225	480	115440	36979280	13326393600
431	93096	26780616	8666865216	481	115921	37210641	13437678241
432	93528	26967240	8747486784	482	116403	37442965	13549658409
433	93961	27154729	8828669521	483	116886	37676254	13662336996
434	94395	27343085	8910416025	484	117370	37910510	13775716900
435	94830	27532310	8992728900	485	117855	38145735	13889801025
436	95266	27722406	9075610756	486	118341	38381931	14004592281
437	95703	27913375	9159064209	487	118828	38619100	14120093584
438	96141	28105219	9243091881	488	119316	38857244	14236307856
439	96580	28297940	9327696400	489	119805	39096365	14353238025
440	97020	28491540	9412880400	490	120295	39336465	14470887025
441	97461	28686021	9498646521	491	120786	39577546	14589257796
442	97903	28881385	9584997409	492	121278	39819610	14708353284
443	98346	29077634	9671935716	493	121771	40062659	14828176441
444	98790	29274770	9759464100	494	122265	40306695	14948730225
445	99235	29472795	9847585225	495	122760	40551720	15070017600
446	99681	29671711	9936301761	496	123256	40797736	15192041536
447	100128	29871520	10025616384	497	123753	41044745	15314805009
448	100576	30072224	10115531776	498	124251	41292749	15438311001
449	101025	30273825	10206050625	499	124750	41541750	15562562500
450	101475	30476325	10297175625	500	125250	41791750	15687562500

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502	126253	42294755	15939820009	552	152628	56217980	23295306384
503	126756	42547764	16067083536	553	153181	56523789	23464418761
504	127260	42801780	16195107600	554	153735	56830705	23634450225
505	127765	43056805	16323895225	555	154290	57138730	23805404100
506	128271	43312841	16453449441	556	154846	57447866	23977283716
507	128778	43569890	16583773284	557	155403	57758115	24150092409
508	129286	43827954	16714869796	558	155961	58069479	24323833521
509	129795	44087035	16846742025	559	156520	58381960	24498510400
510	130305	44347135	16979393025	560	157080	58695560	24674126400
511	130816	44608256	17112825856	561	157641	59010281	24850684881
512	131328	44870400	17247043584	562	158203	59326125	25028189209
513	131841	45133569	17382049281	563	158766	59643094	25206642756
514	132355	45397765	17517846025	564	159330	59961190	25386048900
515	132870	45662990	17654436900	565	159895	60280415	25566411025
516	133386	45929246	17791824996	566	160461	60600771	25747732521
517	133903	46196535	17930013409	567	161028	60922260	25930016784
518	134421	46464859	18069005241	568	161596	61244884	26113267216
519	134940	46734220	18208803600	569	162165	61568645	26297487225
520	135460	47004620	18349411600	570	162735	61893545	26482680225
521	135981	47276061	18490832361	571	163306	62219586	26668849636
522	136503	47548545	18633069009	572	163878	62546770	26855998884
523	137026	47822074	18776124676	573	164451	62875099	27044131401
524	137550	48096650	18920002500	574	165025	63204575	27233250625
525	138075	48372275	19064705625	575	165600	63535200	27423360000
526	138601	48648951	19210237201	576	166176	63866976	27614462976
527	139128	48926680	19356600384	577	166753	64199905	27806563009
528	139656	49205464	19503798336	578	167331	64533989	27999663561
529	140185	49485305	19651834225	579	167910	64869230	28193768100
530	140715	49766205	19800711225	580	168490	65205630	28388880100
531	141246	50048166	19950432516	581	169071	65543191	28585003041
532	141778	50331190	20101001284	582	169653	65881915	28782140409
533	142311	50615279	20252420721	583	170236	66221804	28980295696
534	142845	50900435	20404694025	584	170820	66562860	29179472400
535	143380	51186660	20557824400	585	171405	66905085	29379674025
536	143916	51473956	20711815056	586	171991	67248481	29580904081
537	144453	51762325	20866669209	587	172578	67593050	29783166084
538	144991	52051769	21022390081	588	173166	67938794	29986463556
539	145530	52342290	21178980900	589	173755	68285715	30190800025
540	146070	52633890	21336444900	590	174345	68633815	30396179025
541	146611	52926571	21494785321	591	174936	68983096	30602604096
542	147153	53220335	21654005409	592	175528	69333560	30810078784
543	147696	53515184	21814108416	593	176121	69685209	31018606641
544	148240	53811120	21975097600	594	176715	70038045	31228191225
545	148785	54108145	22136976225	595	177310	70392070	31438836100
546	149331	54406261	22299747561	596	177906	70747286	31650544836
547	149878	54705470	22463414884	597	178503	71103695	31863321009
548	150426	55005774	22627981476	598	179101	71461299	32077168201
549	150975	55307175	22793450625	599	179700	71820100	32292090000
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602	181503	72903705	32943339009	652	212878	92601930	45317042884
603	182106	73267314	33162595236	653	213531	93028339	45595487961
604	182710	73632130	33382944100	654	214185	93456055	45875214225
605	183315	73998155	33604389225	655	214840	93885080	46156225600
606	183921	74365391	33826934241	656	215496	94315416	46438526016
607	184528	74733840	34050582784	657	216153	94747065	46722119409
608	185136	75103504	34275338496	658	216811	95180029	47007009721
609	185745	75474385	34501205025	659	217470	95614310	47293200900
610	186355	75846485	34728186025	660	218130	96049910	47580696900
611	186966	76219806	34956285156	661	218791	96486831	47869501681
612	187578	76594350	35185506084	662	219453	96925075	48159619209
613	188191	76970119	35415852481	663	220116	97364644	48451053456
614	188805	77347115	35647328025	664	220780	97805540	48743808400
615	189420	77725340	35879936400	665	221445	98247765	49037888025
616	190036	78104796	36113681296	666	222111	98691321	49333296321
617	190653	78485485	36348566409	667	222778	99136210	49630037284
618	191271	78867409	36584595441	668	223446	99582434	49928114916
619	191890	79250570	36821772100	669	224115	100029995	50227533225
620	192510	79634970	37060100100	670	224785	100478895	50528296225
621	193131	80020611	37299583161	671	225456	100929136	50830407936
622	193753	80407495	37540225009	672	226128	101380720	51133872384
623	194376	80795624	37782029376	673	226801	101833649	51438693601
624	195000	81185000	38025000000	674	227475	102287925	51744875625
625	195625	81575625	38269140625	675	228150	102743550	52052422500
626	196251	81967501	38514455001	676	228826	103200526	52361338276
627	196878	82360630	38760946884	677	229503	103658855	52671627009
628	197506	82755014	39008620036	678	230181	104118539	52983292761
629	198135	83150655	39257478225	679	230860	104579580	53296339600
630	198765	83547555	39507525225	680	231540	105041980	53610771600
631	199396	83945716	39758764816	681	232221	105505741	53926592841
632	200028	84345140	40011200784	682	232903	105970865	54243807409
633	200661	84745829	40264836921	683	233586	106437354	54562419396
634	201295	85147785	40519677025	684	234270	106905210	54882432900
635	201930	85551010	40775724900	685	234955	107374435	55203852025
636	202566	85955506	41032984356	686	235641	107845031	55526680881
637	203203	86361275	41291459209	687	236328	108317000	55850923584
638	203841	86768319	41551153281	688	237016	108790344	56176584256
639	204480	87176640	41812070400	689	237705	109265065	56503667025
640	205120	87586240	42074214400	690	238395	109741165	56832176025
641	205761	87997121	42337589121	691	239086	110218646	57162115396
642	206403	88409285	42602198409	692	239778	110697510	57493489284
643	207046	88822734	42868046116	693	240471	111177759	57826301841
644	207690	89237470	43135136100	694	241165	111659395	58160557225
645	208335	89653495	43403472225	695	241860	112142420	58496259600
646	208981	90070811	43673058361	696	242556	112626836	58833413136
647	209628	90489420	43943898384	697	243253	113112645	59172022009
648	210276	90909324	44215996176	698	243951	113599849	59512090401
649	210925	91330525	44489355625	699	244650	114088450	59853622500
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702	246753	115562655	60887043009	752	283128	142035880	80161464384
703	247456	116056864	61234471936	753	283881	142602889	80588422161
704	248160	116552480	61583385600	754	284635	143171405	81017083225
705	248865	117049505	61933788225	755	285390	143741430	81447452100
706	249571	117547941	62285684041	756	286146	144312966	81879533316
707	250278	118047790	62639077284	757	286903	144886015	82313331409
708	250986	118549054	62993972196	758	287661	145460579	82748850921
709	251695	119051735	63350373025	759	288420	146036660	83186096400
710	252405	119555835	63708284025	760	289180	146614260	83625072400
711	253116	120061356	64067709456	761	289941	147193381	84065783481
712	253828	120568300	64428653584	762	290703	147774025	84508234209
713	254541	121076669	64791120681	763	291466	148356194	84952429156
714	255255	121586465	65155115025	764	292230	148939890	85398372900
715	255970	122097690	65520640900	765	292995	149525115	85846070025
716	256686	122610346	65887702596	766	293761	150111871	86295525121
717	257403	123124435	66256304409	767	294528	150700160	86746742784
718	258121	123639959	66626450641	768	295296	151289984	87199727616
719	258840	124156920	66998145600	769	296065	151881345	87654484225
720	259560	124675320	67371393600	770	296835	152474245	88111017225
721	260281	125195161	67746198961	771	297606	153068686	88569331236
722	261003	125716445	68122566009	772	298378	153664670	89029430884
723	261726	126239174	68500499076	773	299151	154262199	89491320801
724	262450	126763350	68880002500	774	299925	154861275	89955005625
725	263175	127288975	69261080625	775	300700	155461900	90420490000
726	263901	127816051	69643737801	776	301476	156064076	90887778576
727	264628	128344580	70027978384	777	302253	156667805	91356876009
728	265356	128874564	70413806736	778	303031	157273089	91827786961
729	266085	129406005	70801227225	779	303810	157879930	92300516100
730	266815	129938905	71190244225	780	304590	158488330	92775068100
731	267546	130473266	71580862116	781	305371	159098291	93251447641
732	268278	131009090	71973085284	782	306153	159709815	93729659409
733	269011	131546379	72366918121	783	306936	160322904	94209708096
734	269745	132085135	72762365025	784	307720	160937560	94691598400
735	270480	132625360	73159430400	785	308505	161553785	95175335025
736	271216	133167056	73558118656	786	309291	162171581	95660922681
737	271953	133710225	73958434209	787	310078	162790950	96148366084
738	272691	134254869	74360381481	788	310866	163411894	96637669956
739	273430	134800990	74763964900	789	311655	164034415	97128839025
740	274170	135348590	75169188900	790	312445	164658515	97621878025
741	274911	135897671	75576057921	791	313236	165284196	98116791696
742	275653	136448235	75984576409	792	314028	165911460	98613584784
743	276396	137000284	76394748816	793	314821	166540309	99112262041
744	277140	137553820	76806579600	794	315615	167170745	99612828225
745	277885	138108845	77220073225	795	316410	167802770	100115288100
746	278631	138665361	77635234161	796	317206	168436386	100619646436
747	279378	139223370	78052066884	797	318003	169071595	101125908009
748	280126	139782874	78470575876	798	318801	169708399	101634077601
749	280875	140343875	78890765625	799	319600	170346800	102144160000
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801	321201	171628401	103170082401	851	362526	205793926	131425100676
802	322003	172271605	103685932009	852	363378	206519830	132043570884
803	322806	172916414	104203713636	853	364231	207247439	132664221361
804	323610	173562830	104723432100	854	365085	207976755	133287057225
805	324415	174210855	105245092225	855	365940	208707780	133912083600
806	325221	174860491	105768698841	856	366796	209440516	134539305616
807	326028	175511740	106294256784	857	367653	210174965	135168728409
808	326836	176164604	106821770896	858	368511	210911129	135800357121
809	327645	176819085	107351246025	859	369370	211649010	136434196900
810	328455	177475185	107882687025	860	370230	212388610	137070252900
811	329266	178132906	108416098756	861	371091	213129931	137708530281
812	330078	178792250	108951486084	862	371953	213872975	138349034209
813	330891	179453219	109488853881	863	372816	214617744	138991769856
814	331705	180115815	110028207025	864	373680	215364240	139636742400
815	332520	180780040	110569550400	865	374545	216112465	140283957025
816	333336	181445896	111112888896	866	375411	216862421	140933418921
817	334153	182113385	111658227409	867	376278	217614110	141585133284
818	334971	182782509	112205570841	868	377146	218367534	142239105316
819	335790	183453270	112754924100	869	378015	219122695	142895340225
820	336610	184125670	113306292100	870	378885	219879595	143553843225
821	337431	184799711	113859679761	871	379756	220638236	144214619536
822	338253	185475395	114415092009	872	380628	221398620	144877674384
823	339076	186152724	114972533776	873	381501	222160749	145543013001
824	339900	186831700	115532010000	874	382375	222924625	146210640625
825	340725	187512325	116093525625	875	383250	223690250	146880562500
826	341551	188194601	116657085601	876	384126	224457626	147552783876
827	342378	188878530	117222694884	877	385003	225226755	148227310009
828	343206	189564114	117790358436	878	385881	225997639	148904146161
829	344035	190251355	118360081225	879	386760	226770280	149583297600
830	344865	190940255	118931868225	880	387640	227544680	150264769600
831	345696	191630816	119505724416	881	388521	228320841	150948567441
832	346528	192323040	120081654784	882	389403	229098765	151634696409
833	347361	193016929	120659664321	883	390286	229878454	152323161796
834	348195	193712485	121239758025	884	391170	230659910	153013968900
835	349030	194409710	121821940900	885	392055	231443135	153707123025
836	349866	195108606	122406217956	886	392941	232228131	154402629481
837	350703	195809175	122992594209	887	393828	233014900	155100493584
838	351541	196511419	123581074681	888	394716	233803444	155800720656
839	352380	197215340	124171664400	889	395605	234593765	156503316025
840	353220	197920940	124764368400	890	396495	235385865	157208285025
841	354061	198628221	125359191721	891	397386	236179746	157915632996
842	354903	199337185	125956139409	892	398278	236975410	158625365284
843	355746	200047834	126555216516	893	399171	237772859	159337487241
844	356590	200760170	127156428100	894	400065	238572095	160052004225
845	357435	201474195	127759779225	895	400960	239373120	160768921600
846	358281	202189911	128365274961	896	401856	240175936	161488244736
847	359128	202907320	128972920384	897	402753	240980545	162209979009
848	359976	203626424	129582720576	898	403651	241786949	162934129801
849	360825	204347225	130194680625	899	404550	242595150	163660702500
850	361675	205069725	130808805625	900	405450	243405150	164389702500

p	$S_1(p)$	$S_2(p)$	$S_3(p)$	p	$S_1(p)$	$S_2(p)$	$S_3(p)$
901	406351	244216951	165121135201	951	452676	287147476	204915560976
902	407253	245030555	165855006009	952	453628	288053780	205778362384
903	408156	245845964	166591320336	953	454581	288961989	206643885561
904	409060	246663180	167330083600	954	455535	289872105	207512136225
905	409965	247482205	168071301225	955	456490	290784130	208383120100
906	410871	248303041	168814978641	956	457446	291698066	209256842916
907	411778	249125690	169561121284	957	458403	292613915	210133310409
908	412686	249950154	170309734596	958	459361	293531679	211012528321
909	413595	250776435	171060824025	959	460320	294451360	211894502400
910	414505	251604535	171814395025	960	461280	295372960	212779238400
911	415416	252434456	172570453056	961	462241	296296481	213666742081
912	416328	253266200	173329003584	962	463203	297221925	214557019209
913	417241	254099769	174090052081	963	464166	298149294	215450075556
914	418155	254935165	174853604025	964	465130	299078590	216345916900
915	419070	255772390	175619664900	965	466095	300009815	217244549025
916	419986	256611446	176388240196	966	467061	300942971	218145977721
917	420903	257452335	177159335409	967	468028	301878060	219050208784
918	421821	258295059	177932956041	968	468996	302815084	219957248016
919	422740	259139620	178709107600	969	469965	303754045	220867101225
920	423660	259986020	179487795600	970	470935	304694945	221779774225
921	424581	260834261	180269025561	971	471906	305637786	222695272836
922	425503	261684345	181052803009	972	472878	306582570	223613602884
923	426426	262536274	181839133476	973	473851	307529299	224534770201
924	427350	263390050	182628022500	974	474825	308477975	225458780625
925	428275	264245675	183419475625	975	475800	309428600	226385640000
926	429201	265103151	184213498401	976	476776	310381176	227315354176
927	430128	265962480	185010096384	977	477753	311335705	228247929009
928	431056	266823664	185809275136	978	478731	312292189	229183370361
929	431985	267686705	186611040225	979	479710	313250630	230121684100
930	432915	268551605	187415397225	980	480690	314211030	231062876100
931	433846	269418366	188222351716	981	481671	315173391	232006952241
932	434778	270286990	189031909284	982	482653	316137715	232953918409
933	435711	271157479	189844075521	983	483636	317104004	233903780496
934	436645	272029835	190658856025	984	484620	318072260	234856544400
935	437580	272904060	191476256400	985	485605	319042485	235812216025
936	438516	273780156	192296282256	986	486591	320014681	236770801281
937	439453	274658125	193118939209	987	487578	320988850	237732306084
938	440391	275537969	193944232881	988	488566	321964994	238696736356
939	441330	276419690	194772168900	989	489555	322943115	239664098025
940	442270	277303290	195602752900	990	490545	323923215	240634397025
941	443211	278188771	196435990521	991	491536	324905296	241607639296
942	444153	279076135	197271887409	992	492528	325889360	242583830784
943	445096	279965384	198110449216	993	493521	326875409	243562977441
944	446040	280856520	198951681600	994	494515	327863445	244545085225
945	446985	281749545	199795590225	995	495510	328853470	245530160100
946	447931	282644461	200642180761	996	496506	329845486	246518208036
947	448878	283541270	201491458884	997	497503	330839495	247509235009
948	449826	284439974	202343430276	998	498501	331835499	248503247001
949	450775	285340575	203198100625	999	499500	332833500	249500250000
950	451725	286243075	204055475625	1000	500500	333833500	250500250000

EULER POLYNOMIALS AND EULER NUMBERS

EULER POLYNOMIALS AND EULER NUMBERS

1. *Definition.* By the Euler polynomial of the nth degree we mean the polynomial defined by the following:

$$E_n(x) = \{2^{n+1}/(n+1)\}\{B_{n+1}[\tfrac{1}{2}(x+1)] - B_{n+1}(\tfrac{1}{2}x)\},$$

where $B_r(x)$ is the Bernoulli polynomial of first order and rth degree.

Explicitly we obtain,

$$E_0(x) = 1,$$

$$E_1(x) = x - \tfrac{1}{2},$$

$$E_2(x) = x(x-1),$$

$$E_3(x) = (x - \tfrac{1}{2})(x^2 - x - \tfrac{1}{2}),$$

$$E_4(x) = x(x-1)(x^2 - x - 1),$$

$$E_5(x) = (x - \tfrac{1}{2})(x^4 - 2x^3 - x^2 + 2x + 1),$$

$$E_6(x) = x(x-1)(x^4 - 2x^3 - 2x^2 + 3x + 3),$$

$$E_7(x) = (x - \tfrac{1}{2})(x^6 - 3x^5 - 3x^4/2 + 8x^3 + 4x^2 - 17x/2 - 17/4),$$

$$E_8(x) = x(x-1)(x^6 - 3x^5 - 3x^4 + 11x^3 + 11x^2 - 17x - 17),$$

$$E_9(x) = (x - \tfrac{1}{2})(x^8 - 4x^7 - 2x^6 + 20x^5 + 10x^4 - 58x^3 - 29x^2 + 62x + 31),$$

$$E_{10}(x) = x(x-1)(x^8 - 4x^7 - 4x^6 + 26x^5 + 26x^4 - 100x^3 - 100x^2 + 155x + 155).$$

Polynomials of higher degree can be computed from the expansion,

$$E_n(x) = (x - \tfrac{1}{2})^n + \sum_{s=1}^{\leq \frac{n}{2}} {}_nC_{2s}(-1)^s E_s (x - \tfrac{1}{2})^{n-2s}/2^{2s},$$

where E_s are the *Euler numbers,*

$$E_0 = 1, \quad E_1 = 1, \quad E_2 = 5, \quad E_3 = 61, \quad E_4 = 1385, \quad E_5 = 50521,$$

$$E_6 = 2702765, \cdots \text{ (See table 37) },^*$$

or from the equivalent expansion,

$$E_n(x) = x^n + \sum_{s=1}^{\leq \frac{n+1}{2}} (-1)^s (2^{2s} - 1) B_s x^{n-2s+1} {}_nC_{2s-1}/s \ ,$$

where B_s are the *Bernoulli numbers,*

$$B_1 = 1/6, \quad B_2 = 1/30, \quad B_3 = 1/42, \quad B_4 = 1/30, \quad B_5 = 5/66,$$

$$\cdots \text{ (See table 30)}.$$

The Euler numbers, sometimes referred to as *secant numbers,* are defined by the equation

$$\sec x = 1 + \sum_{n=1}^{\infty} E_n x^{2n}/(2n)!$$

2. Properties of the Euler Polynomials. The Euler polynomials satisfy the following difference equations:

$$\begin{aligned} &E_n(x) + E_n(1+x) = 2x^n \ , \\ &E_n(1-x) - (-1)^n E_n(x) = 0 \ . \end{aligned} \tag{2.1}$$

The transformation of the independent variable from x to mx, m an integer, can be accomplished by means of the formulas,

$$E_n(mx) = m^n \sum_{s=0}^{m-1} (-1)^s E_n(x+s/m) \ , \qquad \text{m an odd integer,}$$

$$E_n(mx) = -[2m^n/(n+1)] \sum_{s=0}^{m-1} (-1)^s B_{n+1}(x+s/m) \ ,$$

m an even integer,

where $B_r(x)$ is the Bernoulli polynomial of first order and rth degree.

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*N. E. Nörlund (Differenzenrechnung, *loc. cit.*, p. 25) defines these numbers as the sequence: $E_0 = 1$, $E_2 = -1$, $E_4 = 5$, $E_6 = -61$, . . . , $E_{2n} = (-1)^n E_n$. Other authors define the sequence, 1, 5, 61, etc. as the Bernoulli numbers of even order and write, $B_{2n} = E_n$.

The first derivative of $E_n(x)$ is given by,

$$d\,E_n(x)/dx = n\,E_{n-1}(x) \quad ,$$

and the first integration by,

$$\int_x^y E_n(t)\,dt = [E_{n+1}(y) - E_{n+1}(x)]/(n+1) \quad .$$

Out of the Taylor's expansion,

$$E_n(x+h) = \sum_{s=0}^{n} {}_nC_s\, h^s\, E_{n-s}(x) \quad ,$$

and the first of the difference equations (2.1), we obtain for $h = 1$, the following formula:

$$E_n(x) + \sum_{s=0}^{n} {}_nC_s\, E_s(x) = 2x^n \quad .$$

The following special values are to be particularly noted:

$$E_{2n-1}(\tfrac{1}{2}) = 0, \quad E_{2n-1}(0) = -E_{2n-1}(1) = C_{2n-1}/2^{2n-1} \quad ,$$

where

$$C_0 = 1, \quad C_1 = -1, \quad C_3 = 2, \quad C_5 = -16,$$
$$C_7 = 272, \quad C_9 = -7936, \cdots ,$$

and in general,

$$C_{2n-1} = (-1)^{n-1}\, 2^{2n}(1 - 2^{2n})\, B_n/2n \quad ,$$

in which B_n is the nth Bernoulli number. (For other values of C_n see the table in section 2, The Bernoulli Polynomials and Bernoulli Numbers). For even subscripts we have $C_{2n} = 0$.

$$E_{2n}(0) = E_{2n}(1) = 0, \quad E_{2n}(\tfrac{1}{2}) = (-1)^n\, E_n/2^{2n} \quad ;$$

$$E_{2n-1}(1/3) = -E_{2n-1}(2/3) = (1 - 1/3^{2n-1})\; C_{2n-1}/2^{2n} \quad ;$$

$$E_{2n}(1/6) = E_{2n}(5/6) = (1 + 1/3^{2n})\;(-1)^n\, E_n/2^{2n+1} \quad .$$

The Euler polynomial may be derived from the following limit:

$$E_n(x) = \lim_{a=0} 2 \sum_{s=0}^{\infty} (-1)^s\;(x+s)^n\, e^{-a(x+s)} \quad ,$$

and from the following difference sum:

$$E_n(x) = \sum_{s=0}^{n} (-1)^s \Delta^s (x^n/2^s) \quad ,$$

where

$$\Delta f(x) = f(x+1) - f(x)$$

and

$$\Delta^s f(x) = \Delta[\Delta^{s-1} f(x)] \quad .$$

These polynomials do not possess the property of orthogonality, but the following integrals of their product are often useful:

$$\int_0^1 E_m(t)\, E_n(t)\, dt$$

$$= (-1)^{m+1}\, m!\, n!\, C_{m+n+1}/[2^{m+n}\ (m+n+1)\,!],\ m, n \geqq 0 \quad ;$$

$$\int_0^{1/2} E_m(t+1/2)\, E_n(t)\, dt$$

$$= (-1)^{m+r}\, m!\, n!\, E_r/[(2r)\,!\, 2^r] \quad ,\ m+n+1 = 2r \quad .$$

In view of the fact that $C_{2r} = 0$, we have a semi-orthogonality of the polynomials over the interval (0,1), since the first integral is zero, when $m + n$ is an odd integer.

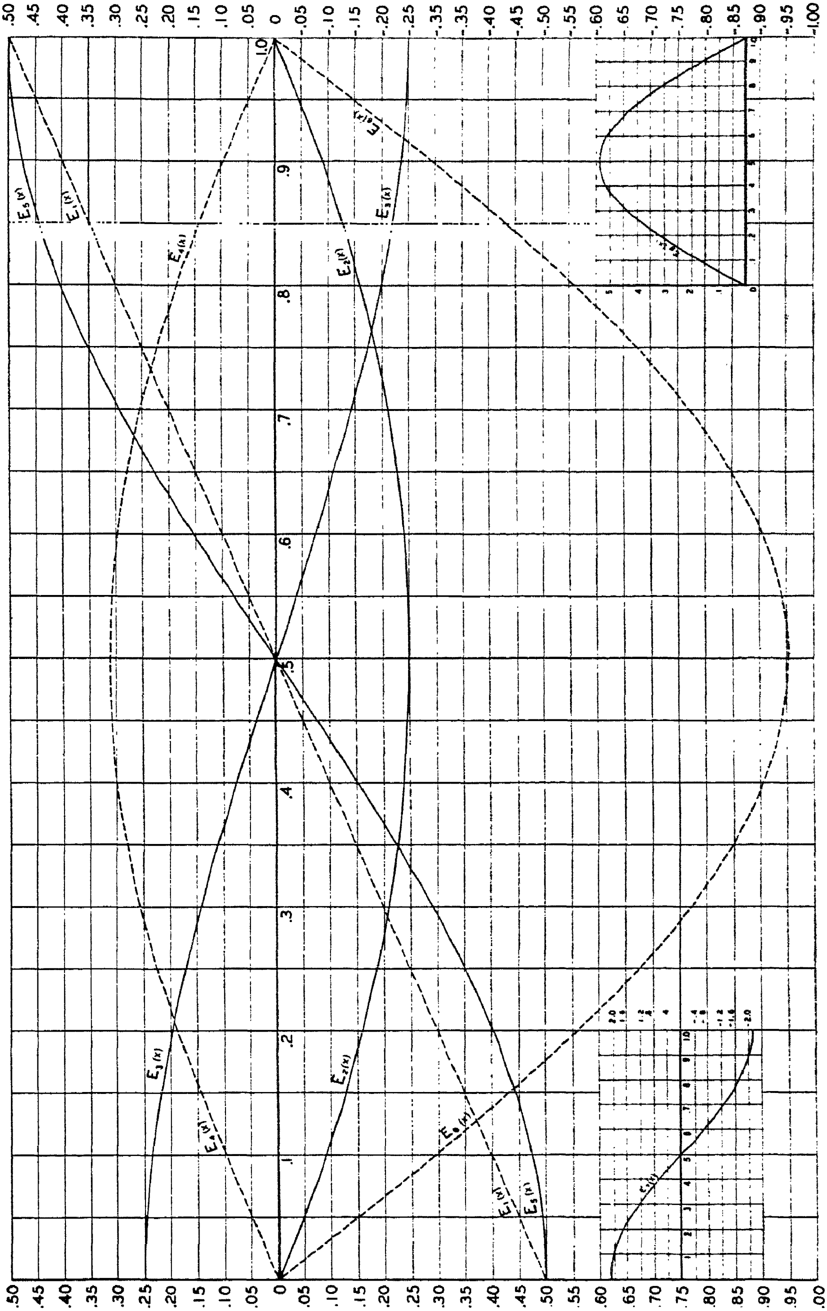
3. The Computation of Tables of E_n. The first eight values of E_n were computed by L. Euler [Bibliography, Euler (2), p. 419]. A ninth value given by him was wrong, but the corrected value, was computed by Rothe and published by Ohm in 1840.* Another value, E_{10}, was added by Stern† in 1875, who referred to a table of the first fourteen values published by H. F. Scherk, in his *Mathematische Abhandlungen* in 1825. This table was republished in N. Nielsen's: *Traité des Nombres de Bernoulli.* Paris (1923), p. 178. The values of the numbers up to $n = 27$ were published in 1914 by J. W. L. Glaisher [See Supplementary Bibliography I, Glaisher (20)] and the values to $n = 50$ by S. A. Joffe in 1916 and 1920 [See Supplementary Bibliography I, Joffe (2) and (3)].

In his computations Glaisher made use of the following equations:

$$E_n - (2n)_2 E_{n-1} + (2n)_4 E_{n-2} - \cdots + (-1)^{n-1} (2n)_2 E_1 + (-1)^n = 0 \quad , \qquad (3.1) \quad ;$$

**Journal für Mathematik*, vol. 20 (1840), p. 12.

†Zur Theorie der Eulerschen Zahlen. *Journal für Mathematik*, vol. 79 (1875) pp. 67-98.



THE EUL. POLYNOM

$$E_n - 2^2(2n)_4E_{n-2} + 2^4(2n)_8E_{n-4} - \cdots = 1\ , \qquad (3.2)\ ;$$

$$E_n - 3(2n)_6 2^4 E_{n-3} + 3(2n)_{12}2^{10}E_{n-6} - \cdots$$
$$= [3^n + (-1)^n]/2\ , \qquad (3.3)\ ;$$

$$E_n - 2^6(2n)_8 u_n E_{n-4} + 2^{12}(2n)_{16}u_8 E_{n-8} - 2^{18}(2n)_{24}\, u_{12}E_{n-12}$$
$$+ \cdots = \tfrac{1}{2}(u_{2n} + v_{2n})\ , \qquad (3.4)\ ;$$

where u_n and v_n are the rational parts of $(1+\sqrt{2})^n$ and $(1+i\sqrt{2})^n$ and may be calculated from the recurrence formulas:

$$u_n = 2u_{n-1} + u_{n-2}\ , \qquad v_n = 2v_{n-1} - 3v_{n-2}\ ,$$

the initial values being $u_1 = 1$, $u_2 = 3$ and $v_1 = 1$, $v_2 = -1$.

$$E_n - 2(2n)_{12}2^9(w_6 + 2^5 - 1)\ E_{n-6} + 3(2n)_{24}2^{21}(w_{12} + 2^{11}$$
$$+ 1)E_{n-12} - \cdots + (-1)^k 3(2n)_{12k}2^{12k-3}\{w_{6k} + 2^{6k-1}$$
$$+ (-1)^k\}\ E_{n-6k} + \cdots$$
$$= \tfrac{1}{4}\{1 + (-1)^n\}\{3^n + (-1)^n\} + (2n)_2\{3^{n-1} + (-1)^{n-1}\}\tfrac{1}{2}$$
$$+ (2n)_4\{3^{n-2} + (-1)^{n-2}\}\,2 + (2n)_6\{3^{n-3} + (-1)^{n-3}\}\,2^3$$
$$+ (2n)_8\{3^{n-4} + (-1)^{n-4}\}\,2^5 + \cdots + (2n)_{2k}\,\{3^{n-k}$$
$$+ (-1)^{n-k}\}\,2^{2k-3} + \cdots + (-1)^{n-1}[(2n)_2\{3^{n-1}$$
$$+ (-1)^n\}\tfrac{1}{2}\,w_1 - (2n)_4\{3^{n-2} + (-1)^n\}\,w_2$$
$$+ (2n)_6\{3^{n-3} + (-1)^n\}\,2w_3 + \cdots + (-1)^{k-1}\,\{3^{n-k}$$
$$+ (-1)^n\}2^{k-2}w_k + \cdots]\ , \qquad (3.5)\ ;$$

in which,

$$w_n = \tfrac{1}{2}\{(2+\sqrt{3})^n + (2-\sqrt{3})^n\}\ .$$

Values of w_n can be calculated from the recurrence formula:

$$w_n = 4w_{n-1} - w_{n-2}\ , \quad w_1 = 2, \quad w_2 = 7\ .$$

$$E_n - (n)_2 2^2 E_{n-1} + (n)_4 2^4 E_{n-2} - \cdots = 1, \qquad (3.6)\ ;$$

$$E_n - (n)_2 6^2 E_{n-1} + (n)_4 6^4 E_{n-2} - \cdots = 2\cdot 5^n - 9, \qquad (3.7)\ ;$$

$$E_n - (n)_2 10^2 E_{n-1} + (n)_4 10^4 E_{n-2} - \cdots = 25^n - 2(21^n - 9^n),$$
$$(3.8)\ ;$$

these being special cases of the more general formula:

$$E_n - (n)_2(2r)^2E_{n-1} + (n)_4(2r)^4E_{n-2} - \cdots$$
$$= 2\{1^n(2r-1)^n - 3^n(2r-3)^n + 5^n(2r-5) - \cdots$$
$$+ (-1)^{\frac{1}{2}(r-3)}(r-2)^n(r+2)^n\} + (-1)^{\frac{1}{2}(r-1)}r^{2n}, \quad (3.9) \quad ;$$

We note the abbreviation,

$$(n)_r = [n(n-1)\cdots(n-r+1)]/r!\ .$$

The first formula, (3.1), is found in the early theory of Euler numbers; the next four, (3.2) to (3.5), are due to Glaisher who developed them from suggestions obtained through a paper relating to Bernoulli numbers by S. Ramanujan*; the next three are special cases of formula (3.9) which Glaisher attributes to A. Radicke.†

By means of these formulas Glaisher computed the first 27 Euler numbers. The values of E_n from $n = 9$ to $n = 23$ were calculated by means of formulas (3.6) and (3.8) and a few re-checked by a recalculation as follows: E_9, E_{11}, and E_{20} by (3.3); E_{10} by (3.4), E_{12} by (3.5), E_{21} by (3.1), and (3.3), and E_{22} by (3.2). The values from $n = 23$ to $n = 27$ were computed by means of formula (3.8) and, excepting for the case $n = 24$, where the value was recalculated by means of (3.4), the check was effected by means of the following congruence, which he derived from formula (3.9):

$$E_n \equiv (-1)^n\,2\{1^{2n} - 3^{2n} + 5^{2n} - \cdots$$
$$+ (-1)^{\frac{1}{2}(r-3)}(r-2)^{2n}\}, \bmod r.$$

Setting $r = 3, 7, 9, 11, 13, 17, 19, 23$ in this formula, Glaisher obtained special congruences of whiich the following (for the modulus $r = 17$) are typical:

$$E_{8k+1} \equiv 1, \bmod 17 \qquad E_{8k+2} \equiv 5, \bmod 17$$
$$E_{8k+3} \equiv 10, \bmod 17 \qquad E_{8k+4} \equiv 8, \bmod 17$$

*Some Properties of Bernoulli's Numbers. *Journal of the Indian Math. Soc.*, vol. 3 (1911), pp. 219-234. Collected Papers, Cambridge, (1927) pp. 1-14.

†See Glaisher: On a Class of Relations Connecting any n Consecutive Bernoullian Functions. *Quarterly Journal of Math.*, vol. 42 (1910), pp. 86-157; in particular, p. 147.

$$E_{8k+5} \equiv 14 \text{, mod } 17 \text{ ,} \qquad E_{8k+6} \equiv 3 \text{, mod } 17 \text{ ,}$$

$$E_{8k+7} \equiv 9 \text{, mod } 17 \text{ ,} \qquad E_{8k} \equiv 0 \text{, mod } 17 \text{ .}$$

A check of the values of E_n was effected by verifying each value for each of the residues to the eight moduli given above.

In extending the table of Euler numbers to $n = 50$, S. A. Joffe made use of the following simple relation:

$$E_n = (-1)^n \sum_{m=1}^{n} (-1)^m e_{m,n} \text{ ,}$$

where we abbreviate,

$$e_{m,n} = \delta^{2m} 0^{2n}/2^m \text{ .}$$

These values are connected by the recurrence formula:

$$e_{m,n} = m(2m-1)\, e_{m-1,n-1} + m^2 e_{m,n-1} \text{ ,}$$

which is immediately derivable from the formula relating to central differences which is found in section 8 of *Bernoulli Polynomials and Bernoulli Numbers.*

Joffe's computation of the first twelve numbers is reproduced below.

4. *Origin and Computation of the Tables.* In the preceding section a description has been given of the methods used in computing the Euler numbers. We shall now include a short account of the origin and computation of the other tables.

Tables of the Euler Polynomials. Table 36 gives the values of the polynomials $E_n(x)$, $n = 2, 3, 4, 5, 6, 7$ and 8, from $x = 0.00$ to $x = 1.00$ at intervals of .01. These values were computed by Esther Kantz, a direct evaluation over the entire range being first achieved. The fact that $E_n(x) = (-1)^n E_n(1-x)$ was then employed in order to check the computations, the values over one half the range being compared with the values over the other half.

. *Tables of the Series T_n and $\log_{10} T_n$.* The eighteen place values of the series T_n given in table 39 were computed by J. W. L. Glaisher, [See Supplementary Bibliography, I, Glaisher (17)], who employed the following series:

Values of e_m

m/n	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2		6	30	126	510	2 046	8 190
3			90	1 260	13 230	126 720	1 171 170
4				2 520	75 600	1 580 040	28 828 800
5					113 400	6 237 000	227 026 800
6						7 484 400	681 080 400
7							681 080 400
E_n	1	5	61	1 385	50 521	2 702 765	199 360 981

m/n	8	9	10
1	1	1	1
2	32 766	131 070	524 286
3	10 663 380	96 461 910	870 123 240
4	494 053 560	8 203 431 600	133 955 839 080
5	6 972 966 000	196 556 560 200	5 283 068 427 000
6	39 502 663 200	1 882 311 631 200	80 735 951 696 400
7	95 351 256 000	8 266 953 895 200	576 371 099 304 000
8	81 729 648 000	16 672 848 192 000	2 059 096 751 712 000
9		12 504 636 144 000	3 563 821 301 040 000
10			2 375 880 867 360 000
E_n	19 391 512 145	2 404 879 675 441	370 371 188 237 525

m/n	11	12
1	1	1
2	2 097 150	8 388 606
3	7 838 973 450	70 582 218 300
4	2 167 656 876 000	34 902 001 272 600
5	138 104 723 433 600	3 550 162 645 260 000
6	3 255 176 777 252 400	126 301 275 727 704 000
7	35 589 155 470 268 400	2 040 089 704 773 120 000
8	200 946 724 026 048 000	17 131 288 994 099 280 000
9	603 711 328 396 176 000	79 645 466 376 075 600 000
10	914 714 133 933 600 000	206 176 565 788 633 440 000
11	548 828 480 360 160 000	277 707 211 062 240 960 000
12		151 476 660 579 404 160 000
E_n	69 348 874 393 137 901	15 514 534 163 557 086 905

$$T_n = 1 - 1/3^n + 1/5^n - T_n'' = 1 - 1/3^n + 1/5^n - 1/7^n + 1/9^n - T_n''' \;,$$

$$T_n'' = 2\,\{n^{(1)}\, S_{n+1}'/4^{n+1} + n^{(3)}\, S_{n+3}/4^{n+3}\, S_{n+3}'/4^{n+3} + n^{(5)}\, S_{n+5}'/4^{n+5} + \cdots\} \;,$$

$$T_n''' = 2\,\{n^{(1)}\, S_{n+1}''/4^{n+1} + n^{(3)}\, S_{n+3}''/4^{n+3} + n^{(5)}\, S_{n+5}''/4^{n+5} + \cdots\} \;,$$

where we abbreviate, $n^{(r)} = n(n+1)(n+2) \cdots (n+r-1)/r!$, $S_n' = S_n - 1$. $S_n'' = S_n - 1 - 1/2^n$.

For odd powers the series were easily calculated from the formula.

$$T_{2n+1} = E_n\, \pi^{2n+1}/[(2n)!\, 2^{2n+2}] \;.$$

Previously* Glaisher had computed the value of T_2 by means of the following asymptotic series derived from Euler's summation formula:

$$1/1^2 + 1/5^2 + 1/9^2 + \cdots + 1/(4x-3)^2$$
$$\infty\, C_1 - 1/[4\cdot(4x+1)] - 1/[2\cdot(4x+1)^2] - 2/[3\cdot(4x+1)^3] + 32/[15\cdot(4x+1)^5] - 512/[21\cdot(4x+1)^7] + \cdots \;,$$

$$1/3^2 + 1/7^2 + 1/11^2 + \cdots + 1/(4x-1)^2$$
$$\infty\, C_2 - 1/[4\cdot(4x+3)] - 1/[2\cdot(4x+3)^2] - 2/[3\cdot(4x+3)^3] + 32/[15\cdot(4x+3)^5] - \cdots \;.$$

Computing the left members directly by means of Barlow's and Oakes' tables of reciprocals, Glaisher obtained the values:

$$C_1 = 1.074833072, \qquad C_2 = 0.158867478 \;,$$

from which he computed T_2 correct to nine places.

Messenger of Math., vol. 6 (1876), pp. 71-76.

Later, in connection with the limit,*

$$\lim_{n=\infty}\{[3^3\,7^7\,11^{11}\cdots(4n-1)^{4n-1}\cdots]/[5^5\,9^9\,13^{13}\cdots(4n-3)^{4n-3}(4n+1)^{2n}\quad]\}=\exp\left(-\tfrac{1}{2}+\frac{2}{\pi}\mathrm{T}_2\right)$$

Glaisher found the equation,

$$T_2=\frac{\pi}{2}\,[\tfrac{1}{2}+t_2/(3\cdot 2^2)+t_4/(5\cdot 2^4)+t_6/(7\cdot 2^6)+\cdots]$$

from which he computed T_2 to 20 decimals places.†

Employing the formulas given at the beginning of this section, Glaisher exhibited their power by evaluating T_2, T_4, and T_6 to 32, 30, and 28 places respectively. These values are given in table 39. [See Supplementary Bibliography I, Glaisher (18)].

The values of $\log_{10}T_n$ were computed by Miss Lucy Kantz, devices being employed in this calculation similar to those described in section 9 of Bernoulli Polynomials etc. in connection with the evaluation of $\log_e S_n$. These values were checked by duplicate calculation by E. W. Scotten.

In connection with the calculation of the values of the series T_n, we might record as of interest the following sums also due to Glaisher [See Supplementary Bibliography I, Glaisher (19)]:

$$1-1/3-1/5+1/7+1/9-\cdots=.623\;225\;240\;140\;230\;51340;$$

$$1+1/3^2-1/5^2-1/7^2+1/9^2+1/11^2+\cdots$$
$$=1.064\;734\;171\;043\;503\;3704\;.$$

$$1+1/3-1/5-1/7+1/9+1/11-\cdots=\pi/2^{3/2}$$
$$=1.110\;720\;734\;539\;591\;561\;753\;970\;247\;5151\;.$$

Proceedings of the London Math. Soc., vol. 8 (1877), pp. 200-201.

†We note that, $t_n=1-1/2^n+1/3^n-1/4^n+\cdots$.

TABLE 36

THE EULER POLYNOMIAL

Description: Values of $E_n(x)$, $n = 2, 3, 4, 5, 6, 7$ and 8, from $x = .00$ to $x = 1.00$ at intervals of $.01$.

x	$E_2(x)$	$E_3(x)$	$E_4(x)$	$E_5(x)$	x
.00	−.0000	.250000	.0000 0000	−.50000 00000	.00
.01	−.0099	.249851	.0099 9801	−.49975 00249	.01
.02	−.0196	.249408	.0199 8416	−.49900 03968	.02
.03	−.0291	.248677	.0299 4681	−.49775 20007	.03
.04	−.0384	.247664	.0398 7456	−.49600 62976	.04
.05	−.0475	.246375	.0497 5625	−.49376 53125	.05
.06	−.0564	.244816	.0595 8096	−.49103 16224	.06
.07	−.0651	.242993	.0693 3801	−.48780 83443	.07
.08	−.0736	.240912	.0790 1696	−.48409 91232	.08
.09	−.0819	.238579	.0886 0761	−.47990 81201	.09
.10	−.0900	.236000	.0981 0000	−.47524 00000	.10
.11	−.0979	.233181	.1074 8441	−.47009 99199	.11
.12	−.1056	.230128	.1167 5136	−.46449 35168	.12
.13	−.1131	.226847	.1258 9161	−.45842 68957	.13
.14	−.1204	.223344	.1348 9616	−.45190 66176	.14
.15	−.1275	.219625	.1437 5625	−.44493 96875	.15
.16	−.1344	.215696	.1524 6336	−.43753 35424	.16
.17	−.1411	.211563	.1610 0921	−.42969 60393	.17
.18	−.1476	.207232	.1693 8576	−.42143 54432	.18
.19	−.1539	.202709	.1775 8521	−.41276 04151	.19
.20	−.1600	.198000	.1856 0000	−.40368 00000	.20
.21	−.1659	.193111	.1934 2281	−.39420 36149	.21
.22	−.1716	.188048	.2010 4656	−.38434 10368	.22
.23	−.1771	.182817	.2084 6441	−.37410 23907	.23
.24	−.1824	.177424	.2156 6976	−.36349 81376	.24
.25	−.1875	.171875	.2226 5625	−.35253 90625	.25
.26	−.1924	.166176	.2294 1776	−.34123 62624	.26
.27	−.1971	.160333	.2359 4841	−.32960 11343	.27
.28	−.2016	.154352	.2422 4256	−.31764 53632	.28
.29	−.2059	.148239	.2482 9481	−.30538 09101	.29
.30	−.2100	.142000	.2541 0000	−.29282 00000	.30
.31	−.2139	.135641	.2596 5321	−.27997 51099	.31
.32	−.2176	.129168	.2649 4976	−.26685 89568	.32
.33	−.2211	.122587	.2699 8521	−.25348 44857	.33
.34	−.2244	.115904	.2747 5536	−.23986 48576	.34
.35	−.2275	.109125	.2792 5625	−.22601 34375	.35
.36	−.2304	.102256	.2834 8416	−.21194 37824	.36
.37	−.2331	.095303	.2874 3561	−.19766 96293	.37
.38	−.2356	.088272	.2911 0736	−.18320 48832	.38
.39	−.2379	.081169	.2944 9641	−.16856 36051	.39
.40	−.2400	.074000	.2976 0000	−.15376 00000	.40
.41	−.2419	.066771	.3004 1561	−.13880 84049	.41
.42	−.2436	.059488	.3029 4096	−.12372 32768	.42
.43	−.2451	.052157	.3051 7401	−.10851 91807	.43
.44	−.2464	.044784	.3071 1296	−.09321 07776	.44
.45	−.2475	.037375	.3087 5625	−.07781 28125	.45
.46	−.2484	.029936	.3101 0256	−.06234 01024	.46
.47	−.2491	.022473	.3111 5081	−.04680 75243	.47
.48	−.2496	.014992	.3119 0016	−.03123 00032	.48
.49	−.2499	.007499	.3123 5001	−.01562 25001	.49
.50	−.2500	.000000	.3125 0000	−.00000 00000	.50

x	$E_2(x)$	$E_3(x)$	$E_4(x)$	$E_5(x)$	x
.50	–.2500	.000000	.3125 0000	.00000 00000	.50
.51	–.2499	–.007499	.3123 5001	.01562 25001	.51
.52	–.2496	–.014992	.3119 0016	.03123 00032	.52
.53	–.2491	–.022473	.3111 5081	.04680 75243	.53
.54	–.2484	–.029936	.3101 0256	.06234 01024	.54
.55	–.2475	–.037375	.3087 5625	.07781 28125	.55
.56	–.2464	–.044784	.3071 1296	.09321 07776	.56
.57	–.2451	–.052157	.3051 7401	.10851 91807	.57
.58	–.2436	–.059488	.3029 4096	.12372 32768	.58
.59	–.2419	–.066771	.3004 1561	.13880 84049	.59
.60	–.2400	–.074000	.2976 0000	.15376 00000	.60
.61	–.2379	–.081169	.2944 9641	.16856 36051	.61
.62	–.2356	–.088272	.2911 0736	.18320 48832	.62
.63	–.2331	–.095303	.2874 3561	.19766 96293	.63
.64	–.2304	–.102256	.2834 8416	.21194 37824	.64
.65	–.2275	–.109125	.2792 5625	.22601 34375	.65
.66	–.2244	–.115904	.2747 5536	.23986 48576	.66
.67	–.2211	–.122587	.2699 8521	.25348 44857	.67
.68	–.2176	–.129168	.2649 4976	.26685 89568	.68
.69	–.2139	–.135641	.2596 5321	.27997 51099	.69
.70	–.2100	–.142000	.2541 0000	.29282 00000	.70
.71	–.2059	–.148239	.2482 9481	.30538 09101	.71
.72	–.2016	–.154352	.2422 4256	.31764 53632	.72
.73	–.1971	–.160333	.2359 4841	.32960 11343	.73
.74	–.1924	–.166176	.2294 1776	.34123 62624	.74
.75	–.1875	–.171875	.2226 5625	.35253 90625	.75
.76	–.1824	–.177424	.2156 6976	.36349 81376	.76
.77	–.1771	–.182817	.2084 6441	.37410 23907	.77
.78	–.1716	–.188048	.2010 4656	.38434 10368	.78
.79	–.1659	–.193111	.1934 2281	.39420 36149	.79
.80	–.1600	–.198000	.1856 0000	.40368 00000	.80
.81	–.1539	–.202709	.1775 8521	.41276 04151	.81
.82	–.1476	–.207232	.1693 8576	.42143 54432	.82
.83	–.1411	–.211563	.1610 0921	.42969 60393	.83
.84	–.1344	–.215696	.1524 6336	.43753 35424	.84
.85	–.1275	–.219625	.1437 5625	.44493 96875	.85
.86	–.1204	–.223344	.1348 9616	.45190 66176	.86
.87	–.1131	–.226847	.1258 9161	.45842 68957	.87
.88	–.1056	–.230128	.1167 5136	.46449 35168	.88
.89	–.0979	–.233181	.1074 8441	.47009 99199	.89
.90	–.0900	–.236000	.0981 0000	.47524 00000	.90
.91	–.0819	–.238579	.0886 0761	.47990 81201	.91
.92	–.0736	–.240912	.0790 1696	.48409 91232	.92
.93	–.0651	–.242993	.0693 3801	.48780 83443	.93
.94	–.0564	–.244816	.0595 8096	.49103 16224	.94
.95	–.0475	–.246375	.0497 5625	.49376 53125	.95
.96	–.0384	–.247664	.0398 7456	.49600 62976	.96
.97	–.0291	–.248677	.0299 4681	.49775 20007	.97
.98	–.0196	–.249408	.0199 8416	.49900 03968	.98
.99	–.0099	–.249851	.0099 9801	.49975 00249	.99
1.00	–.0000	–.250000	.0000 0000	.50000 00000	1.00

x	$E_6(x)$	$E_7(x)$	$E_8(x)$	x
.00	.00000 00000	2.12500 00000	0.00000 00000	.00
.01	−.02999 50003	2.12395 00875	0.16997 20014	.01
.02	−.05996 00095	2.12080 13997	0.33977 60448	.02
.03	−.08986 50722	2.11555 70850	0.50924 43401	.03
.04	−.11968 03031	2.10822 23858	0.67820 94330	.04
.05	−.14937 59219	2.09880 46336	0.84650 43719	.05
.06	−.17892 22861	2.08731 32395	1.01396 28754	.06
.07	−.20828 99245	2.07375 96840	1.18041 94974	.07
.08	−.23744 95683	2.05815 75035	1.34570 97930	.08
.09	−.26637 21833	2.04052 22753	1.50967 04816	.09
.10	−.29502 90000	2.02087 16000	1.67213 96100	.10
.11	−.32339 15437	1.99922 50819	1.83295 67133	.11
.12	−.35143 16636	1.97560 43074	1.99196 29745	.12
.13	−.37912 15611	1.95003 28212	2.14900 13818	.13
.14	−.40643 38177	1.92253 61008	2.30391 68846	.14
.15	−.43334 14219	1.89314 15289	2.45655 65469	.15
.16	−.45981 77956	1.86187 83641	2.60676 96985	.16
.17	−.48583 68195	1.82877 77094	2.75440 80838	.17
.18	−.51137 28582	1.79387 24794	2.89932 60084	.18
.19	−.53640 07838	1.75719 73656	3.04138 04821	.19
.20	−.56089 60000	1.71878 88000	3.18043 13600	.20
.21	−.58483 44642	1.67868 49170	3.31634 14801	.21
.22	−.60819 27097	1.63692 55139	3.44897 67981	.22
.23	−.63094 78670	1.59355 20101	3.57820 65183	.23
.24	−.65307 76842	1.54860 74043	3.70390 32223	.24
.25	−.67456 05469	1.50213 62305	3.82594 29932	.25
.26	−.69537 54970	1.45418 45129	3.94420 55367	.26
.27	−.71550 22516	1.40479 97193	4.05857 42986	.27
.28	−.73492 12201	1.35403 07132	4.16893 65783	.28
.29	−.75361 35214	1.30192 77046	4.27518 36382	.29
.30	−.77156 10000	1.24854 22000	4.37721 08100	.30
.31	−.78874 62416	1.19392 69508	4.47491 75959	.31
.32	−.80515 25878	1.13813 59010	4.56820 77664	.32
.33	−.82076 41499	1.08122 41340	4.65698 94533	.33
.34	−.83556 58228	1.02324 78180	4.74117 52390	.34
.35	−.84954 32969	.96426 41508	4.82068 22407	.35
.36	−.86268 30705	.90433 13035	4.89543 21908	.36
.37	−.87497 24607	.84350 83638	4.96535 15127	.37
.38	−.88639 96140	.78185 52785	5.03037 13911	.38
.39	−.89695 35159	.71943 27944	5.09042 78393	.39
.40	−.90662 40000	.65630 24000	5.14546 17600	.40
.41	−.91540 17561	.59252 62654	5.19541 90030	.41
.42	−.92327 83379	.52816 71822	5.24025 04167	.42
.43	−.93024 61699	.46328 85029	5.27991 18958	.43
.44	−.93629 85533	.39795 40796	5.31436 44236	.44
.45	−.94142 96719	.33222 82023	5.34357 41094	.45
.46	−.94563 45959	.26617 55374	5.36751 22213	.46
.47	−.94890 92868	.19986 10651	5.38615 52136	.47
.48	−.95125 05999	.13335 00168	5.39948 47496	.48
.49	−.95265 62875	.06670 78130	5.40748 77187	.49
.50	−.95312 50000	.00000 00000	5.41015 62500	.50

x	$E_6(x)$	$E_7(x)$	$E_8(x)$	x
.50	−.95312 50000	.00000 00000	5.41015 62500	.50
.51	−.95265 62875	−.06670 78130	5.40748 77187	.51
.52	−.95125 05999	−.13335 00168	5.39948 47496	.52
.53	−.94890 92868	−.19986 10651	5.38615 52136	.53
.54	−.94563 45959	−.26617 55374	5.36751 22213	.54
.55	−.94142 96719	−.33222 82023	5.34357 41094	.55
.56	−.93629 85533	−.39795 40796	5.31436 44236	.56
.57	−.93024 61699	−.46328 85029	5.27991 18958	.57
.58	−.92327 83379	−.52816 71822	5.24025 04167	.58
.59	−.91540 17561	−.59252 62654	5.19541 90030	.59
.60	−.90662 40000	−.65630 24000	5.14546 17600	.60
.61	−.89695 35159	−.71943 27944	5.09042 78393	.61
.62	−.88639 96140	−.78185 52785	5.03037 13911	.62
.63	−.87497 24607	−.84350 83638	4.96535 15127	.63
.64	−.86268 30705	−.90433 13035	4.89543 21908	.64
.65	−.84954 32969	−.96426 41508	4.82068 22407	.65
.66	−.83556 58228	−1.02324 78180	4.74117 52390	.66
.67	−.82076 41499	−1.08122 41340	4.65698 94533	.67
.68	−.80515 25878	−1.13813 59010	4.56820 77664	.68
.69	−.78874 62416	−1.19392 69508	4.47491 75959	.69
.70	−.77156 10000	−1.24854 22000	4.37721 08100	.70
.71	−.75361 35214	−1.30192 77046	4.27518 36382	.71
.72	−.73492 12201	−1.35403 07132	4.16893 65783	.72
.73	−.71550 22516	−1.40479 97193	4.05857 42986	.73
.74	−.69537 54970	−1.45418 45129	3.94420 55367	.74
.75	−.67456 05469	−1.50213 62305	3.82594 29932	.75
.76	−.65307 76842	−1.54860 74043	3.70390 32223	.76
.77	−.63094 78670	−1.59355 20101	3.57820 65183	.77
.78	−.60819 27097	−1.63692 55139	3.44897 67981	.78
.79	−.58483 44642	−1.67868 49170	3.31634 14801	.79
.80	−.56089 60000	−1.71878 88000	3.18043 13600	.80
.81	−.53640 07838	−1.75719 73656	3.04138 04821	.81
.82	−.51137 28582	−1.79387 24794	2.89932 60084	.82
.83	−.48583 68195	−1.82877 77094	2.75440 80838	.83
.84	−.45981 77956	−1.86187 83641	2.60676 96985	.84
.85	−.43334 14219	−1.89314 15289	2.45655 65469	.85
.86	−.40643 38177	−1.92253 61008	2.30391 68846	.86
.87	−.37912 15611	−1.95003 28212	2.14900 13818	.87
.88	−.35143 16636	−1.97560 43074	1.99196 29745	.88
.89	−.32339 15437	−1.99922 50819	1.83295 67133	.89
.90	−.29502 90000	−2.02087 16000	1.67213 96100	.90
.91	−.26637 21833	−2.04052 22753	1.50967 04816	.91
.92	−.23744 95683	−2.05815 75035	1.34570 97930	.92
.93	−.20828 99245	−2.07375 96840	1.18041 94974	.93
.94	−.17892 22861	−2.08731 32395	1.01396 28754	.94
.95	−.14937 59219	−2.09880 46336	0.84650 43719	.95
.96	−.11968 03031	−2.10822 23858	0.67820 94330	.96
.97	−.08986 50722	−2.11555 70850	0.50924 43401	.97
.98	−.05996 00095	−2.12080 13997	0.33977 60448	.98
.99	−.02999 50003	−2.12395 00875	0.16997 20014	.99
1.00	−.00000 00000	−2.12500 00000	0.00000 00000	1.00

TABLE 37

Euler Numbers

Description: Values of the first 50 Euler numbers.

n	E_n
1	1
2	5
3	61
4	1 385
5	50 521
6	2 702 765
7	199 360 981
8	19 391 512 145
9	2 404 879 675 441
10	370 371 188 237 525
11	69 348 874 393 137 901
12	15 514 534 163 557 086 905
13	4 087 072 509 293 123 892 361
14	1 252 259 641 403 629 865 468 285
15	441 543 893 249 023 104 553 682 821
16	177 519 391 579 539 289 436 664 789 665
17	80 723 299 235 887 898 062 168 247 453 281
18	41 222 060 339 517 702 122 347 079 671 259 045
19	23 489 580 527 043 108 252 017 828 576 198 947 741
20	14 851 150 718 114 980 017 877 156 781 405 826 684 425
21	10 364 622 733 519 612 119 397 957 304 745 185 976 310 201
22	7 947 579 422 597 592 703 608 040 510 088 070 619 519 273 805
23	6 667 537 516 685 544 977 435 028 474 773 748 197 524 107 684 661
24	6 096 278 645 568 542 158 691 685 742 876 843 153 976 539 044 435 185
25	6 053 285 248 188 621 896 314 383 785 111 649 088 103 498 225 146 815 121
26	6 506 162 486 684 608 847 715 870 634 080 822 983 483 644 236 765 385 576 565
27	7 546 659 939 008 739 098 061 432 565 889 736 744 212 240 024 711 699 858 645 581
28	9 420 321 896 420 241 204 202 286 237 690 583 227 209 388 852 599 646 009 394 905 945
29	12 622 019 251 806 218 719 903 409 237 287 489 255 482 341 061 191 825 594 069 964 920 041
30	18 108 911 496 579 230 496 545 807 741 652 158 688 733 487 349 236 314 106 008 095 454 231 325
31	27 757 101 702 071 580 597 366 980 908 371 527 449 233 019 594 800 917 578 033 782 766 889 782 501
32	45 358 103 330 017 889 174 746 887 871 567 762 366 351 861 519 470 368 881 468 843 837 919 695 760 705
33	78 862 842 066 617 894 181 007 207 422 399 904 239 478 162 972 003 768 932 709 757 494 857 167 945 376 961
34	145 618 443 801 396 315 007 150 470 094 942 326 661 860 812 858 314 932 986 447 697 768 064 595 488 862 902 085
35	285 051 783 223 697 718 732 198 729 556 739 339 504 255 241 778 255 239 879 353 211 106 980 427 546 235 397 447 421

n	E_n
36	590 574 720 777 544 365 455 135 032 296 439 571 372 033 016 181 822 954 929 765 972 153 659 805 050 264 501 891 063 465
37	1 292 973 664 187 864 170 497 603 235 938 698 754 076 170 519 123 672 606 411 370 597 343 787 035 331 808 195 731 850 937 881
38	2 986 928 183 284 576 950 930 743 652 217 140 605 692 922 369 370 680 702 813 812 833 466 898 038 172 015 655 808 960 288 452 845
39	7 270 601 714 016 864 143 803 280 651 699 281 851 647 234 288 049 207 905 108 309 583 687 335 688 017 641 546 191 095 009 395 592 341
40	18 622 915 758 412 697 044 482 492 303 043 126 011 920 010 194 518 556 063 577 101 095 681 956 123 546 201 442 832 293 837 005 396 878 225
41	50 131 049 408 109 796 612 908 693 678 881 009 420 083 336 722 220 539 765 973 596 236 561 571 401 154 699 761 552 253 189 084 809 951 554 801
42	141 652 557 597 856 259 916 722 069 410 021 670 405 475 845 492 837 912 390 700 146 845 374 567 994 390 844 977 125 987 675 020 436 380 612 547 605
43	419 664 316 404 024 471 322 573 414 069 418 891 818 962 628 391 683 907 039 212 228 549 032 921 853 217 838 146 608 053 808 786 365 440 570 254 969 261
44	1 302 159 590 524 046 398 125 858 691 330 818 681 356 757 613 986 610 030 678 095 758 242 404 286 633 729 262 297 123 677 199 743 591 748 006 204 646 868 985
45	4 227 240 686 139 909 064 705 589 929 214 593 102 933 845 388 672 369 082 676 644 542 650 248 228 369 590 525 634 078 984 302 153 217 507 945 782 396 923 579 721
46	14 343 212 791 976 583 406 133 682 640 578 565 858 579 882 148 843 159 111 106 574 955 509 790 196 812 618 254 848 857 854 461 550 714 631 444 034 921 517 907 250 365
47	50 817 990 724 580 425 164 559 757 643 090 736 003 482 435 671 513 413 926 813 239 886 828 210 876 247 074 897 752 122 164 140 484 881 907 534 297 068 189 565 042 330 181
48	187 833 293 645 293 026 402 007 579 184 179 892 539 001 444 997 005 361 637 080 870 116 823 642 645 755 601 678 579 681 159 136 078 780 812 233 831 035 373 097 528 077 899 745
49	723 653 438 103 385 777 657 187 661 736 782 292 986 259 565 181 067 232 760 712 431 055 015 669 043 224 647 591 792 236 141 452 770 950 810 842 191 949 814 198 134 897 708 964 641
50	2 903 528 346 661 097 497 054 603 834 764 435 875 077 553 006 646 158 945 080 492 319 146 997 643 370 625 023 889 353 447 129 967 354 174 648 294 748 510 553 528 692 457 632 980 625 125

TABLE 38

Logarithms of the Euler Numbers

Description: Logarithms of the first 250 Euler numbers computed to twelve decimal places.

Also the first ten significant figures in the first 250 Euler numbers.

(The numbers in parentheses denote the number of additional figures before the decimal point.)

n	$\log_{10} E_n$	E_n
1	0.0000 0000 0000	1.00000 00000
2	0.6989 7000 4336	5.00000 00000
3	1.7853 2983 5011	61.0000 00000
4	3.1414 4977 3400	1385.00 00000
5	4.7034 7193 8284	50521.0 00000
6	6.4318 0828 6305	2702765. 0000
7	8.2996 4016 2027	1993609 81.00
8	10.2876 1167 6568	1939151 2145
9	12.3810 9335 1978	2404879 675(3)
10	14.5686 3719 4867	3703711 882(5)
11	16.8410 3941 6358	6934887 439(7)
12	19.1907 3874 0073	1551453 416(10)
13	21.6114 1234 2656	4087072 509(12)
14	24.0976 9438 4097	1252259 641(15)
15	26.6449 7388 2655	4415438 932(17)
16	29.2492 4580 0749	1775193 916(20)
17	31.9069 9890 3609	8072329 924(22)
18	34.6151 2969 4666	4122206 034(25)
19	37.3708 7526 1289	2348958 053(28)
20	40.1717 6010 5584	1485115 072(31)
21	43.0155 5349 8641	1036462 273(34)
22	45.9002 3487 6646	7947579 423(36)
23	48.8239 6546 8043	6667537 517(39)
24	51.7850 6480 9294	6096278 646(42)
25	54.7819 9113 9598	6053285 248(45)
26	57.8133 2490 5271	6506162 487(48)
27	60.8777 5478 0634	7546659 939(51)
28	63.9740 6574 3074	9420321 896(54)
29	67.1011 2883 8249	1262201 925(58)
30	70.2578 9234 6215	1810891 150(61)
31	73.4433 7411 6664	2775710 170(64)
32	76.6566 5488 6041	4535810 333(67)
33	79.8968 7242 4165	7886284 207(70)
34	83.1632 1638 5512	1456184 438(74)
35	86.4549 2376 2203	2850517 832(77)
36	89.7712 7485 3293	5905747 208(80)
37	93.1115 8967 9184	1292973 664(84)
38	96.4752 2478 0696	2986928 183(87)
39	99.8615 7035 4499	7270601 714(90)
40	103.2700 4767 8721	1862291 576(94)
41	106.7001 0679 5923	5013104 941(97)
42	110.1512 2442 0201	1416525 576(101)
43	113.6229 0204 3198	4196643 164(104)
44	117.1146 6421 3908	1302159 591(108)
45	120.6260 5697 5931	4227240 686(111)
46	124.1566 4644 1537	1434321 279(115)
47	127.7060 1748 9631	5081799 072(118)
48	131.2737 7257 3899	1878332 936(122)
49	134.8595 3062 9797	7236534 381(125)
50	138.4629 2607 0334	2903528 347(129)

n	$\log_{10} E_n$	E_n
51	142.0836 0786 1819	1212293 738(133)
52	145.7212 3867 1762	5263064 250(136)
53	149.3754 9408 2037	2374073 072(140)
54	153.0460 6186 1149	1111890 095(144)
55	156.7326 4129 0187	5403078 660(147)
56	160.4349 4253 7584	2722341 086(151)
57	164.1526 8607 8343	1421301 055(155)
58	167.8856 0215 3863	7684261 820(158)
59	171.6334 3026 8855	4299621 926(162)
60	175.3959 1872 2235	2488391 575(166)
61	179.1728 2416 9166	1488758 210(170)
62	182.9639 1121 1707	9202614 119(173)
63	186.7689 5201 5773	5874244 458(177)
64	190.5877 2595 2316	3870133 554(181)
65	194.4200 1926 0862	2630384 646(185)
66	198.2656 2473 3663	1843421 862(189)
67	202.1243 4141 8935	1331500 761(193)
68	205.9959 7434 1740	9907734 079(196)
69	209.8803 3424 1237	7591616 154(200)
70	213.7772 3732 3109	5987386 904(204)
71	217.6865 0502 6087	4858531 537(208)
72	221.6079 6380 1587	4054747 378(212)
73	225.5414 4490 5546	3478923 713(216)
74	229.4867 8420 1629	3067497 388(220)
75	233.4438 2197 5037	2778574 048(224)
76	237.4124 0275 6215	2584656 040(228)
77	241.3923 7515 3808	2468170 481(232)
78	245.3835 9169 6273	2418753 977(236)
79	249.3859 0868 1576	2431692 647(240)
80	253.3991 8603 4492	2507183 001(244)
81	257.4232 8717 1006	2650252 001(248)
82	261.4580 7886 9398	2871301 974(252)
83	265.5034 3114 7591	3187360 217(256)
84	269.5592 1714 6405	3624241 646(260)
85	273.6253 1301 8336	4220005 513(264)
86	277.7015 9782 1576	5030345 579(268)
87	281.7879 5341 8927	6136961 785(272)
88	285.8842 6438 1367	7660628 139(276)
89	289.9904 1789 5977	9781780 113(280)
90	294.1063 0367 8000	1277331 664(285)
91	298.2318 1388 6794	1705351 419(289)
92	302.3668 4304 5474	2327250 035(293)
93	306.5112 8796 4034	3245547 459(297)
94	310.6650 4766 5774	4624317 726(301)
95	314.8280 2331 6840	6730127 887(305)
96	319.0001 1815 8731	1000272 109(310)
97	323.1812 3744 3609	1517880 018(314)
98	327.3712 8837 2267	2351193 499(318)
99	331.5701 8003 4630	3716892 792(322)
100	335.7778 2335 2644	5995471 635(326)

n	$\log_{10} E_n$	E_n
101	339.9941 3102 5450	9865770 883 (330)
102	344.2190 1747 6729	1655836 596 (335)
103	348.4523 9880 4094	2833993 202 (339)
104	352.6941 9273 0453	4945301 000 (343)
105	356.9443 1855 7238	8796675 209 (347)
106	361.2026 9711 9404	1594766 557 (352)
107	365.4692 5074 2132	2946122 102 (356)
108	369.7439 0319 9138	5545021 050 (360)
109	374.0265 7967 2531	1063113 597 (365)
110	378.3172 0671 4133	2075901 363 (369)
111	382.6157 1220 8208	4127738 805 (373)
112	386.9220 2533 5530	8356517 664 (377)
113	391.2360 7653 8729	1722172 059 (382)
114	395.5577 9748 8862	3612413 768 (386)
115	399.8871 2105 3159	7711183 769 (390)
116	404.2239 8126 3882	1674870 619 (395)
117	408.5683 1328 8258	3700950 601 (399)
118	412.9200 5339 9439	8318660 484 (403)
119	417.2791 3894 8445	1901686 610 (408)
120	421.6455 0833 7045	4420876 034 (412)
121	426.0191 0099 1540	1044963 190 (417)
122	430.3998 5733 7417	2511061 431 (421)
123	434.7877 1877 4824	6133646 954 (425)
124	439.1826 2765 4850	1522746 658 (430)
125	443.5845 2725 6557	3841733 694 (434)
126	447.9933 6176 4760	9848311 226 (438)
127	452.4090 7624 8495	2564934 319 (443)
128	456.8316 1664 0181	6786043 515 (447)
129	461.2609 2971 5415	1823600 554 (452)
130	465.6969 6307 3406	4976947 659 (456)
131	470.1396 6511 8004	1379320 269 (461)
132	474.5889 8503 9303	3881369 951 (465)
133	479.0448 7279 5811	1108849 988 (470)
134	483.5072 7909 7144	3215726 446 (474)
135	487.9761 5538 7245	9465757 783 (478)
136	492.4514 5382 8094	2827833 455 (483)
137	496.9331 2728 3894	8572890 643 (487)
138	501.4211 2930 5730	2637116 438 (492)
139	505.9154 1411 6652	8230270 642 (496)
140	510.4159 3659 7207	2605773 105 (501)
141	514.9226 5227 1371	8368589 619 (505)
142	519.4355 1729 2882	2725946 280 (510)
143	523.9544 8843 1960	9005097 762 (514)
144	528.4795 2306 2393	3016637 062 (519)
145	533.0105 7914 8988	1024658 506 (524)
146	537.5476 1523 5362	3528704 045 (528)
147	542.0905 9043 2068	1231942 484 (533)
148	546.6394 6440 5045	4359778 297 (537)
149	551.1941 9736 4378	1563858 175 (542)
150	555.7547 5005 3362	5685256 369 (546)

n	$\log_{10} E_n$	E_n
151	560.3210 8373 7852	2094516 268(551)
152	564.8931 6019 5903	7819161 727(555)
153	569.4709 4170 7671	2957615 460(560)
154	574.0543 9104 5588	1133420 454(565)
155	578.6434 7146 4787	4400190 355(569)
156	583.2381 4669 3772	1730400 747(574)
157	587.8383 8092 5331	6892565 872(578)
158	592.4441 3880 7679	2780601 852(583)
159	597.0553 8543 5821	1136018 584(588)
160	601.6720 8634 3138	4699875 387(592)
161	606.2942 0749 3178	1968826 713(597)
162	610.9217 1527 1656	8350553 670(601)
163	615.5545 7647 8642	3585720 868(606)
164	620.1927 5832 0954	1558684 875(611)
165	624.8362 2840 4721	6858488 341(615)
166	629.4849 5472 8141	3054602 678(620)
167	634.1389 0567 4398	1376910 382(625)
168	638.7980 5000 4765	6281306 777(629)
169	643.4623 5685 1853	2899725 261(634)
170	648.1317 9571 3038	1354552 098(639)
171	652.8063 3644 4027	6402306 244(643)
172	657.4859 4925 2581	3061605 664(648)
173	662.1706 0469 2387	1481169 268(653)
174	666.8602 7365 7064	7248925 846(657)
175	671.5549 2737 4313	3588619 183(662)
176	676.2545 3740 0197	1796955 823(667)
177	680.9590 7561 3550	9100717 084(671)
178	685.6685 1421 0517	4661376 803(676)
179	690.3828 2569 9213	2414491 602(681)
180	695.1019 8289 4499	1264686 535(686)
181	699.8259 5891 2877	6698212 369(690)
182	704.5547 2716 7502	3586965 235(695)
183	709.2882 6136 3293	1942054 277(700)
184	714.0265 3549 2158	1063005 453(705)
185	718.7695 2382 8324	5881983 858(709)
186	723.5172 0092 3760	3290038 074(714)
187	728.2695 4160 3709	1860122 748(719)
188	733.0265 2096 2304	1062969 890(724)
189	737.7881 1435 8287	6139236 421(728)
190	742.5542 9741 0811	3583417 504(733)
191	747.3250 4599 5338	2113712 888(738)
192	752.1002 4623 9614	1259639 410(743)
193	756.8801 4451 9734	7588300 481(747)
194	761.6644 4745 6287	4617931 179(752)
195	766.4532 2191 0579	2839369 485(757)
196	771.2464 4498 0935	1763782 304(762)
197	776.0440 9399 9076	1106863 329(767)
198	780.8461 4652 6567	7016920 023(771)
199	785.6525 8035 1344	4493454 530(776)
200	790.4633 7348 4298	2906521 128(781)

n	$\log_{10} E_n$	E_n
201	795.2785 0415 5943	1898909 014 (786)
202	800.0979 5081 3134	1252999 256 (791)
203	804.9216 9211 5866	8350108 446 (795)
204	809.7497 0693 4120	5619619 802 (800)
205	814.5819 7434 4787	3819217 088 (805)
206	819.4184 7362 8636	2621039 879 (810)
207	824.2591 8426 7353	1816286 134 (815)
208	829.1040 8594 0632	1270825 559 (820)
209	833.9531 5852 3320	8977564 278 (824)
210	838.8063 8208 2624	6402979 077 (829)
211	843.6637 3687 5361	4610381 622 (834)
212	848.5252 0334 5268	3351223 133 (839)
213	853.3907 6212 0361	2459020 337 (844)
214	858.2603 9401 0339	1821352 518 (849)
215	863.1340 8000 4043	1361695 506 (854)
216	868.0118 0126 6958	1027545 987 (859)
217	872.8935 3913 8764	7825987 299 (863)
218	877.7792 7513 0927	6015547 092 (868)
219	882.6689 9092 4341	4666496 284 (873)
220	887.5626 6836 7009	3653157 252 (878)
221	892.4602 8947 1766	2885954 447 (883)
222	897.3618 3641 4043	2300575 095 (888)
223	902.2672 9152 9676	1850510 398 (893)
224	907.1766 3731 2746	1501887 189 (898)
225	912.0898 5641 3464	1229862 087 (903)
226	917.0069 3163 6093	1016088 734 (908)
227	921.9278 4593 6903	8469269 192 (912)
228	926.8525 8242 2164	7121679 453 (917)
229	931.7811 2434 6177	6041215 753 (922)
230	936.7134 5510 9336	5169578 201 (927)
231	941.6495 5825 6221	4462294 778 (932)
232	946.5894 1747 3734	3885236 622 (937)
233	951.5330 1658 9254	3412059 448 (942)
234	956.4803 3956 8833	3022313 903 (947)
235	961.4313 7051 5424	2700041 973 (952)
236	966.3860 9366 7127	2432728 636 (957)
237	971.3444 9339 5478	2210514 640 (962)
238	976.3065 5420 3763	2025602 404 (967)
239	981.2722 6072 5355	1871805 527 (972)
240	986.2415 9772 2085	1744205 783 (977)
241	991.2145 5008 2637	1638891 047 (982)
242	996.1911 0282 0973	1552754 588 (987)
243	1001.1712 4107 4777	1483341 253 (992)
244	1006.1549 5010 3934	1428729 803 (997)
245	1011.1422 1528 9026	1387443 445 (1002)
246	1016.1330 2212 9856	1358382 663 (1007)
247	1021.1273 5624 3997	1340776 052 (1012)
248	1026.1252 0336 5360	1334146 022 (1017)
249	1031.1265 4934 2793	1338287 258 (1022)
250	1036.1313 8013 8692	1353256 554 (1027)

TABLE 39

THE SERIES T_n

Description: Values of T_n to 18 decimal places, $n = 1$ to $n = 38$, and the common logarithms, $\log_{10} T_n$, over the same range.

n	T_n	$\log_{10} T_n$	n
1	.785 398 163 397 448 310	9.895 089 881 366 171 464	1
2	.915 965 594 177 219 015	9.961 879 160 851 683 560	2
3	.968 946 146 259 369 380	9.986 299 639 762 495 587	3
4	.988 944 551 741 105 336	9.995 171 942 205 934 634	4
5	.996 157 828 077 088 064	9.998 328 152 111 194 882	5
6	.998 685 222 218 438 135	9.999 428 623 565 655 428	6
7	.999 554 507 890 539 909	9.999 806 482 126 585 992	7
8	.999 849 990 246 829 657	9.999 934 846 705 029 415	8
9	.999 949 684 187 220 090	9.999 978 147 570 391 219	9
10	.999 983 164 026 196 877	9.999 992 688 167 928 757	10
11	.999 994 374 973 823 699	9.999 997 557 075 300 307	11
12	.999 998 122 350 587 882	9.999 999 184 546 455 800	12
13	.999 999 373 583 771 841	9.999 999 727 950 803 528	13
14	.999 999 791 087 248 735	9.999 999 909 270 335 449	14
15	.999 999 930 340 842 624	9.999 999 969 747 411 284	15
16	.999 999 976 775 950 903	9.999 999 989 913 923 513	16
17	.999 999 992 257 782 104	9.999 999 996 637 597 477	17
18	.999 999 997 419 086 745	9.999 999 998 879 123 614	18
19	.999 999 999 139 660 745	9.999 999 999 626 359 409	19
20	.999 999 999 713 213 274	9.999 999 999 875 450 107	20
21	.999 999 999 904 403 029	9.999 999 999 958 482 763	21
22	.999 999 999 968 134 064	9.999 999 999 986 160 800	22
23	.999 999 999 989 377 965	9.999 999 999 953 869 088	23
24	.999 999 999 996 459 311	9.999 999 999 998 462 298	24
25	.999 999 999 998 819 768	9.999 999 999 999 487 432	25
26	.999 999 999 999 606 589	9.999 999 999 999 829 144	26
27	.999 999 999 999 868 863	9.999 999 999 999 943 048	27
28	.999 999 999 999 956 288	9.999 999 999 999 981 016	28
29	.999 999 999 999 985 429	9.999 999 999 999 993 672	29
30	.999 999 999 999 995 143	9.999 999 999 999 997 891	30
31	.999 999 999 999 998 381	9.999 999 999 999 999 297	31
32	.999 999 999 999 999 460	9.999 999 999 999 999 765	32
33	.999 999 999 999 999 820	9.999 999 999 999 999 922	33
34	.999 999 999 999 999 940	9.999 999 999 999 999 974	34
35	.999 999 999 999 999 980	9.999 999 999 999 999 991	35
36	.999 999 999 999 999 993	9.999 999 999 999 999 997	36
37	.999 999 999 999 999 998	9.999 999 999 999 999 999	37
38	.999 999 999 999 999 999	9.999 999 999 999 999 999	38

$T_2 =$ .915 965 594 177 219 015 054 603 514 932 38
$T_4 =$ .988 944 551 741 105 336 108 422 633 228
$T_6 =$ .998 685 222 218 438 135 441 600 787 9

GRAM POLYNOMIALS
POLYNOMIAL APPROXIMATION

GRAM POLYNOMIALS — POLYNOMIAL APPROXIMATION*

1. Description. Gram polynomials, named in honor of J. P. Gram who was one of the first to apply polynomials orthogonal over discrete intervals to the problem of polynomial approximation, arise in connection with the problem of fitting a polynomial to data given as a series of equally spaced items.† It is this question which we shall first consider.

Two cases are to be distinguished (I) where the number of items is odd and (II) where the number is even. Let us consider case (I) first, the number of items being equal to $2p + 1$. It is obvious that no restriction in generality is imposed if the items are put into correspondence with the integers, $-p$, $-p+1$, $\cdots$, -2, -1, 0, 1, 2, $\cdots$, p as follows:

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*The material contained in this and the next section (Functions of Polynomial Approximation) is derived mainly from two papers (1) (with V. V. Latshaw) Formulas for the Fitting of Polynomials by the Method of Least Squares, *Annals of Mathematics*, vol. 31 (series 2) (1930), pp. 52-78; (2) Polynomial Approximation by the Method of Least Squares, *Annals of Mathematical Statistics*, vol. 4 (1933), pp. 154-196.

†Über die Entwickelung reeler Functionen in Reihen mittelst der Methode der kleinsten Quadrate. *Journal für Math.*, vol. 94 (1883), pp. 41-73.

The reader is also referred to the following papers:

A. C. Aitken: On the Graduation of Data by the Orthogonal Polynomials of Least Squares. *Proc. of the Royal Soc. of Edinburgh*, vol. 53 (1932-33), pp. 54-78. On Fitting Polynomials to Weighted Data by Least Squares. *Ibid.*, vol. 54 (1933-1934), pp. 1-11. On Fitting Polynomials to Data with Weighted and Correlated Errors. *Ibid.*, vol. 54, pp. 13-16.

V. Pareto: Tables pour faciliter l'application de la méthode des moindres carrés. *Zeitschrift für Schweizerische Statistik, Jhr.* 35, vol. 1, (1899), pp. 121-150.

Edward Condon: The Rapid Fitting of a Certain Class of Empirical Formulae by the Method of Least Squares. *University of California Publications in Mathematics*, vol. 2 (1927), pp. 55-66.

R. T. Birge and J. D. Shea: A Rapid Method for Calculating the Least Square Solution of a Polynomial of Any Degree. *Ibid.*, pp. 67-118.

Karl Jordan: Berechnung der Trendlinie auf Grund der Theorie der kleinsten Quadrate, Budapest (*Hungarian National Committee on Economic Statistics*), (1930).

A. Sipos: Praktische Anwendung der Trendberechungs-Methode von Jordan, Budapest (*ibid.*), (1930).

Karl Jordan: Approximation and Graduation According to the Principle of Least Squares by Orthogonal Polynomials. *Annals of Math. Statistics*, vol. 3 (1932), pp. 257-357.

Max Sasuly: *Trend Analysis of Statistics.* Washington, D. C., (1934), 421 p.

y	y_{-p} ,	y_{-p+1} ,	y_{-p+2} ,	$\cdots$,	y_{-2} ,	y_{-1} ,	y_0 ,	y_1 ,	y_2 ,	$\cdots$,	y_p
x	$-p$	$-p+1$	$-p+2$	$\cdots$	-2	-1	0	1	2	$\cdots$	p

If we designate the r-th moment by M_r,

$$M_r = \sum_{i=-p}^{p} y_i i^r ,$$

and by s_r the series,

$$s_r = 1^r + 2^r + 3^r + \cdots + p^r ,$$

the coefficients of the polynomial,

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n ,$$

fitted to the data by the method of least squares, will be determined by the following sets of equations:

$$\begin{aligned}
(2p+1)\, a_0 + 2s_2 a_2 + 2s_4\, a_4 + \cdots\cdots &= M_0 , \\
2s_2 a_0 + 2s_4 a_2 + 2s_6\, a_4 + \cdots\cdots &= M_2 , \\
2s_4 a_0 + 2s_6 a_2 + 2s_8\, a_4 + \cdots\cdots &= M_4 ,
\end{aligned}$$

$$\begin{aligned}
2s_2 a_1 + 2s_4 a_3 + 2s_6\, a_5 + \cdots\cdots &= M_1 , \\
2s_4 a_1 + 2s_6 a_3 + 2s_8\, a_5 + \cdots\cdots &= M_3 , \\
2s_6 a_1 + 2s_8 a_3 + 2s_{10} a_5 + \cdots\cdots &= M_5 ,
\end{aligned}$$

2. *The Functions of Approximation.* It will be found convenient in setting forth the solutions of the sets of equations given in the last section to employ the following abbreviations:

$$p_{2n} = k_{2n} s_{2n} / p(p+1)(2p+1), \ n = 1, 2, \cdots, 7, \ k_2 = 6,$$

$$k_4 = 30, \ k_6 = 42, \ k_8 = 90, \ k_{10} = 66, \ k_{12} = 2730, \ k_{14} = 90,$$

or explicitly,

$$p_0 = 1/p(p+1), \ p_2 = 1, \ p_4 = 3p^2 + 3p - 1, \ p_6 = 3p^4 + 6p^3$$
$$- 3p + 1, \ p_8 = 5p^6 + 15p^5 + 5p^4 - 15p^3 - p^2 + 9p - 3,$$

$p_{10} = 3p^8 + 12p^7 + 8p^6 - 18p^5 - 10p^4 + 24p^3 + 2p^2 - 15p + 5$, $p_{12} = 105p^{10} + 525p^9 + 525p^8 - 1050p^7 - 1190p^6 + 2310p^5 + 1420p^4 - 3285p^3 - 287p^2 + 2073p - 691$,
$p_{14} = 3p^{12} + 18p^{11} + 24p^{10} - 45p^9 - 81p^8 + 144p^7 + 182p^6 - 345p^5 - 217p^4 + 498p^3 + 44p^2 - 315p + 105$.

In terms of these functions the solutions of the equations of the last section can be expressed as follows:

The straight line, $y = a_0 + a_1 x$.

$$a_0 = AM_0 \ , \quad a_1 = A'M_1 \ ,$$

where we abbreviate,

$$A = 1/(2p+1) \ , \quad A' = 3/p(p+1)(2p+1) \ .$$

The parabola, $y = a_0 + a_1 x + a_2 x^2$.

$$a_0 = AM_0 + BM_2 \ ,$$
$$a_2 = BM_0 + CM_2 \ ,$$

a_1 determined from the straight line,

where we have,

$$A = 3p_4/P \ , \quad B = -15p_2/P \ , \quad C = 45p_0/P \ ,$$
$$P = (2p-1)(2p+3)(2p+1) \ .$$

The cubic, $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$.

$$a_1 = A'M_1 + B'M_3,$$
$$a_3 = B'M_1 + C'M_3 \ ,$$

a_0, a_2 determined from the parabola,

where we use the abbreviations,

$$A' = 5^2 p_6/P \ , \quad B' = -5 \cdot 7\, p_4/P \ , \quad C' = 5^2 \cdot 7\, p_2/P \ ,$$
$$P = p(p+1)(2p+1)(2p+3)(2p-1)(p-1)(p+2) \ .$$

The quartic, $y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$.

$$a_0 = AM_0 + BM_2 + CM_4 \ ,$$

$$a_2 = BM_0 + DM_2 + EM_4 \ ,$$

$$a_4 = CM_0 + EM_2 + FM_4 \ ,$$

a_1, and a_3 computed from the cubic,

where the coefficients of the moments are,

$$A = 5P_{11}/4P \ , \quad B = -5^2 \cdot 7P_{12}/4P \ , \quad C = 15 \cdot 63P_{13}/4P \ , \quad D = 5 \cdot 147P_{22}/4PP' \ ,$$

$$E = -15 \cdot 105P_{23}/4PP' \ , \quad F = 15 \cdot 735P_{13}/4PP' \ ;$$

$$P = (2p-1)(2p-3)(2p+1)(2p+3)(2p+5) \ , \quad P' = p(p-1)(p+1)(p+2) \ ;$$

$$P_{11} = 15p_6 - 35p_4 - 14p_2 \ , \quad P_{12} = 2p_4 - 7p_2 \ , \quad P_{13} = p_2 \ ,$$

$$P_{22} = 4p_6 - 4p_4 + 7p_2 \ , \quad P_{23} = 2p_4 - 3p_2 \ .$$

The quintic, $y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$.

$$a_1 = A'M_1 + B'M_3 + C'M_5 \ ,$$

$$a_3 = B'M_1 + D'M_3 + E'M_5 \ ,$$

$$a_5 = C'M_1 + E'M_3 + F'M_5 \ ,$$

a_0, a_2 and a_4 computed from the quartic,

where we employ the abbreviations,

$$A' = 7^2 P_{11}'/4P, \quad B' = -3 \cdot 7^2 P_{12}'/4P, \quad C' = 3 \cdot 7 \cdot 11 P'_{13}/4P \ ,$$

$$D' = 7 \cdot 3^2 \cdot 5^2 P_{22}'/4P \ , \quad E' = -3 \cdot 5 \cdot 7^2 \cdot 11 P_{23}'/4P \ ,$$

$$F' = 3^4 \cdot 7^2 \cdot 11 P_{33}'/4P \ ,$$

$$P = p(p+1)(2p+1)(2p+3)(2p-1)(p-1)(p+2)(2p+5)(2p-3)(p-2)(p+3) \ ,$$

$$P_{11}' = 25p_{10} - 70p_8 + 105p_6 + 140p_4 - 84p_2 \ , \quad P_{12}' = 18p_8 - 105p_6 + 91p_4 + 70p_2 \ , \quad P_{13}' = 15p_6 - 35p_4 - 14p_2 \ ,$$

$$P_{22}' = 12p_6 - 28p_4 + 77p_2 \ , \quad P_{23}' = 2p_4 - 7p_2 \ , \quad P_{33}' = p_2$$

The sextic,

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 \;.$$

$$a_0 = AM_0 + BM_2 + CM_4 + DM_6 \;,$$
$$a_2 = BM_0 + EM_2 + FM_4 + GM_6 \;,$$
$$a_4 = CM_0 + FM_2 + HM_4 + IM_6 \;,$$
$$a_6 = DM_0 + GM_2 + IM_4 + JM_6 \;,$$

a_1, a_3 and a_5 computed from the quintic,

where the coefficients of the moments are,

$$A = 5 \cdot 7\, P_{11}/4P' \;,$$
$$B = -3 \cdot 5 \cdot 7^2 P_{12}/4P' \;,$$
$$C = 3 \cdot 5 \cdot 7^2 \cdot 11 P_{13}/4P' \;,$$
$$D = -3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 P_{14}/4P' \;,$$
$$E = 3^2 \cdot 7^2 P_{22}/4P \;,$$
$$F = -3^2 \cdot 5 \cdot 7 \cdot 11 P_{23}/4P \;,$$
$$G = 3 \cdot 7^2 \cdot 11 \cdot 13 P_{24}/4P \;,$$
$$H = 3 \cdot 5^2 \cdot 7^2 \cdot 11^2 P_{33}/4P \;,$$
$$I = -3^2 \cdot 5 \cdot 7^2 \cdot 11 \cdot 13 P_{34}/4P \;,$$
$$J = 3^2 \cdot 7^2 \cdot 11^2 \cdot 13 P_{44}/4P \;;$$

$$P = p\,(p+1)\,(2p+1)\,(2p+3)\,(2p-1)\,(p-1)\,(p+2)\,(2p-3) \times (2p+5)\,(p+3)\,(2p+7)\,(p-2)\,(2p-5) \;,$$

$$P' = (2p+1)\,(2p+3)\,(2p-1)\,(2p-3)\,(2p+5)\,(2p+7)\,(2p-5),$$

$$P_{11} = 7p_8 - 105p_6 + 168p_4 + 114p_2 \;,$$
$$P_{12} = 5p_6 - 30p_4 + 66p_2 \;,$$
$$P_{13} = p_4 - 9p_2 \;,$$
$$P_{14} = p_2 \;,$$

$$P_{22} = 135p_{10} - 648p_8 + 2610p_6 - 1096p_4 + 1473p_2 \;,$$
$$P_{23} = 27p_8 - 270p_6 + 483p_4 - 454p_2 \;, \quad P_{24} = 5p_6 - 20p_4 + 17p_2 \;,$$
$$P_{33} = p_6 - 4p_4 + 16p_2 \;, \quad P_{34} = p_4 - 6p_2 \;, \quad P_{44} = p_2 \;.$$

The septimic,

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 \ .$$

$$a_1 = A'M_1 + B'M_3 + C'M_5 + D'M_7 \ ,$$
$$a_3 = B'M_1 + E'M_3 + F'M_5 + G'M_7 \ ,$$
$$a_5 = C'M_1 + F'M_3 + H'M_5 + I'M_7 \ ,$$
$$a_7 = D'M_1 + G'M_3 + I'M_5 + J'M_7 \ ,$$

a_0, a_2, a_4, and a_6 computed from the sextic.

where we use the following abbreviations,

$$A' = 3^2 P'_{11}/4P \ ,$$
$$B' = -3^3 \cdot 7 \cdot 11\, P'_{12}/4P \ ,$$
$$C' = 3^3 \cdot 7 \cdot 11 \cdot 13\, P'_{13}/4P \ ,$$
$$D' = -3^2 \cdot 5 \cdot 11 \cdot 13\, P'_{14}/4P \ ,$$
$$E' = 3^3 \cdot 5 \cdot 7 \cdot 11^2\, P'_{22}/4P \ ,$$
$$F' = -3^2 \cdot 7^2 \cdot 11^2 \cdot 13\, P'_{23}/4P \ ,$$
$$G' = 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13\, P'_{24}/4P \ ,$$
$$H' = 3^3 \cdot 7^2 \cdot 11 \cdot 13^2\, P'_{33}/4P \ ,$$
$$I' = -3^3 \cdot 5 \cdot 7 \cdot 11^2\, 13\, P'_{34}/4P \ ,$$
$$J' = 3^3 \cdot 5 \cdot 11^2 \cdot 13^2\, P'_{44}/4P \ ;$$

$$P = p(p+1)(2p+1)(2p+3)(2p-1)(p-1)(p+2)(2p+5)$$
$$\times (2p-3)(p-2)(p+3)(2p+7)(2p-5)(p-3)(p+4) \ ,$$

$$P'_{11} = 1225p_{14} - 630p_{12} + 130{,}095p_{10} - 144{,}144p_8$$
$$- 115{,}962p_6 + 635{,}964p_4 - 347{,}016p_2 \ ,$$

$$P'_{12} = p_{12} - 525p_{10} + 1554p_8 - 4220p_6 + 1032p_4 + 5130p_2 \ ,$$
$$P'_{13} = 5p_{10} - 36p_8 + 219p_6 - 236p_4 - 228p_2 \ ,$$
$$P'_{14} = 7p_8 - 105p_6 + 168p_4 + 114p_2 \ ,$$
$$P'_{22} = 5p_{10} - 36p_8 + 240p_6 - 312p_4 + 582p_2 \ ,$$
$$P'_{23} = p_8 - 15p_6 + 42p_4 - 90p_2 \ ,$$
$$P'_{24} = 5p_6 - 30p_4 + 66p_2 \ , \quad P'_{33} = p_6 - 6p_4 + 33p_2 \ ,$$
$$P'_{34} = p_4 - 9p_2 \ , \quad P'_{44} = p_2 \ .$$

These functions, i. e. the coefficients of the moments which we have designated by means of capital letters, have been tabulated for various ranges of the variable p.

3. *Numerical Application.* As an example illustrating the application of the formulas of the last section, let us consider the polynomial approximation to the following data which are taken from T. N. Thiele and consist of a system of frequencies obtained from a game of patience (solitaire) :*

Value of Character	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Frequency (y)	0	3	7	35	101	89	94	70	46	30	15	4	5	1	0
Class Marks (x)	—7	—6	—5	—4	—3	—2	—1	0	1	2	3	4	5	6	7

Computing the moments, using values of x for this purpose, we get:

$$M_0 = 500\,, \quad M_3 = -5508\,, \quad M_6 = 626188\,,$$
$$M_1 = -570\,, \quad M_4 = 34108\,, \quad M_7 = -1419708\,.$$
$$M_2 = 2728\,, \quad M_5 = -76380\,,$$

Substituting these values in the respective formulas we obtain:

$$y = 33.3333 - 2.03571x\,,$$
$$y = 63.2217 - 2.03571x - 1.601163x^2\,,$$
$$y = 63.2217 - 9.92462x - 1.601163x^2 + .2361953x^3\,,$$

***Forelaesninger over Almindelig Iagttagelseslaere, Copenhagan,* (1889), p. 12. These data are selected for illustration because they were similarly employed by Karl Pearson in testing his theory of polynomial curve fitting based on the method of moments: On the Systematic Fitting of Curves to Observations and Measurements. *Biometrika*, vol. I, (1902), pp. 265-303; vol. II, (1903), pp. 1-23, in particular, p. 18.

$$y = 75.0584 - 9.92462x - 3.760517x^2 + .2361953x^3 + .0456661x^4 \ ,$$

$$y = 75.0584 - 20.40589x - 3.760517x^2 + 1.1397932x^3 + .0456661x^4 - .01492181x^5 \ ,$$

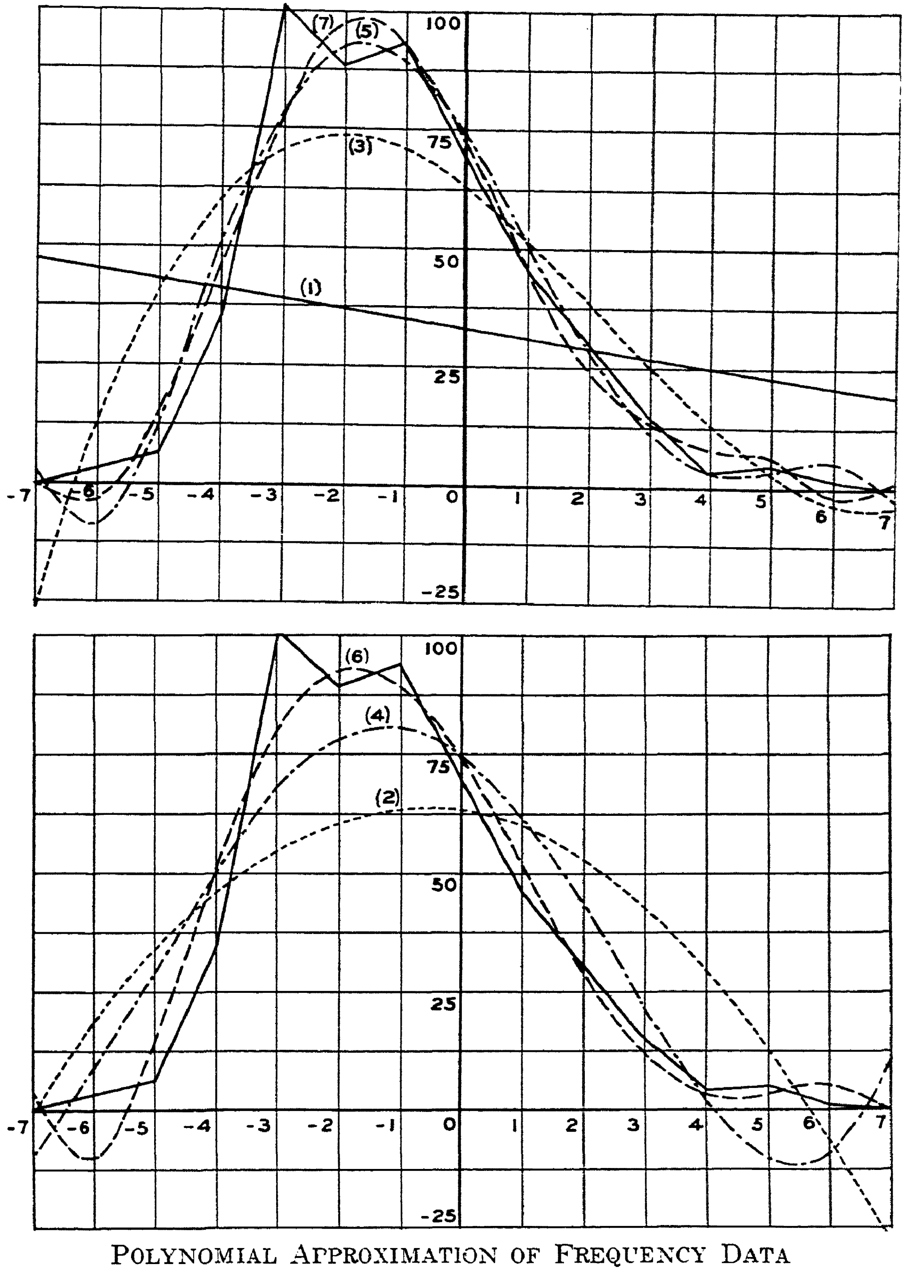
$$y = 73.9504 - 20.40589x - 3.3205859x^2 + 1.1397932x^3 + .02089034x^4 - .01492181x^5 + .000338551x^6 \ ,$$

$$y = 73.9504 - 25.76303x - 3.3205859x^2 + 2.0703223x^3 - .02089034x^4 - .054117647x^5 + .000338551x^6 + .0004607106x^7 \ .$$

We compute from these equations the following approximations

x	Data (y)	Straight line	Parabola	Cubic	Quartic	Quintic	Sextic	Septimic
—7	0	47.58	—.99	—26.78	—11.11	3.12	3.92	.59
—6	3	45.55	17.79	14.11	7.39	—8.86	—10.45	—3.48
—5	7	43.51	33.37	43.29	29.69	15.77	15.47	12.43
—4	35	41.48	45.75	62.19	51.16	50.54	51.51	45.98
—3	101	39.44	54.92	72.21	68.31	78.98	80.07	79.54
—2	89	37.41	60.89	74.78	78.71	92.92	93.19	97.66
—1	94	35.37	63.66	71.31	81.03	90.63	89.93	94.40
0	70	33.33	63.22	63.22	75.06	75.06	73.95	73.95
1	46	31.30	59.59	51.93	61.66	52.06	51.37	46.91
2	30	29.26	52.75	38.86	42.79	28.58	28.85	24.40
3	15	27.23	42.70	25.42	21.52	10.84	11.94	12.47
4	4	25.19	29.46	13.02	2.00	2.62	3.60	9.14
5	5	23.16	13.01	3.09	—10.51	3.40	3.10	6.13
6	1	21.12	—6.63	—2.95	—9.67	6.59	5.01	—1.96
7	0	19.08	—29.49	—3.69	11.98	—2.25	—1.46	.87

The goodness of fit is graphically illustrated in the accompanying figures.



POLYNOMIAL APPROXIMATION OF FREQUENCY DATA
(The numbers refer to the degrees of the polynomials used in the approximations.)

4. Case Where the Number of Items is Even. In the previous sections we discussed the problem of fitting the polynomials to data in which the number of items was odd. We now assume that the number is even, $N = 2p$, and arrange the data according to the following scheme:

$$\begin{array}{cccccccc} y_{-p} & & y_{-2} & y_{-1} & y_1 & y_2 & \cdots & y_p \\ -(2p-1)/2 & \cdots & -3/2 & -1/2 & 1/2 & 3/2 & \cdots & (2p-1)/2 \end{array}$$

The moments which apply to the data are computed from the formula:

$$M' = (1/2)^r \sum_{s=1}^{p} \{(2s-1)^r [y_s + (-1)^r y_{-s}]\} .$$

The values of the constants which multiply the moments in the formulas of section 2 are taken from the tables which correspond to the argument $(2p-1)/2$.

As an example consider the following data which are identical with the series employed in section 3 except for the deletion of the last item.

Value of Character	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Frequency (y)	0	3	7	35	101	89	94	70	46	30	15	4	5	1
Class Marks (x)	$-\frac{13}{2}$	$-\frac{11}{2}$	$-\frac{9}{2}$	$-\frac{7}{2}$	$-\frac{5}{2}$	$-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$	$\frac{11}{2}$	$\frac{13}{2}$

Computing the first five moments from these data, we obtain:

$$M_0 = 500 , \quad M_2 = 2233 , \quad M_4 = 26930.25 ,$$
$$M_1 = -320 , \quad M_3 = -1781 , \quad M_5 = -1632.5 .$$

Substituting these values in the formulas of section 2 for the cases of the parabola and the quintic, using the tabulated values corresponding to $p = 13/2$, we obtain the following least square polynomials:

$$y = 68.3147 - 1.40659x - 2.006181x^2 \ ,$$

$$y = 83.3725 - 16.63150x - 5.172034x^2 + 1.1492205x^3 + .0770816x^4 - .0178009x^5 \ .$$

From these equations the following table of values was computed:

x	Data (y)	Parabola	Quintic	x	Data (y)	Parabola	Quintic
—6.5	0	—7.304	1.492	0.5	70	67.110	73.912
—5.5	3	15.364	—12.686	1.5	46	61.691	50.922
—4.5	7	34.019	13.214	2.5	30	52.260	28.698
—3.5	35	48.662	49.869	3.5	15	38.816	13.295
—2.5	101	59.293	79.419	4.5	4	21.360	7.280
—1.5	89	65.911	93.329	5.5	5	—.108	7.592
—0.5	94	68.516	90.257	6.5	1	—25.589	3.407

5. *The Gram Polynomials.* The Gram polynomials are defined to be the members of a set of polynomials,

$$\varphi_0(x) \ , \ \varphi_1(x) \ , \ \varphi_2(x) \ , \ \cdots \ , \ \varphi_m(x) \ ,$$

of degrees 0, 1, 2, $\cdots$, m, respectively, such that,

$$\sum_{x=-p}^{p} \varphi_s(x) \, \varphi_t(x) = 0 \ , \ \text{when } s \neq t \ ,$$

$$\sum_{x=-p}^{p} \varphi_s^2(x) = S_s \ ,$$

where $S_s = \delta_{s-2}/\delta_s$, in which we employ the abbreviation,

$$\delta_{-2} = \delta_{-1} = 1 \ ,$$

$$\delta_0 = 2p+1 \ , \ \delta_{2n} = \begin{vmatrix} 2p+1 & , 2s_2 & & , 2s_{2n} \\ 2s_2 & , 2s_4 & & , 2s_{2n+2} \\ & & & \\ 2s_2 & , 2s_{2n-2} & , \cdots & , 2s_{4n} \end{vmatrix}$$

$$\delta_1 = 2s_2 \ , \qquad \delta_{2n-1} = \begin{vmatrix} 2s_2 & , 2s_4 & , \cdots & , 2s_{2n} \\ 2s_4 & , 2s_6 & , \cdots & , 2s_{2n+2} \\ \cdot & \cdot & \cdot & \cdot \\ 2s_{2n} & , 2s_{2n+2} & , \cdots & , 2s_{4n-2} \end{vmatrix}$$

The explicit values of these determinants are given below:

$$\delta_{-2} = \delta_{-1} = 1 \ ,$$

$$\delta_0 = 2p+1 \ ,$$

$$\delta_1 = p(p+1)(2p+1)/3 \ ,$$

$$\delta_2 = p(p+1)(2p+1)^2(2p+3)(2p-1)/3^2 \cdot 5 \ ,$$

$$\delta_3 = p^2(p+1)^2(2p+1)^2(2p+3)(2p-1)(p-1) \times (p+2)/3 \cdot 5^2 \cdot 7 \ ,$$

$$\delta_4 = 4p^2(p+1)^2(p+2)(p-1)(2p-1)^2(2p-3)(2p+1)^3 \times (2p+3)^2(2p+5)/3^4 \cdot 5^3 \cdot 7^2 \ ,$$

$$\delta_5 = 4p^3(p+1)^3(2p+1)^3(2p+3)^2(2p-1)^2(p-1)^2(p+2)^2 \times (2p+5)(2p-3)(p-2)(p+3)/3^5 \cdot 5^2 \cdot 7^3 \cdot 11 \ ,$$

$$\delta_6 = 16p^3(p+1)^3(2p+1)^4(2p+3)^3(2p-1)^3(p-1)^2(p+2)^2 \times (2p-3)^2(2p+5)^2(p+3)(2p+7)(p-2) \times (2p-5)/3^6 \cdot 5^3 \cdot 7^4 \cdot 11^2 \cdot 13 \ ,$$

$$\delta_7 = 16p^4(p+1)^4(2p+1)^4(2p+3)^3(2p-1)^3(p-1)^3 \times (p+2)^3(2p+5)^2(2p-3)^2(p-2)^2(p+3)^2 \times (2p+7)(2p-5)(p-3)/3^8 \cdot 5^3 \cdot 7^3 \cdot 11^3 \cdot 13^2 \ .^*$$

*The factorization of the general determinant δ_n can be accomplished by means of an adaptation to the present case of a method given by A. C.Aitken: Note on a Special Persymmetric Determinant. *Annals of Mathematics*, vol. 32 (2) (1931), pp. 461-462.

It will be found by explicit calculation that the Gram polynomials are obtained from the *functions of approximation* given in section 2 by replacing the moments, M_r, in the last formulas of each case(straight line, parabola, cubic, etc.) by x^r. We thus have the following expressions for the first eight polynomials:

$$\varphi_0(x) = 1/(2p+1) = A \ ,$$

$$\varphi_1(x) = 3x/p(p+1)(2p+1) = A'x \ ,$$

$$\varphi_2(x) = \{3^2 \cdot 5/p(p+1)(4p^2-1)(2p+3)\} \{x^2 - p(p+1)/3\} = Cx^2 + B \ ,$$

$$\varphi_3(x) = \{5^2 \cdot 7/p(p^2-1)(4p-1)(2p+3)(p+2)\}\{x^3 - (3p^2+3p-1)x/5\} = C'x^3 + B'x \ ,$$

$$\varphi_4(x) = \{15^2\, 7^2/4p(p^2-1)(4p^2-1)(4p^2-9)(2p+5) \times (p+2)\}\{x^4 - (6p^2+6p-5)\,x^2/7 + 3p(p^2-1)(p+2)/35\} = Fx^4 + Ex^2 + C \ ,$$

$$\varphi_5(x) = \{3^4 \cdot 7^2 \cdot 11/4p(p^2-1)(4p^2-1)(4p^2-9)(p^2-4) \times (2p+5)(p+3)\}\{(x^5 - 5(2p^2+2p-3)x^3/9 + (15p^4+30p^3-35p^2-50p+12)x/63\} = F'x^5 + E'x^3 + C'x \ ,$$

$$\varphi_6(x) = \{3^2 \cdot 7^2 \cdot 11^2 \cdot 13/4p(p^2-1)(p^2-4)(4p^2-9) \times (4p^2-25)(p+3)(2p+7)\}\{x^6 - 5(3p^2+3p-7)x^4/11 + (5p^4+10p^3-20p^2-25p+14)x^2/11 - 5p(p^2-1)(p^2-4)(p+3)/3 \cdot 7 \cdot 11\} = Jx^6 + Ix^4 + Gx^2 + D \ ,$$

$$\varphi_7(x) = \{3^2 \cdot 5 \cdot 11^2 \cdot 13^2/4p(p^2-1)(p^2-4)(p^2-9)(4p^2-1) \times (4p^2-9)(4p^2-25)(p+4)(2p+7)\}\{x^7 - 7(3p^2+3p-10)\,x^5/13 + 7(15p^4+30p^3-90p^2-105p+101)x^3/11 \cdot 13 - (35p^6+105p^5-280p^4-735p^3+497p^2+882p-180)x \cdot 3 \cdot 11 \cdot 13\} = J'x^7 + I'x^5 + G'x^3 + D'x \ .$$

It will be found that the following recursion relationship holds between successive members of the set of polynomials:*

$$(n+1)^2(2p-n)(2p+n+2)\,\varphi_{n+1}(x) - 4(2n+1)(2n+3)\,x\,\varphi_n(x)$$
$$+4(2n+1)(2n+3)\,\varphi_{n-1}(x) = 0\ .$$

It is also possible to show that these polynomials satisfy the following difference equation:†

$$[(p-1)(p+2) - 3x - x^2]\Delta^2\varphi_n(x) + [(n-1)(n-2)$$
$$-2x]\Delta\varphi_n(x) + n(n+1)\,\varphi_n = 0\ ,$$

where we employ the customary difference notation,

$$\Delta\,\varphi_n(x) = \varphi_n(x+1) - \varphi_n(x)\ ,$$
$$\Delta^2\,\varphi_n(x) = \Delta[\Delta\,\varphi_n(x)]$$
$$= \varphi_n(x+2) - 2\,\varphi_n(x+1) + \varphi_n(x)\ .$$

6. Use of the Polynomials in Approximation. The Gram polynomials are employed in the problem of approximation as follows:

Let $f(x)$ be a function defined over the discrete interval from $x = -p$ to $x = p$, and let us consider the following expression:

$$J = \sum^{p} \{f(x) - A_0\,\varphi_0(x) - A_1\,\varphi_1(x) - \cdots - A_m\varphi_m(x)\}^2\ .$$

*The definition of the $\phi_n(x)$ which has been given above was chosen for the obvious connection which the functions in that form have with the problem of curve fitting and with the computed values in the tables. If, however, the coefficient of x^n were reduced to unity, i.e. $\phi_0(x) = 1$, $\phi_1(x) = x$, $\phi_2(x) = x^2 - p(p+1)/3$, etc., then the recursion formula just given would have been,

$$4(4n^2-1)\,\phi_{n+1}(x) - 4(4n^2-1)\,x\,\phi_n(x)$$
$$+ n^2\,(2p-n+1)(2p+n+1)\,\phi_{n-1}(x) = 0\ .$$

If, moreover, $\phi_n(x)$ as just defined were multiplied by the coefficient $1\cdot 3\cdot 5\cdots(2n-1)/n!$, which is the multiplier of the corresponding and analogous Legendre polynomials, then the recursion formula becomes,

$$4(n+1)\,\phi_{n+1}(x) - 4(2n+1)\,x\,\phi_n(x)$$
$$+ n(2p-n+1)(2p+n+1)\,\phi_{n-1}(x) = 0\ .$$

†This equation was discovered by H. E. H. Greenleaf in connection with work on a doctor's dissertation.

This function can be minimized from the zero values of the derivatives with respect to A_r,

$$\frac{\partial J}{\partial A_r} = 0 \ ,$$

from which we obtain,

$$= \sum^{p} f(x)\varphi_r(x)/S_r \ ,$$

where

$$S_r = \sum_{x=-p}^{p} \varphi^2{}_r(x) = \delta_{r-2}/\delta_r$$

Explicit evaluation of the coefficient is easily attained. Thus if we represent $\varphi_r(x)$ by the series:

$$\varphi_r(x) = \varphi_{r,0} + \varphi_{r,1}x + \varphi_{r,2}\,x^2 + \cdots + \varphi_{r,r}\,x^r \ ,$$

then we obtain,

$$A_r = \sum^{p} f(x)\ \varphi_r(x)/S_r \ ,$$

$$= (\varphi_{r,0}/S_r)M_0 + (\varphi_{r,1}/S_r)M_1 + \cdots + (\varphi_{r,r}/S_r)M_r \ .$$

Since S_r is equal to the coefficient of M_r in the formula for the computation of a_r for the case where $n = r$ and since the $\varphi_{r,i}$ are the coefficients of M_i in the same expression for a_r, $\varphi_{r,r} = S_r$, the value of A_r may be written down at once from the formulas of previous sections.

For example, in the case of the parabola, we have,

$$\varphi_2(x) = B + Cx^2 \ ,$$

$$A_2 = \sum_{x=-p}^{p} f(x)\varphi^2(x)/S_2 = (B/C)\ M_0 + M_1 \ .$$

Hence, if we represent a function $f(x)$ by means of a parabolic approximation, we shall attain,

$$f(x) = A_0\varphi_0(x) + A_1\varphi_1(x) + A_2\varphi_2(x) \ ,$$

where $A_0 = M_0$, $A_1 = M_1$, $A_2 = (BM_0/C + M_1)$.

Explicit calculation will show that this expansion is identical

7. *Transformation of Results to the Range from $x = 1$ to $x = p'$.* The results of previous sections can be applied to the curve-fitting problem associated with the range from $x = 1$ to $x = p'$, which is discussed in the chapter of this volume entitled: "Functions of Polynomial Approximation".

The coefficients of the least-square polynomial,

$$y = A_0 + A_1 x + A_2 x^2 + \cdots + A_m x^n$$

fitted to data over the discrete range from $x = 1$ to $x = p'$, can be obtained from the coefficients, $a_0, a_1, a_2, \cdots, a_m$, of section 2, by means of the following substitution:

$$p = (p' - 1)/2 \ ,$$

$$M_r = m_r - r\left(\frac{p'+1}{2}\right) m_{r-1}$$

$$+ \frac{r(r-1)}{2!}\left(\frac{p'+1}{2}\right)^2 m_{r-2} - \cdots (-1)^r \left(\frac{p'+1}{2}\right)^r m_0$$

where M_r are the moments defined in section 1 and m_r are the moments,

$$m_r = \sum_{s=1}^{p'} s^r y_s \ .$$

Conversely we can pass from the range $x = -p$ to $x = p$, to the range $x = 1$ to $x = p'$ by means of the substitution:

$$p' = 2p + 1 \ ,$$

$$m_r = M_r + r\left(\frac{p'+1}{2}\right) M_{r-1}$$

$$+ \frac{r(r-1)}{2!}\left(\frac{p'+1}{2}\right)^2 M_{r-2} + \cdots + \left(\frac{p'+1}{2}\right)^r M_0 \ .$$

By means of these transformations a new set of polynomials $\varphi_m(x)$, which are orthogonal over the discrete range from $x = 1$ to $x = p'$, may be obtained from

$$\varphi_m(x) = \varphi_{m\,0} + \varphi_{m,1}\, x + \varphi_{m,2} x^2 + \cdots + \varphi_{m,m} x^m$$

as follows:

$$\psi'_m(x) = \varphi'_{m,0} + \varphi'_{m,1}(x-b) + \varphi'_{m,2}(x^2 - 2bx + b^2) + \cdots$$
$$+ \varphi'_{m,m}\{x^m - mbx^{m-1} + m(m-1)b^2x^{m-2}/2! + \cdots\} ,$$

where $b = (p'+1)/2$ and $\varphi'_{m,r}$ denotes the value of $\varphi_{m,r}$ after the substitution $p = (p'-1)/2$.

These polynomials can be proved to be orthogonal over the discrete range $x = 1$ to $x = p'$,

$$\sum_{x=1}^{p'} \psi'_m(x)\ \psi'_n(x) = 0 ,$$

$$\sum_{x=1}^{p'} \psi'^2_m(x) = \varphi_m'(p') .$$

They are identifiable with the last lines of the formulas of section 2 (Functions of Polynomial Approximation) where the m_r are replaced by x^r.

The following explicit formula, in the notation of the present paper where the range is from $x = 1$ to $x = p$, was derived by Gram:

$$\Psi_m(x) = \{1/(p-m-1!)\}\{(p-1)!/m! - (m+1)(p-2)!$$
$$\times (x-1)/(m-1)!\cdot 1!^2 + (m+1)(m+2)\cdot(p-3)!$$
$$\times (x-1)(x-2)/(m-2)!\cdot 2!^2 - (m+1)(m+2)$$
$$\times (m+3)\cdot(p-4)!(x-1)(x-2)$$
$$\times (x-3)/(m-3)!\cdot 3!^2 + \cdots\} .$$

The following relationship holds between $\Psi_m(x)$ and $\psi'_m(x)$:

$$\Psi_m(x) = \{(-1)^m p(p^2-1)(p^2-4)\cdots(p^2-m^2)$$
$$(m!)/(2m)!\cdot(2m+1)\}\ \psi'_m(x) .$$

The first four polynomials are given below explicitly:*

$$\psi_0(\mathrm{x}) = 1/p ,$$

$$\psi'_1(x) = [12/p(p^2-1)][x - (p+1)/2] ,$$

*These polynomials are essentially the same as those given by Jordan (loc. cit.) except that the summation in his work is taken over the range from $x = 0$ to $x = n-1$. The polynomials are also expressed in terms of the Newton polynomials: $x(x-1)\cdots(x-n)$.

with the corresponding case in section 2.

$$\psi_2(x) = [180/p(p^2-1)(p^2-4)][x^2 - (p+1)x + p(p+1)(p+2)/6] \; ,$$

$$\psi_3(x) = [2800/p(p^2-1)(p^2-4)(p^2-9)][x^3 - 3(p+1)x^2/2 + (6p^2+15p+11)x/10 - (p+1)(p+2)(+3)/20] \; .^*$$

8. Computation of the Tables. The tables which accompany this chapter were computed in the statistics laboratory of Indiana University directly from the formulas of section 2 by Dr. Irene Price, Miss Anna Lescisin, and Marion B. Shelley.† They were checked by duplicate calculation.

The computation of the formulas themselves was the combined work of Dr. V. V. Latshaw and the director of this work. They were assisted by L. G. Mitten. The problem at one stage required the factorization of a polynomial of thirty-sixth degree.

*Employing the abbreviation, $z = x - (p+1)/2$, Pareto, (*loc. cit.*) has given the following explicit values for the next five of these polynomials with the leading coefficient reduced to unity:

$$P_4(z) = z^4 - (3p^2-13)z^2/14 + 3(p^2-1)(p^2-9)/560 \; ,$$
$$P_5(z) = z^5 - 5(p^2-7)z^3/18 + 15p^4 - 230p^2 + 407)z/1008 \; ,$$
$$P_6(z) = z^6 - 5(3p^2-31)z^4/44 + (5p^4 - 110p^2 + 329)z^2/176 - 5(p^2-1)(p^2-9)(p^2-25)/14784 \; ,$$
$$P_7(z) = z^7 - 7(3p^2-43)z^5/52 + 7(15p^4 - 450p^2 + 2051)z^3/2288 - (35p^6 - 1645p^4 + 17297p^2 - 27027)z/27456,$$
$$P_8(z) = z^8 - 7(p^2-19)z^6/15 + 7(3p^4 - 118p^2 + 763)z^4/312 - (105p^6 - 6405p^4 + 91679p^2 - 231491)z^2/34320 + 35(p^2-1)(p^2-9)(p^2-25)(p^2-49)/1647360 \; .$$

It has been shown by Tschebytscheff that these polynomials satisfy the difference equation:

$$4(4n^2-1)P_{n+1}(z) - 4(4n^2-1)zP_n(z) + n^2(p^2-n^2)P_{n-1}(z) = 0 \; .$$

†P. W. Overman computed the last fifty values in the first table.

TABLE 40

Coefficients for the Straight Line for the Range From $x = -p$ to $x = +p$

Description: The coefficients A and A' are computed to ten significant figures for the range $p = .5$ to $p = 150.00$ by increments of .5.

The straight line,

$$y = a_0 + a_1 x \ ,$$

is obtained from the equations,

$$a_0 = A\, M_0 \ ,$$

$$a_1 = A' M_1 \ .$$

where M_0, M_1 are the moments computed over the range $x = -p$ to $x = +p$.

(The numbers in parentheses denote the number of ciphers between the decimal point and the first significant figure.)

p	A	A'	p	A
0.5	.500 0000 000	2.000 0000 000	25.5	.(1)192 3076 923
1.0	.333 3333 333	.500 0000 000	26.0	.(1)188 6792 453
1.5	.250 0000 000	.200 0000 000	26.5	.(1)185 1851 852
2.0	.200 0000 000	.100 0000 000	27.0	.(1)181 8181 818
2.5	.166 6666 667	.(1)571 4285 714	27.5	.(1)178 5714 286
3.0	.142 8571 429	.(1)357 1428 571	28.0	.(1)175 4385 965
3.5	.125 0000 000	.(1)238 0952 381	28.5	.(1)172 4137 931
4.0	.111 1111 111	.(1)166 6666 667	29.0	.(1)169 4915 254
4.5	.100 0000 000	.(1)121 2121 212	29.5	.(1)166 6666 667
5.0	.(1)909 0909 091	.(2)909 0909 091	30.0	.(1)163 9344 262
5.5	.(1)833 3333 333	.(2)699 3006 993	30.5	.(1)161 2903 226
6.0	.(1)769 2307 692	.(2)549 4505 495	31.0	.(1)158 7301 587
6.5	.(1)714 2857 143	.(2)439 5604 396	31.5	.(1)156 2500 000
7.0	.(1)666 6666 667	.(2)357 1428 571	32.0	.(1)153 8461 538
7.5	.(1)625 0000 000	.(2)294 1176 471	32.5	.(1)151 5151 515
8.0	.(1)588 2352 941	.(2)245 0980 392	33.0	.(1)149 2537 313
8.5	.(1)555 5555 556	.(2)206 3983 488	33.5	.(1)147 0588 235
9.0	.(1)526 3157 895	.(2)175 4385 965	34.0	.(1)144 9275 362
9.5	.(1)500 0000 000	.(2)150 3759 398	34.5	.(1)142 8571 429
10.0	.(1)476 1904 762	.(2)129 8701 299	35.0	.(1)140 8450 704
10.5	.(1)454 5454 545	.(2)112 9305 477	35.5	.(1)138 8888 889
11.0	.(1)434 7826 087	.(3)988 1422 925	36.0	.(1)136 9863 014
11.5	.(1)416 6666 667	.(3)869 5652 174	36.5	.(1)135 1351 351
12.0	.(1)400 0000 000	.(3)769 2307 692	37.0	.(1)133 3333 333
12.5	.(1)384 6153 846	.(3)683 7606 838	37.5	.(1)131 5789 474
13.0	.(1)370 3703 704	.(3)610 5006 105	38.0	.(1)129 8701 299
13.5	.(1)357 1428 571	.(3)547 3453 749	38.5	.(1)128 2051 282
14.0	.(1)344 8275 862	.(3)492 6108 374	39.0	.(1)126 5822 785
14.5	.(1)333 3333 333	.(3)444 9388 209	39.5	.(1)125 0000 000
15.0	.(1)322 5806 452	.(3)403 2258 064	40.0	.(1)123 4567 901
15.5	.(1)312 5000 000	.(3)366 5689 150	40.5	.(1)121 9512 195
16.0	.(1)303 0303 030	.(3)334 2245 989	41.0	.(1)120 4819 277
16.5	.(1)294 1176 471	.(3)305 5767 762	41.5	.(1)119 0476 190
17.0	.(1)285 7142 857	.(3)280 1120 448	42.0	.(1)117 6470 588
17.5	.(1)277 7777 778	.(3)257 4002 574	42.5	.(1)116 2790 698
18.0	.(1)270 2702 703	.(3)237 0791 844	43.0	.(1)114 9425 287
18.5	.(1)263 1578 947	.(3)218 8423 241	43.5	.(1)113 6363 636
19.0	.(1)256 4102 564	.(3)202 4291 498	44.0	.(1)112 3595 506
19.5	.(1)250 0000 000	.(3)187 6172 608	44.5	.(1)111 1111 111
20.0	.(1)243 9024 390	.(3)174 2160 279	45.0	.(1)109 8901 099
20.5	.(1)238 0952 381	.(3)162 0614 213	45.5	.(1)108 6956 522
21.0	.(1)232 5581 395	.(3)151 0117 789	46.0	.(1)107 5268 817
21.5	.(1)227 2727 273	.(3)140 9443 270	46.5	.(1)106 3829 787
22.0	.(1)222 2222 222	.(3)131 7523 057	47.0	.(1)105 2631 579
22.5	.(1)217 3913 043	.(3)123 3425 840	47.5	.(1)104 1666 667
23.0	.(1)212 7659 574	.(3)115 6336 725	48.0	.(1)103 0927 835
23.5	.(1)208 3333 333	.(3)108 5540 599	48.5	.(1)102 0408 163
24.0	.(1)204 0816 327	.(3)102 0408 163	49.0	.(1)101 0101 010
24.5	.(1)200 0000 000	.(4)960 3841 537	49.5	.(1)100 0000 000
25.0	.(1)196 0784 314	.(4)904 9773 756	50.0	.(2)990 0990 099

p	A'	p	A	A'
25.5	.(4)853 7522 411	50.5	.(2)980 3921 569	.(4)113 0895 500
26.0	.(4)806 3215 610	51.0	.(2)970 8737 864	.(4)109 8273 514
26.5	.(4)762 3403 850	51.5	.(2)961 5384 615	.(4)106 6894 271
27.0	.(4)721 5007 215	52.0	.(2)952 3809 524	.(4)103 6699 150
27.5	.(4)683 5269 993	52.5	.(2)943 3962 264	.(4)100 7632 819
28.0	.(4)648 1721 545	53.0	.(2)934 5794 393	.(5)979 6430 181
28.5	.(4)615 2142 484	53.5	.(2)925 9259 259	.(5)952 6803 662
29.0	.(4)584 4535 359	54.0	.(2)917 4311 927	.(5)926 6981 744
29.5	.(4)555 7099 194	54.5	.(2)909 0909 091	.(5)901 6522 778
30.0	.(4)528 8207 298	55.0	.(2)900 9009 009	.(5)877 5008 775
30.5	.(4)503 6387 903	55.5	.(2)892 8571 429	.(5)854 2043 940
31.0	.(4)480 0307 220	56.0	.(2)884 9557 522	.(5)831 7253 310
31.5	.(4)457 8754 579	56.5	.(2)877 1929 825	.(5)810 0281 485
32.0	.(4)437 0629 371	57.0	.(2)869 5652 174	.(5)789 0791 446
32.5	.(4)417 4929 548	57.5	.(2)862 0689 655	.(5)768 8463 461
33.0	.(4)399 0741 480	58.0	.(2)854 7008 547	.(5)749 2994 051
33.5	.(4)381 7230 981	58.5	.(2)847 4576 271	.(5)730 4095 041
34.0	.(4)365 3635 367	59.0	.(2)840 3361 345	.(5)712 1492 665
34.5	.(4)349 9256 408	59.5	.(2)833 3333 333	.(5)694 4926 731
35.0	.(4)335 3454 058	60.0	.(2)826 4462 810	.(5)677 4149 844
35.5	.(4)321 5640 877	60.5	.(2)819 6721 311	.(5)660 8926 677
36.0	.(4)308 5277 058	61.0	.(2)813 0081 301	.(5)644 9033 290
36.5	.(4)296 1865 976	61.5	.(2)806 4516 129	.(5)629 4256 491
37.0	.(4)284 4950 213	62.0	.(2)800 0000 000	.(5)614 4393 241
37.5	.(4)273 4107 997	62.5	.(2)793 6507 937	.(5)599 9250 094
38.0	.(4)262 8949 997	63.0	.(2)787 4015 748	.(5)585 8642 670
38.5	.(4)252 9116 453	63.5	.(2)781 2500 000	.(5)572 2395 166
39.0	.(4)243 4274 586	64.0	.(2)775 1937 984	.(5)559 0339 893
39.5	.(4)234 4116 268	64.5	.(2)769 2307 692	.(5)546 2316 842
40.0	.(4)225 8355 917	65.0	.(2)763 3587 786	.(5)533 8173 277
40.5	.(4)217 6728 595	65.5	.(2)757 5757 576	.(5)521 7763 354
41.0	.(4)209 8988 288	66.0	.(2)751 8796 992	.(5)510 0947 756
41.5	.(4)202 4906 348	66.5	.(2)746 2686 567	.(5)498 7593 361
42.0	.(4)195 4270 080	67.0	.(2)740 7407 407	.(5)487 7572 920
42.5	.(4)188 6881 457	67.5	.(2)735 2941 176	.(5)477 0764 754
43.0	.(4)182 2555 952	68.0	.(2)729 9270 073	.(5)466 7052 476
43.5	.(4)176 1121 482	68.5	.(2)724 6376 812	.(5)456 6324 725
44.0	.(4)170 2417 433	69.0	.(2)719 4244 604	.(5)446 8474 910
44.5	.(4)164 6293 781	69.5	.(2)714 2857 143	.(5)437 3400 975
45.0	.(4)159 2610 288	70.0	.(2)709 2198 582	.(5)428 1005 180
45.5	.(4)154 1235 763	70.5	.(2)704 2253 521	.(5)419 1193 883
46.0	.(4)149 2047 387	71.0	.(2)699 3006 993	.(5)410 3877 343
46.5	.(4)144 4930 102	71.5	.(2)694 4444 444	.(5)401 8969 536
47.0	.(4)139 9776 036	72.0	.(2)689 6551 724	.(5)393 6387 970
47.5	.(4)135 6483 993	72.5	.(2)684 9315 068	.(5)385 6053 522
48.0	.(4)131 4958 973	73.0	.(2)680 2721 088	.(5)377 7890 275
48.5	.(4)127 5111 732	73.5	.(2)675 6756 757	.(5)370 1825 370
49.0	.(4)123 6858 380	74.0	.(2)671 1409 396	.(5)362 7788 863
49.5	.(4)120 0120 012	74.5	.(2)666 6666 667	.(5)355 5713 587
50.0	.(4)116 4822 365	75.0	.(2)662 2516 556	.(5)348 5535 030

p	A	A′
75.5	.(2)657 8947 368	.(5)341 7191 206
76.0	.(2)653 5947 712	.(5)335 0622 546
76.5	.(2)649 3506 494	.(5)328 5771 787
77.0	.(2)645 1612 903	.(5)322 2583 868
77.5	.(2)641 0256 410	.(5)316 1005 832
78.0	.(2)636 9426 752	.(5)310 0986 734
78.5	.(2)632 9113 924	.(5)304 2477 550
79.0	.(2)628 9308 176	.(5)298 5431 096
79.5	.(2)625 0000 000	.(5)292 9801 945
80.0	.(2)621 1180 124	.(5)287 5546 354
80.5	.(2)617 2839 506	.(5)282 2622 188
81.0	.(2)613 4969 325	.(5)277 0988 855
81.5	.(2)609 7560 976	.(5)272 0607 240
82.0	.(2)606 0606 061	.(5)267 1439 639
82.5	.(2)602 4096 386	.(5)262 3449 705
83.0	.(2)598 8023 952	.(5)257 6602 389
83.5	.(2)595 2380 952	.(5)253 0863 885
84.0	.(2)591 7159 763	.(5)248 6201 581
84.5	.(2)588 2352 941	.(5)244 2584 010
85.0	.(2)584 7953 216	.(5)239 9980 800
85.5	.(2)581 3953 488	.(5)235 8362 636
86.0	.(2)578 0346 821	.(5)231 7701 211
86.5	.(2)574 7126 437	.(5)227 7969 190
87.0	.(2)571 4285 714	.(5)223 9140 170
87.5	.(2)568 1818 182	.(5)220 1188 642
88.0	.(2)564 9717 514	.(5)216 4089 957
88.5	.(2)561 7977 528	.(5)212 7820 293
89.0	.(2)558 6592 179	.(5)209 2356 621
89.5	.(2)555 5555 556	.(5)205 7676 677
90.0	.(2)552 4861 878	.(5)202 3758 930
90.5	.(2)549 4505 495	.(5)199 0582 554
91.0	.(2)546 4480 874	.(5)195 8127 404
91.5	.(2)543 4782 609	.(5)192 6373 986
92.0	.(2)540 5405 405	.(5)189 5303 438
92.5	.(2)537 6344 086	.(5)186 4897 501
93.0	.(2)534 7593 583	.(5)183 5138 498
93.5	.(2)531 9148 936	.(5)180 6009 315
94.0	.(2)529 1005 291	.(5)177 7493 379
94.5	.(2)526 3157 895	.(5)174 9574 635
95.0	.(2)523 5602 094	.(5)172 2237 531
95.5	.(2)520 8333 333	.(5)169 5466 999
96.0	.(2)518 1347 150	.(5)166 9248 438
96.5	.(2)515 4639 175	.(5)164 3567 692
97.0	.(2)512 8205 128	.(5)161 8411 044
97.5	.(2)510 2040 816	.(5)159 3765 191
98.0	.(2)507 6142 132	.(5)156 9617 233
98.5	.(2)505 0505 051	.(5)154 5954 662
99.0	.(2)502 5125 628	.(5)152 2765 342
99.5	.(2)500 0000 000	.(5)150 0037 501
100.0	.(2)497 5124 378	.(5)147 7759 716

p	A	A'	p
101	.(2)492 6108 374	.(5)143 4510 301	101
102	.(2)487 8048 780	.(5)139 2932 262	102
103	.(2)483 0917 874	.(5)135 2945 633	103
104	.(2)478 4688 995	.(5)131 4474 999	104
105	.(2)473 9336 493	.(5)127 7449 189	105
106	.(2)469 4835 681	.(5)124 1801 009	106
107	.(2)465 1162 791	.(5)120 7466 976	107
108	.(2)460 8294 931	.(5)117 4387 087	108
109	.(2)456 6210 046	.(5)114 2504 599	109
110	.(2)452 4886 878	.(5)111 1765 818	110
111	.(2)448 4304 933	.(5)108 2119 916	111
112	.(2)444 4444 444	.(5)105 3518 753	112
113	.(2)440 5286 344	.(5)102 5916 708	113
114	.(2)436 6812 227	.(6)999 2705 325	114
115	.(2)432 9004 329	.(6)973 5392 044	115
116	.(2)429 1845 494	.(6)948 6837 961	116
117	.(2)425 5319 149	.(6)924 6673 509	117
118	.(2)421 9409 283	.(6)901 4547 677	118
119	.(2)418 4100 418	.(6)879 0126 929	119
120	.(2)414 9377 593	.(6)857 3094 201	120
121	.(2)411 5226 337	.(6)836 3147 956	121
122	.(2)408 1632 653	.(6)816 0001 306	122
123	.(2)404 8582 996	.(6)796 3381 188	123
124	.(2)401 6064 257	.(6)777 3027 594	124
125	.(2)398 4063 745	.(6)758 8692 848	125
126	.(2)395 2569 170	.(6)741 0140 926	126
127	.(2)392 1568 627	.(6)723 7146 827	127
128	.(2)389 1050 584	.(6)706 9495 973	128
129	.(2)386 1003 861	.(6)690 6983 651	129
130	.(2)383 1417 625	.(6)674 9414 488	130
131	.(2)380 2281 369	.(6)659 6601 958	131
132	.(2)377 3584 906	.(6)644 8367 918	132
133	.(2)374 5318 352	.(6)630 4542 170	133
134	.(2)371 7472 119	.(6)616 4962 055	134
135	.(2)369 0036 900	.(6)602 9472 059	135
136	.(2)366 3003 663	.(6)589 7923 459	136
137	.(2)363 6363 636	.(6)577 0173 971	137
138	.(2)361 0108 303	.(6)564 6087 431	138
139	.(2)358 4229 391	.(6)552 5533 490	139
140	.(2)355 8718 861	.(6)540 8387 327	140
141	.(2)353 3568 905	.(6)529 4529 375	141
142	.(2)350 8771 930	.(6)518 3845 065	142
143	.(2)348 4320 558	.(6)507 6224 588	143
144	.(2)346 0207 612	.(6)497 1562 662	144
145	.(2)343 6426 117	.(6)486 9758 314	145
146	.(2)341 2969 283	.(6)477 0714 682	146
147	.(2)338 9830 508	.(6)467 4338 815	147
148	.(2)336 7003 367	.(6)458 0541 493	148
149	.(2)334 4481 605	.(6)448 9237 054	149
150	.(2)332 2259 136	.(6)440 0343 227	150

TABLE 41

Coefficients for the Parabola for the Range from $x = -p$ to $x = +p$

Description: The coefficients A, B, C, are computed to ten significant figures for the range $p = 1.0$ to $p = 100.0$ by increments of .5.

The parabola,

$$y = a_0 + a_1 x + a_2 x^2 \ ,$$

is obtained from the equations,

$$a_0 = A\,M_0 + B\,M_2 \ ,$$
$$a_2 = B\,M_0 + C\,M_2 \ ,$$

combined with the value for a_1 given by the tables for the straight line. The moments, M_0, M_1, M_2 are computed over the range $x = -p$

(The numbers in parentheses denote the number of ciphers between the decimal point and the first significant figure.)

p	A	B	C
1.0	1.000 0000 000	—1.000 0000 000	1.500 0000 000
1.5	.640 6250 000	—.312 5000 000	.250 0000 000
2.0	.485 7142 857	—.142 8571 429	.(1)714 2857 143
2.5	.394 5312 500	—.(1)781 2500 000	.(1)267 8571 429
3.0	.333 3333 333	—.(1)476 1904 762	.(1)119 0476 190
3.5	.289 0625 000	—.(1)312 5000 000	.(2)595 2380 952
4.0	.255 4112 554	—.(1)216 4502 165	.(2)324 6753 247
4.5	.228 9062 500	—.(1)156 2500 000	.(2)189 3939 394
5.0	.207 4592 075	—.(1)116 5501 166	.(2)116 5501 166
5.5	.189 7321 429	—.(2)892 8571 429	.(3)749 2507 493
6.0	.174 8251 748	—.(2)699 3006 993	.(3)499 5004 995
6.5	.162 1093 750	—.(2)558 0357 143	.(3)343 4065 934
7.0	.151 1312 217	—.(2)452 4886 878	.(3)242 4046 542
7.5	.141 5550 595	—.(2)372 0238 095	.(3)175 0700 280
8.0	.133 1269 350	—.(2)309 5975 232	.(3)128 9989 680
8.5	.125 6510 417	—.(2)260 4166 667	.(4)967 4922 601
9.0	.118 9739 054	—.(2)221 1410 880	.(4)737 1369 601
9.5	.112 9734 848	—.(2)189 3939 394	.(4)569 6058 328
10.0	.107 5514 874	—.(2)163 4521 085	.(4)445 7784 778
10.5	.102 6278 409	—.(2)142 0454 545	.(4)352 9079 616
11.0	.(1)981 3664 596	—.(2)124 2236 025	.(4)282 3263 693
11.5	.(1)940 2316 434	—.(2)109 2657 343	.(4)228 0328 367
12.0	.(1)902 4154 589	—.(3)966 1835 749	.(4)185 8045 336
12.5	.(1)867 5309 066	—.(3)858 5164 835	.(4)152 6251 526
13.0	.(1)835 2490 421	—.(3)766 2835 249	.(4)126 3104 711
13.5	.(1)805 2884 615	—.(3)686 8131 868	.(4)105 2587 260
14.0	.(1)777 4069 954	—.(3)617 9705 846	.(5)882 8151 209
14.5	.(1)751 3950 893	—.(3)558 0357 143	.(5)744 8752 582
15.0	.(1)727 0704 824	—.(3)505 6122 965	.(5)632 0153 706
15.5	.(1)704 2738 971	—.(3)459 5588 235	.(5)539 0719 338
16.0	.(1)682 8655 216	—.(3)418 9359 028	.(5)462 0616 575
16.5	.(1)662 7221 201	—.(3)382 9656 863	.(5)397 8864 273
17.0	.(1)643 7346 437	—.(3)351 0003 510	.(5)344 1179 912
17.5	.(1)625 8062 436	—.(3)322 4974 200	.(5)298 8393 081
18.0	.(1)608 8506 089	—.(3)297 0002 970	.(5)260 5265 763
18.5	.(1)592 7905 702	—.(3)274 1228 070	.(5)227 9607 543
19.0	.(1)577 5569 190	—.(3)253 5368 389	.(5)200 1606 623
19.5	.(1)563 0874 060	—.(3)234 9624 060	.(5)176 3320 120
20.0	.(1)549 3258 868	—.(3)218 1596 056	.(5)155 8282 897
20.5	.(1)536 2215 909	—.(3)202 9220 779	.(5)138 1205 295
21.0	.(1)523 7284 931	—.(3)189 0716 582	.(5)122 7738 040
21.5	.(1)511 8047 713	—.(3)176 4539 808	.(5)109 4288 253
22.0	.(1)500 4123 371	—.(3)164 9348 507	.(6)977 8746 091
22.5	.(1)489 5164 279	—.(3)154 3972 332	.(6)876 0126 706
23.0	.(1)479 0852 511	—.(3)144 7387 466	.(6)785 2017 048
23.5	.(1)469 0896 739	—.(3)135 8695 652	.(6)707 9612 604
24.0	.(1)459 5029 501	—.(3)127 7106 587	.(6)638 5532 937
24.5	.(1)450 3004 808	—.(3)120 1923 077	.(6)577 1539 385
25.0	.(1)441 4596 027	—.(3)113 2528 483	.(6)522 7054 537

p	A	B	C
25.5	.(1)432 9594 017	—.(3)106 8376 068	.(6)474 3068 006
26.0	.(1)424 7805 469	—.(3)100 8979 921	.(6)431 1880 006
26.5	.(1)416 9051 435	—.(4)953 9072 039	.(6)392 6890 719
27.0	.(1)409 3166 020	—.(4)902 7715 085	.(6)358 2426 621
27.5	.(1)401 9995 211	—.(4)855 2271 483	.(6)327 3596 740
28.0	.(1)394 9395 832	—.(4)810 9642 365	.(6)299 6173 287
28.5	.(1)388 1234 606	—.(4)769 7044 335	.(6)274 6492 180
29.0	.(1)381 5387 315	—.(4)731 1972 624	.(6)252 1369 870
29.5	.(1)375 1738 042	—.(4)695 2169 077	.(6)231 8033 590
30.0	.(1)369 0178 489	—.(4)661 5594 279	.(6)213 4062 671
30.5	.(1)363 0607 359	—.(4)630 0403 226	.(6)196 7339 024
31.0	.(1)357 2929 802	—.(4)600 4924 038	.(6)181 6005 253
31.5	.(1)351 7056 910	—.(4)572 7639 296	.(6)167 8429 098
32.0	.(1)346 2905 254	—.(4)546 7169 646	.(6)155 3173 195
32.5	.(1)341 0396 474	—.(4)522 2259 358	.(6)143 8969 284
33.0	.(1)335 9456 896	—.(4)499 1763 590	.(6)133 4696 147
33.5	.(1)331 0017 189	—.(4)477 4637 128	.(6)123 9360 708
34.0	.(1)326 2012 046	—.(4)456 9924 414	.(6)115 2081 785
34.5	.(1)321 5379 902	—.(4)437 6750 700	.(6)107 2076 105
35.0	.(1)317 0062 663	—.(4)419 4314 188	.(7)998 6462 352
35.5	.(1)312 6005 470	—.(4)402 1879 022	.(7)931 1701 382
36.0	.(1)308 3156 473	—.(4)385 8769 053	.(7)869 0921 290
36.5	.(1)304 1466 631	—.(4)370 4362 257	.(7)811 9150 152
37.0	.(1)300 0889 521	—.(4)355 8085 750	.(7)759 1932 610
37.5	.(1)296 1381 169	—.(4)341 9411 314	.(7)710 5270 263
38.0	.(1)292 2899 885	—.(4)328 7851 389	.(7)665 5569 614
38.5	.(1)288 5406 123	—.(4)316 2955 466	.(7)623 9596 513
39.0	.(1)284 8862 343	—.(4)304 4306 842	.(7)585 4436 234
39.5	.(1)281 3232 880	—.(4)293 1519 700	.(7)550 4855 968
40.0	.(1)277 8483 837	—.(4)282 4236 468	.(7)516 6286 221
40.5	.(1)274 4582 970	—.(4)272 2125 436	.(7)485 8769 184
41.0	.(1)271 1499 593	—.(4)262 4878 599	.(7)457 2959 232
41.5	.(1)267 9204 481	—.(4)253 2209 708	.(7)430 7052 407
42.0	.(1)264 7669 787	—.(4)244 3852 489	.(7)405 9555 630
42.5	.(1)261 6868 960	—.(4)235 9559 046	.(7)382 8899 060
43.0	.(1)258 6776 671	—.(4)227 9098 389	.(7)361 3792 371
43.5	.(1)255 7368 746	—.(4)220 2255 109	.(7)341 3026 128
44.0	.(1)252 8622 095	—.(4)212 8828 165	.(7)322 5542 998
44.5	.(1)250 0514 657	—.(4)205 8629 776	.(7)305 0198 458
45.0	.(1)247 3025 344	—.(4)199 1484 413	.(7)288 6209 294
45.5	.(1)244 6133 981	—.(4)192 7227 875	.(7)273 2687 523
46.0	.(1)241 9821 265	—.(4)186 5706 450	.(7)258 8861 864
46.5	.(1)239 4068 715	—.(4)180 6776 133	.(7)245 4025 308
47.0	.(1)236 8858 628	—.(4)175 0301 927	.(7)232 7529 158
47.5	.(1)234 4174 039	—.(4)169 6157 186	.(7)220 8777 671
48.0	.(1)231 9998 685	—.(4)164 4223 022	.(7)209 7223 243
48.5	.(1)229 6316 964	—.(4)159 4387 755	.(7)199 2362 081
49.0	.(1)227 3113 909	—.(4)154 6546 407	.(7)189 3730 295
49.5	.(1)225 0375 150	—.(4)150 0600 240	.(7)180 0900 378
50.0	.(1)222 8086 886	—.(4)145 6456 325	.(7)171 3478 030

p	A	B	C
50.5	.(1)220 6235 860	—.(4)141 4027 149	.(7)163 1099 278
51.0	.(1)218 4809 327	—.(4)137 3230 250	.(7)155 3427 884
51.5	.(1)216 3795 035	—.(4)133 3987 877	.(7)148 0152 984
52.0	.(1)214 3181 200	—.(4)129 6226 684	.(7)141 0986 957
52.5	.(1)212 2956 479	—.(4)125 9877 439	.(7)134 5663 486
53.0	.(1)210 3109 957	—.(4)122 4874 757	.(7)128 3935 804
53.5	.(1)208 3631 123	—.(4)119 1156 852	.(7)122 5575 085
54.0	.(1)206 4509 850	—.(4)115 8665 310	.(7)117 0369 000
54.5	.(1)204 5736 382	—.(4)112 7344 877	.(7)111 8120 384
55.0	.(1)202 7301 313	—.(4)109 7143 258	.(7)106 8646 031
55.5	.(1)200 9195 574	—.(4)106 8010 936	.(7)102 1775 591
56.0	.(1)199 1410 418	—.(4)103 9901 001	.(8)977 3505 652
56.5	.(1)197 3937 403	—.(4)101 2768 991	.(8)935 2233 857
57.0	.(1)195 6768 382	—.(5)986 5727 449	.(8)895 2565 744
57.5	.(1)193 9895 490	—.(5)961 2722 631	.(8)857 3219 737
58.0	.(1)192 3311 130	—.(5)936 8295 813	.(8)821 3000 421
58.5	.(1)190 7007 963	—.(5)913 2086 499	.(8)787 0792 070
59.0	.(1)189 0978 896	—.(5)890 3752 219	.(8)754 5552 728
59.5	.(1)187 5217 074	—.(5)868 2967 491	.(8)723 6308 764
60.0	.(1)185 9715 868	—.(5)846 9422 843	.(8)694 2149 871
60.5	.(1)184 4468 866	—.(5)826 2823 903	.(8)666 2224 473
61.0	.(1)182 9469 865	—.(5)806 2890 546	.(8)639 5735 494
61.5	.(1)181 4712 863	—.(5)786 9356 098	.(8)614 1936 467
62.0	.(1)180 0192 049	—.(5)768 1966 583	.(8)590 0127 944
62.5	.(1)178 5901 798	—.(5)750 0480 031	.(8)566 9654 196
63.0	.(1)177 1836 660	—.(5)732 4665 812	.(8)544 9900 158
63.5	.(1)175 7991 358	—.(5)715 4304 029	.(8)524 0288 613
64.0	.(1)174 4360 776	—.(5)698 9184 935	.(8)504 0277 597
64.5	.(1)173 0939 958	—.(5)682 9108 392	.(8)484 9357 992
65.0	.(1)171 7724 099	—.(5)667 3883 359	.(8)466 7051 300
65.5	.(1)170 4708 538	—.(5)652 3327 419	.(8)449 2907 595
66.0	.(1)169 1888 755	—.(5)637 7266 321	.(8)432 6503 610
66.5	.(1)167 9260 366	—.(5)623 5533 562	.(8)416 7440 977
67.0	.(1)166 6819 116	—.(5)609 7969 986	.(8)401 5344 591
67.5	.(1)165 4560 875	—.(5)596 4423 407	.(8)386 9861 091
68.0	.(1)164 2481 635	—.(5)583 4748 260	.(8)373 0657 455
68.5	.(1)163 0577 503	—.(5)570 8805 261	.(8)359 7419 689
69.0	.(1)161 8844 697	—.(5)558 6461 100	.(8)346 9851 615
69.5	.(1)160 7279 547	—.(5)546 7588 138	.(8)334 7673 741
70.0	.(1)159 5878 482	—.(5)535 2064 131	.(8)323 0622 212
70.5	.(1)158 4638 037	—.(5)523 9771 965	.(8)311 8447 829
71.0	.(1)157 3554 838	—.(5)513 0599 408	.(8)301 0915 145
71.5	.(1)156 2625 611	—.(5)502 4438 871	.(8)290 7801 613
72.0	.(1)155 1847 168	—.(5)492 1187 187	.(8)280 8896 796
72.5	.(1)154 1216 409	—.(5)482 0745 403	.(8)271 4001 634
73.0	.(1)153 0730 320	—.(5)472 3018 576	.(8)262 2927 754
73.5	.(1)152 0385 968	—.(5)462 7915 587	.(8)253 5496 829
74.0	.(1)151 0180 498	—.(5)453 5348 963	.(8)245 1539 980
74.5	.(1)150 0111 131	—.(5)444 5234 708	.(8)237 0897 218
75.0	.(1)149 0175 162	—.(5)435 7492 141	.(8)229 3416 916

p	A	B	C
75.5	.(1)148 0369 959	—.(5)427 2043 746	.(8)221 8955 328
76.0	.(1)147 0692 956	—.(5)418 8815 026	.(8)214 7376 124
76.5	.(1)146 1141 654	—.(5)410 7734 371	.(8)207 8549 966
77.0	.(1)145 1713 622	—.(5)402 8732 923	.(8)201 2354 107
77.5	.(1)144 2406 486	—.(5)395 1744 458	.(8)194 8672 015
78.0	.(1)143 3217 937	—.(5)387 6705 266	.(8)188 7393 021
78.5	.(1)142 4145 722	—.(5)380 3554 041	.(8)182 8411 989
79.0	.(1)141 5187 645	—.(5)373 2231 778	.(8)177 1629 008
79.5	.(1)140 6341 567	—.(5)366 2681 669	.(8)171 6949 101
80.0	.(1)139 7605 399	—.(5)359 4849 013	.(8)166 4281 950
80.5	.(1)138 8977 106	—.(5)352 8681 120	.(8)161 3541 647
81.0	.(1)138 0454 701	—.(5)346 4127 230	.(8)156 4646 446
81.5	.(1)137 2036 248	—.(5)340 1138 429	.(8)151 7518 541
82.0	.(1)136 3719 855	—.(5)333 9667 569	.(8)147 2083 854
82.5	.(1)135 5503 678	—.(5)327 9669 199	.(8)142 8271 834
83.0	.(1)134 7385 917	—.(5)322 1099 490	.(8)138 6015 271
83.5	.(1)133 9364 812	—.(5)316 3916 169	.(8)134 5250 116
84.0	.(1)133 1438 649	—.(5)310 8078 455	.(8)130 5915 317
84.5	.(1)132 3605 750	—.(5)305 3547 000	.(8)126 7952 663
85.0	.(1)131 5864 481	—.(5)300 0283 827	.(8)123 1306 632
85.5	.(1)130 8213 241	—.(5)294 8252 276	.(8)119 5924 258
86.0	.(1)130 0650 470	—.(5)289 7416 953	.(8)116 1754 993
86.5	.(1)129 3174 642	—.(5)284 7743 676	.(8)112 8750 590
87.0	.(1)128 5784 266	—.(5)279 9199 429	.(8)109 6864 980
87.5	.(1)127 8477 885	—.(5)275 1752 316	.(8)106 6054 166
88.0	.(1)127 1254 075	—.(5)270 5371 515	.(8)103 6276 117
88.5	.(1)126 4111 445	—.(5)266 0027 239	.(8)100 7490 669
89.0	.(1)125 7048 632	—.(5)261 5690 691	.(9)979 6594 350
89.5	.(1)125 0064 308	—.(5)257 2334 033	.(9)952 7457 143
90.0	.(1)124 3157 171	—.(5)252 9930 341	.(9)926 7144 106
90.5	.(1)123 6325 948	—.(5)248 8453 575	.(9)901 5319 538
91.0	.(1)122 9569 394	—.(5)244 7878 546	.(9)877 1662 253
91.5	.(1)122 2886 291	—.(5)240 8180 879	.(9)853 5864 881
92.0	.(1)121 6275 450	—.(5)236 9336 988	.(9)830 7633 199
92.5	.(1)120 9735 702	—.(5)233 1324 043	.(9)808 6685 508
93.0	.(1)120 3265 909	—.(5)229 4119 941	.(9)787 2752 029
93.5	.(1)119 6864 953	—.(5)225 7703 284	.(9)766 5574 344
94.0	.(1)119 0531 742	—.(5)222 2053 346	.(9)746 4904 858
94.5	.(1)118 4265 205	—.(5)218 7150 056	.(9)727 0506 294
95.0	.(1)117 8064 296	—.(5)215 2973 968	.(9)708 2151 209
95.5	.(1)117 1927 988	—.(5)211 9506 240	.(9)689 9621 539
96.0	.(1)116 5855 277	—.(5)208 6728 615	.(9)672 2708 166
96.5	.(1)115 9845 180	—.(5)205 4623 396	.(9)655 1210 509
97.0	.(1)115 3896 733	—.(5)202 3173 428	.(9)638 4936 130
97.5	.(1)114 8008 993	—.(5)199 2362 081	.(9)622 3700 369
98.0	.(1)114 2181 034	—.(5)196 2173 225	.(9)606 7325 988
98.5	.(1)113 6411 951	—.(5)193 2591 218	.(9)591 5642 838
99.0	.(1)113 0700 856	—.(5)190 3600 889	.(9)576 8487 544
99.5	.(1)112 5046 880	—.(5)187 5187 519	.(9)562 5703 199
100.0	.(1)111 9449 168	—.(5)184 7336 824	.(9)548 7139 081

TABLE 42

Coefficients for the Cubic for the Range from

$$x = -p \text{ to } x = +p$$

Description: The coefficients A', B', C' are computed to ten significant figures for the range $p = 1.5$ to $p = 50.0$ by increments of .5.

The cubic,

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ ,$$

is obtained from the equations,

$$a_1 = A' M_1 + B' M_3 \ ,$$

$$a_3 = B' M_1 + C' M_3 \ ,$$

combined with the values for a_0, a_2 given by the tables for the parabola. The moments, M_0, M_1, M_2, M_3, are computed for the range $x = -p$ to $x = +p$.

(The numbers in parentheses denote the number of ciphers between the decimal point and the first significant figure.)

p	A'	B'	C'
1.5	2.534 7222 222	--1.138 8888 889	.555 5555 556
2.0	.902 7777 778	—.236 1111 111	.(1)694 4444 444
2.5	.450 6999 559	—.(1)779 3209 877	.(1)154 3209 877
3.0	.262 5661 376	—.(1)324 0740 741	.(2)462 9629 630
3.5	.167 8541 366	—.(1)155 7239 057	.(2)168 3501 684
4.0	.114 3378 227	—.(2)827 7216 611	.(3)701 4590 348
4.5	.(1)816 0531 598	—.(2)474 2942 243	.(3)323 7503 238
5.0	.(1)603 7943 538	—.(2)288 1377 881	.(3)161 8751 619
5.5	.(1)459 7794 181	—.(2)183 4585 167	.(4)863 3341 967
6.0	.(1)358 4609 835	—.(2)121 4063 714	.(4)485 6254 856
6.5	.(1)285 0269 624	—.(3)829 8482 563	.(4)285 6620 504
7.0	.(1)230 4589 927	—.(3)583 0679 850	.(4)174 5712 530
7.5	.(1)189 0399 941	—.(3)419 5222 848	.(4)110 2555 282
8.0	.(1)157 0204 105	—.(3)308 1642 014	.(5)716 6609 334
8.5	.(1)131 8685 978	—.(3)230 5259 336	.(5)477 7739 556
9.0	.(1)111 8316 811	—.(3)175 2561 737	.(5)325 7549 697
9.5	.(2)956 6897 397	—.(3)135 1741 492	.(5)226 6121 528
10.0	.(2)824 8507 009	—.(3)105 6201 476	.(5)160 5169 416
10.5	.(2)716 2246 443	—.(4)835 0091 302	.(5)115 5721 980
11.0	.(2)625 9079 085	—.(4)667 2071 889	.(6)844 5660 620
11.5	.(2)550 1917 791	—.(4)538 3326 639	.(6)625 6044 903
12.0	.(2)486 2354 500	—.(4)438 2359 455	.(6)469 2033 678
12.5	.(2)431 8375 890	—.(4)359 6848 299	.(6)355 9473 824
13.0	.(2)385 2742 263	—.(4)297 4533 626	.(6)272 8929 932
13.5	.(2)345 1820 327	—.(4)247 7164 138	.(6)211 2719 947
14.0	.(2)310 4731 565	—.(4)207 6407 573	.(6)165 0562 459
14.5	.(2)280 2723 203	—.(4)175 1046 701	.(6)130 0443 149
15.0	.(2)253 8698 342	—.(4)148 5029 580	.(6)103 2704 854
15.5	.(2)230 6861 266	—.(4)126 6096 151	.(7)826 1638 832
16.0	.(2)210 2447 093	—.(4)108 4799 077	.(7)665 5209 059
16.5	.(2)192 1513 833	—.(5)933 7977 791	.(7)539 6115 453
17.0	.(2)176 0781 076	—.(5)807 3440 736	.(7)440 2094 185
17.5	.(2)161 7503 875	—.(5)700 9036 937	.(7)361 1974 716
18.0	.(2)148 9373 393	—.(5)610 8752 239	.(7)297 9879 141
18.5	.(2)137 4438 092	—.(5)534 3795 459	.(7)247 1119 288
19.0	.(2)127 1040 798	—.(5)469 1008 114	.(7)205 9266 073
19.5	.(2)117 7768 137	—.(5)413 1653 981	.(7)172 4036 712
20.0	.(2)109 3409 664	—.(5)365 0491 008	.(7)144 9758 144
20.5	(2)101 6924 641	—.(5)323 5054 757	.(7)122 4240 211
21.0	.(3)947 4149 003	—.(5)287 5101 521	.(7)103 7942 787
21.5	.(3)884 1025 513	—.(5)256 2172 813	.(8)883 3555 638
22.0	.(3)826 3115 900	—.(5)228 9252 750	.(8)754 5328 774
22.5	.(3)773 4526 546	—.(5)205 0496 990	.(8)646 7424 663
23.0	.(3)725 0103 342	—.(5)184 1017 105	.(8)556 1985 210
23.5	.(3)680 5325 601	—.(5)165 6708 183	.(8)479 8575 476
24.0	.(3)639 6217 018	—.(5)149 4110 299	.(8)415 2613 392
24.5	.(3)601 9270 658	—.(5)135 0296 678	.(8)360 4155 020
25.0	.(3)567 1385 543	—.(5)122 2783 008	.(8)313 6949 739

p	A'	B'	C'
25.5	.(3)534 9812 832	−.(5)110 9453 570	.(8)273 7701 591
26.0	.(3)505 2110 028	−.(5)100 8500 824	.(8)239 5488 892
26.5	.(3)477 6101 850	−.(6)918 3758 072	.(8)210 1306 046
27.0	.(3)451 9846 731	−.(6)837 7472 451	.(8)184 7700 144
27.5	.(3)428 1608 021	−.(6)765 4677 208	.(8)162 8481 482
28.0	.(3)405 9829 189	−.(6)700 5455 924	.(8)143 8491 976
28.5	.(3)385 3112 389	−.(6)642 1215 945	.(8)127 3419 126
29.0	.(3)366 0199 892	−.(6)589 4492 824	.(8)112 9645 999
29.5	.(3)347 9957 969	−.(6)541 8786 342	.(8)100 4129 777
30.0	.(3)331 1362 854	−.(6)498 8422 596	.(9)894 3030 827
30.5	.(3)315 3488 491	−.(6)459 8437 659	.(9)797 9935 200
31.0	.(3)300 5495 828	−.(6)424 4479 170	.(9)713 3578 436
31.5	.(3)286 6623 418	−.(6)392 2722 841	.(9)638 8279 197
32.0	.(3)273 6179 176	−.(6)362 9801 450	.(9)573 0662 221
32.5	.(3)261 3533 111	−.(6)336 2744 286	.(9)514 9290 691
33.0	.(3)249 8110 923	−.(6)311 8925 371	.(9)463 4361 622
33.5	.(3)238 9388 342	−.(6)289 6019 105	.(9)417 7452 730
34.0	.(3)228 6886 114	−.(6)269 1962 143	.(9)377 1311 492
34.5	.(3)219 0165 553	−.(6)250 4920 591	.(9)340 9678 883
35.0	.(3)209 8824 591	−.(6)233 3261 690	.(9)308 7141 691
35.5	.(3)201 2494 260	−.(6)217 5529 331	.(9)279 9008 467
36.0	.(3)193 0835 554	−.(6)203 0422 839	.(9)254 1205 055
36.5	.(3)185 3536 630	−.(6)189 6778 555	.(9)231 0186 414
37.0	.(3)178 0310 300	−.(6)177 3553 804	.(9)210 2861 992
37.5	.(3)171 0891 785	−.(6)165 9829 799	.(9)191 6532 449
38.0	.(3)164 5036 705	−.(6)155 4715 079	.(9)174 8835 859
38.5	.(3)158 2519 263	−.(6)145 7503 555	.(9)159 7701 896
39.0	.(3)152 3130 623	−.(6)136 7496 434	.(9)146 1312 710
39.5	.(3)146 6677 431	−.(6)128 4078 366	.(9)133 8069 469
40.0	.(3)141 2980 508	−.(6)120 6693 349	.(9)122 6563 680
40.5	.(3)136 1873 638	−.(6)113 4838 362	.(9)112 5552 554
41.0	.(3)131 3202 493	−.(6)106 8057 759	.(9)103 3937 811
41.5	.(3)126 6823 653	−.(6)100 5938 300	.(10)950 7474 123
42.0	.(3)122 2603 712	−.(7)948 1047 667	.(10)875 1197 773
42.5	.(3)118 0418 479	−.(7)894 2160 708	.(10)806 2901 319
43.0	.(3)114 0152 237	−.(7)843 9617 986	.(10)743 5786 772
43.5	.(3)110 1697 079	−.(7)797 0591 436	.(10)686 3803 174
44.0	.(3)106 4952 300	−.(7)753 2501 737	.(10)634 1557 280
44.5	.(3)102 9823 841	−.(7)712 2993 971	.(10)586 4235 765
45.0	.(4)996 2237 840	−.(7)673 9915 889	.(10)542 7537 357
45.5	.(4)964 0698 884	−.(7)638 1298 500	.(10)502 7613 551
46.0	.(4)933 2851 680	−.(7)604 5338 699	.(10)466 1016 730
46.5	.(4)903 7975 033	−.(7)573 0383 707	.(10)432 4654 698
47.0	.(4)875 5392 866	−.(7)543 4917 120	.(10)401 5750 791
47.5	.(4)848 4470 959	−.(7)515 7546 374	.(10)373 1808 816
48.0	.(4)822 4613 955	−.(7)489 6991 482	.(10)347 0582 199
48.5	.(4)797 5262 609	−.(7)465 2074 902	.(10)323 0046 799
49.0	.(4)773 5891 262	−.(7)442 1712 397	.(10)300 8376 920
49.5	.(4)750 6005 505	−.(7)420 4904 806	.(10)280 3924 120
50.0	.(4)728 5140 038	−.(7)400 0730 601	.(10)261 5198 458

TABLE 43

COEFFICIENTS FOR THE QUARTIC FOR THE RANGE FROM

$x = -p$ to $x = +p$

Description: The coe cients A, B, C, D, E, F are computed to ten significant figures for the range $p = 2.0$ to $p = 25.0$ by increments of .5.

The quartic,

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 \;,$$

is obtained from the equations,

$$a_0 = A\,M_0 + B\,M_2 + C\,M_4 \;,$$

$$a_2 = B\,M_0 + D\,M_2 + E\,M_4 \;,$$

$$a_4 = C\,M_0 + E\,M_2 + F\,M_4 \;,$$

combined with the values a_1, a_3 given by the tables for the cubic. The moments, M_0, M_1, M_2, M_3, M_4, are computed over the range $x = -p$ to $x = +p$.

(The numbers in parentheses denote the number of ciphers between the decimal point and the first significant figure.)

p	A	B	C
2.0	1.000 0000 000	—1.250 0000 000	.250 0000 000
2.5	.705 9936 523	—.495 6054 688	.(1)615 2343 750
3.0	.567 0995 671	—.265 1515 152	.(1)227 2727 273
3.5	.479 4006 348	—.162 3535 156	.(1)102 5390 625
4.0	.417 2494 172	—.107 8088 578	.(2)524 4755 245
4.5	.370 3002 930	—.(1)756 8359 375	.(2)292 9687 500
5.0	.333 3333 333	—.(1)553 6130 536	.(2)174 8251 748
5.5	.303 3523 560	—.(1)418 0908 203	.(2)109 8632 813
6.0	.278 4862 197	—.(1)323 9407 651	.(3)719 8683 669
6.5	.257 4942 453	—.(1)256 3476 563	.(3)488 2812 500
7.0	.239 5159 021	—.(1)206 4885 579	.(3)340 9902 791
7.5	.223 9329 020	—.(1)168 8639 323	.(3)244 1406 250
8.0	.210 2881 638	—.(1)139 9142 653	.(3)178 6139 557
8.5	.198 2357 141	—.(1)117 2614 820	.(3)133 1676 136
9.0	.187 5084 130	—.(2)992 7311 886	.(3)100 9557 141
9.5	.177 8964 418	—.(2)848 0187 618	.(4)776 8110 795
10.0	.169 2325 443	—.(2)730 2463 319	.(4)605 7342 846
10.5	.161 3816 481	—.(2)633 3998 033	.(4)478 0375 874
11.0	.154 2334 096	—.(2)553 0129 672	.(4)381 3882 532
11.5	.147 6967 551	—.(2)485 7203 344	.(4)307 3098 776
12.0	.141 6958 188	—.(2)428 9521 906	.(4)249 8750 625
12.5	.136 1668 726	—.(2)380 7227 928	.(4)204 8732 517
13.0	.131 0559 774	—.(2)339 4807 972	.(4)169 2702 036
13.5	.126 3171 605	—.(2)304 0020 282	.(4)140 8503 606
14.0	.121 9109 898	—.(2)273 3115 358	.(4)117 9762 025
14.5	.117 8034 442	—.(2)246 6262 196	.(5)994 2378 394
15.0	.113 9650 116	—.(2)223 3120 976	.(5)842 6871 608
15.5	.110 3699 628	—.(2)202 8521 369	.(5)718 0606 618
16.0	.106 9957 611	—.(2)184 8217 922	.(5)614 9338 741
16.5	.103 8225 802	—.(2)168 8702 311	.(5)529 0973 297
17.0	.100 8329 073	—.(2)154 7057 999	.(5)457 2585 218
17.5	.(1)980 1121 368	—.(2)142 0846 788	.(5)396 8229 973
18.0	.(1)953 4368 071	—.(2)130 8019 601	.(5)345 7320 530
18.5	.(1)928 1796 986	—.(2)120 6845 814	.(5)302 3413 313
19.0	.(1)904 2302 916	—.(2)111 5856 901	.(5)265 3292 500
19.5	.(1)881 4892 929	—.(2)103 3801 211	.(5)233 6273 923
20.0	.(1)859 8672 410	—.(3)959 6074 542	.(5)206 3671 945
20.5	.(1)839 2833 163	—.(3)892 3550 616	.(5)182 8388 288
21.0	.(1)819 6643 189	—.(3)831 2499 365	.(5)162 4592 807
21.5	.(1)800 9437 893	—.(3)775 6048 512	.(5)144 7474 061
22.0	.(1)783 0612 483	—.(3)724 8225 801	.(5)129 3043 255
22.5	.(1)765 9615 377	—.(3)678 3828 434	.(5)115 7979 249
23.0	.(1)749 5942 464	—.(3)635 8307 796	.(5)103 9505 362
23.5	.(1)733 9132 094	—.(3)596 7675 752	.(6)935 2909 319
24.0	.(1)718 8760 691	—.(3)560 8425 626	.(6)843 3722 746
24.5	(1)704 4438 900	—.(3)527 7465 684	.(6)762 0889 075
25.0	.(1)690 5808 198	—.(3)497 2062 910	.(6)690 0318 611

p	D	E	F
2.0	2.454 8611 111	—.538 1944 444	.121 5277 778
2.5	.586 3715 278	—.(1)824 6527 778	.(1)121 5277 778
3.0	.214 3308 081	—.(1)211 4898 990	.(2)220 9595 960
3.5	.(1)962 5552 400	—.(2)706 2815 657	.(3)552 3989 899
4.0	.(1)491 2101 788	—.(2)279 2346 542	.(3)169 9689 200
4.5	.(1)274 0445 318	—.(2)124 4415 307	.(4)607 0318 570
5.0	.(1)163 4129 759	—.(3)607 0318 570	.(4)242 8127 428
5.5	.(1)102 6452 931	—.(3)317 9329 351	.(4)106 2305 750
6.0	.(2)672 3770 510	—.(3)176 3963 161	.(5)499 9085 881
6.5	.(2)455 9791 210	—.(3)102 6597 994	.(5)249 9542 941
7.0	.(2)318 3891 488	—.(4)622 0667 018	.(5)131 5548 916
7.5	.(2)227 9369 385	—.(4)390 2012 053	.(6)723 5519 039
8.0	.(2)166 7477 045	—.(4)252 2095 208	.(6)413 4582 308
8.5	.(2)124 3142 044	—.(4)167 3566 157	.(6)244 3162 273
9.0	.(3)942 4020 458	—.(4)113 6601 579	.(6)148 7142 253
9.5	.(3)725 1166 208	—.(5)788 0526 136	.(7)929 4639 082
10.0	.(3)565 4114 845	—.(5)556 6161 004	.(7)594 8569 012
10.5	.(3)446 2073 095	—.(5)399 7797 905	.(7)388 9448 970
11.0	.(3)355 9882 993	—.(5)291 5234 609	.(7)259 2965 980
11.5	.(3)286 8401 475	—.(5)215 5402 971	.(7)175 9512 629
12.0	.(3)233 2288 596	—.(5)161 3897 791	.(7)121 3456 986
12.5	.(3)191 2235 292	—.(5)122 2557 913	.(8)849 4198 899
13.0	.(3)157 9915 515	—.(6)936 0842 049	.(8)602 8141 154
13.5	.(3)131 4646 693	—.(6)723 8747 984	.(8)433 2726 455
14.0	.(3)110 1142 734	—.(6)564 9425 143	.(8)315 1073 785
14.5	.(4)927 9784 865	—.(6)444 6919 780	.(8)231 6966 018
15.0	.(4)786 5252 285	—.(6)352 8408 251	.(8)172 1174 757
15.5	.(4)670 2028 598	—.(6)282 0575 132	.(8)129 0881 067
16.0	.(4)573 9480 513	—.(6)227 0555 132	.(9)976 8829 699
16.5	.(4)493 8316 295	—.(6)183 9826 864	.(9)745 5159 507
17.0	.(4)426 7803 550	—.(6)150 0043 633	.(9)573 4738 083
17.5	.(4)370 3725 753	—.(6)123 0152 522	.(9)444 4422 014
18.0	.(4)322 6867 183	—.(6)101 4381 253	.(9)346 8817 182
18.5	.(4)282 1879 680	—.(7)840 8165 075	.(9)272 5499 214
19.0	.(4)247 6427 884	—.(7)700 3899 143	.(9)215 5045 890
19.5	.(4)218 0539 471	—.(7)586 1479 930	.(9)171 4241 049
20.0	.(4)192 6107 529	—.(7)492 7218 558	.(9)137 1392 839
20.5	.(4)170 6506 766	—.(7)415 9387 633	.(9)110 3076 849
21.0	.(4)151 6295 539	—.(7)352 5353 142	.(10)891 8493 673
21.5	.(4)135 0983 037	—.(7)299 9440 718	.(10)724 6276 109
22.0	.(4)120 6846 287	—.(7)256 1336 780	.(10)591 5327 436
22.5	.(4)108 0785 483	—.(7)219 4882 245	.(10)485 0568 497
23.0	.(5)970 2089 990	—.(7)188 7156 473	.(10)399 4585 821
23.5	.(5)872 9415 063	—.(7)162 7777 261	.(10)330 3215 199
24.0	.(5)787 1501 985	—.(7)140 8362 750	.(10)274 2291 863
24.5	.(5)711 2852 655	—.(7)122 2115 427	.(10)228 5243 219
25.0	.(5)644 0316 483	—.(7)106 3498 773	.(10)191 1294 329

TABLE 44

Coefficients for the Quintic for the Range from

$$x = -p \text{ to } x = +p$$

Description: The coefficients A', B', C', D', E', F' are computed to ten significant figures for the range $p = 2.5$ to $p = 25.0$ by increments of .5.

The quintic,

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 \ ,$$

is obtained from the equations,

$$a_1 = A' M_1 + B' M_3 + C' M_5 \ ,$$

$$a_3 = B' M_1 + D' M_3 + E' M_5 \ ,$$

$$a_5 = C' M_1 + E' M_3 + F' M_5 \ ,$$

combined with the values a_0, a_2, a_4 given by the tables for the quartic. The moments, M_0, M_1, M_2, M_3, M_4, M_5, are computed over the range $x = -p$ to $x = +p$.

(The numbers in parentheses denote the number of ciphers between the decimal point and the first significant figure.)

p	A'	B'	C'
2.5	2.755 1030 816	−1.695 6163 194	.200 8159 722
3.0	1.170 5555 556	−.456 9444 444	.(1)363 8888 889
3.5	.658 2671 327	−.181 6553 455	.(1)104 8944 979
4.0	.418 6208 236	−.(1)868 9782 440	.(2)382 4786 325
4.5	.286 4246 324	−.(1)466 0481 012	.(2)162 0459 402
5.0	.206 0275 835	−.(1)270 7119 270	.(3)763 8888 889
5.5	.153 7850 411	−.(1)166 9293 642	.(3)390 4384 270
6.0	.118 1431 967	−.(1)107 8702 752	.(3)212 7325 289
6.5	.(1)928 9213 334	−.(2)724 0120 171	.(3)122 1004 174
7.0	.(1)744 5226 377	−.(2)501 4851 420	.(4)731 8541 452
7.5	.(1)606 4555 217	−.(2)356 7180 657	.(4)455 0831 382
8.0	.(1)500 8805 006	−.(2)259 6021 548	.(4)292 0668 942
8.5	.(1)418 6842 624	−.(2)192 7113 758	.(4)192 6724 344
9.0	.(1)353 6828 101	−.(2)145 5695 568	.(4)130 2141 757
9.5	.(1)301 5707 729	−.(2)111 6692 311	.(5)899 1006 061
10.0	.(1)259 2832 573	−.(3)868 5125 822	.(5)632 8140 010
10.5	.(1)224 5954 598	−.(3)683 8979 612	.(5)453 1298 476
11.0	.(1)195 8642 756	−.(3)544 5806 418	.(5)329 5585 675
11.5	.(1)171 8573 720	−.(3)438 0711 370	.(5)243 1030 464
12.0	.(1)151 6376 163	−.(3)355 6755 034	.(5)181 6613 061
12.5	.(1)134 4833 894	−.(3)291 2423 953	.(5)137 3671 277
13.0	.(1)119 8326 681	−.(3)240 3543 455	.(5)105 0128 024
13.5	.(1)107 2431 572	−.(3)199 7956 319	.(6)810 9219 640
14.0	.(2)963 6345 096	−.(3)167 1959 380	.(6)632 0799 809
14.5	.(2)869 1189 432	−.(3)140 7878 615	.(6)496 9749 385
15.0	.(2)786 6089 931	−.(3)119 2394 389	.(6)393 9212 946
15.5	.(2)714 2517 761	−.(3)101 5368 865	.(6)314 6050 442
16.0	.(2)650 5281 970	−.(4)869 0145 535	.(6)253 0429 506
16.5	.(2)594 1846 661	−.(4)747 2977 719	.(6)204 8829 224
17.0	.(2)544 1803 654	−.(4)645 5059 265	.(6)166 9275 426
17.5	.(2)499 6461 807	−.(4)559 9303 930	.(6)136 8055 923
18.0	.(2)459 8524 699	−.(4)487 6318 743	.(6)112 7430 005
18.5	.(2)424 1835 784	−.(4)426 2652 106	.(7)934 0140 025
19.0	.(2)392 1175 513	−.(4)373 9474 673	.(7)777 6318 298
19.5	.(2)363 2098 774	−.(4)329 1578 113	.(7)650 4887 492
20.0	.(2)337 0803 860	−.(4)290 6609 405	.(7)546 5721 085
20.5	.(2)313 4026 161	−.(4)257 4481 065	.(7)461 2129 835
21.0	.(2)291 8951 572	−.(4)228 6913 803	.(7)390 7629 285
21.5	.(2)272 2750 483	−.(4)203 7079 595	.(7)332 3537 754
22.0	.(2)254 4494 241	−.(4)181 9321 409	.(7)283 7178 436
22.5	.(2)238 1156 895	−.(4)162 8931 866	.(7)243 0524 324
23.0	.(2)223 1524 850	−.(4)146 1977 446	.(7)208 9170 140
23.5	.(2)209 4188 368	−.(4)131 5158 166	.(7)180 1547 432
24.0	.(2)196 7908 418	−.(4)118 5694 971	.(7)155 8321 715
24.5	.(2)186 2576 776	−.(4)107 7593 614	.(7)135 9946 487
25.0	.(2)174 4277 221	−.(5)969 7978 581	.(7)117 6202 932

p	D'	E'	F'
2.5	1.151 0416 667	—.140 9722 222	.(1)175 0000 000
3.0	.203 1250 000	—.(1)170 1388 889	.(2)145 8333 333
3.5	.(1)579 2905 012	—.(2)355 2350 427	.(3)224 3589 744
4.0	.(1)210 1544 289	—.(3)988 2478 632	.(4)480 7692 308
4.5	.(2)887 9662 005	—.(3)331 1965 812	.(4)128 2051 282
5.0	.(2)417 9414 336	—.(3)126 8696 582	.(5)400 6410 256
5.5	.(2)213 4163 324	—.(4)538 1158 874	.(5)141 4027 149
6.0	.(2)116 2108 929	—.(4)247 4547 511	.(6)549 8994 470
6.5	.(3)666 7389 843	—.(4)121 5567 199	.(6)231 5366 092
7.0	.(3)399 5246 884	—.(5)630 9372 602	.(6)104 1914 742
7.5	.(3)248 3850 322	—.(5)343 1703 316	.(7)496 1498 770
8.0	.(3)159 3881 480	—.(5)194 3253 685	.(7)248 0749 385
8.5	.(3)105 1352 375	—.(5)113 9706 601	.(7)129 4304 027
9.0	.(4)710 4822 114	—.(6)689 3966 587	.(8)701 0813 479
9.5	.(4)490 5434 324	—.(6)428 5943 973	.(8)392 6055 548
10.0	.(4)345 2433 425	—.(6)273 0621 968	.(8)226 5082 047
10.5	.(4)247 2044 428	—.(6)177 8469 607	.(8)134 2241 213
11.0	.(4)179 7851 483	—.(6)118 1651 639	.(9)814 9321 650
11.5	.(4)132 6177 560	—.(7)799 4765 550	.(9)505 8199 645
12.0	.(5)990 9817 818	—.(7)549 9387 059	.(9)320 3526 442
12.5	.(5)749 3410 036	—.(7)384 0787 078	.(9)206 6791 253
13.0	.(5)572 8402 600	—.(7)272 0198 696	.(9)135 6331 760
13.5	.(5)442 3498 419	—.(7)195 1610 699	.(10)904 2211 731
14.0	.(5)344 7903 612	—.(7)141 7056 417	.(10)611 6790 289
14.5	.(5)271 0905 849	—.(7)104 0436 901	.(10)419 4370 484
15.0	.(5)214 8754 286	—.(8)771 8806 793	.(10)291 2757 280
15.5	.(5)171 6092 639	—.(8)578 2216 817	.(10)204 6802 413
16.0	.(5)138 0280 549	—.(8)437 1000 417	.(10)145 4306 978
16.5	.(5)111 7576 435	—.(8)333 2476 075	.(10)104 4117 830
17.0	.(6)910 5379 546	—.(8)256 1134 028	.(11)756 9854 269
17.5	.(6)746 2299 456	—.(8)198 3240 275	.(11)553 8917 758
18.0	.(6)614 9748 289	—.(8)154 6720 805	.(11)408 8248 821
18.5	.(6)509 4718 636	—.(8)121 4431 743	.(11)304 2417 727
19.0	.(6)424 1700 765	—.(9)959 6292 582	.(11)228 1813 296
19.5	.(6)354 8175 306	—.(9)762 8862 452	.(11)172 4036 712
20.0	.(6)298 1344 148	—.(9)609 9716 846	.(11)131 1767 064
20.5	.(6)251 5739 152	—.(9)490 3776 023	.(11)100 4757 751
21.0	.(6)213 1458 799	—.(9)396 2862 254	.(12)774 5007 663
21.5	.(6)181 2857 886	—.(9)321 8393 150	.(12)600 6332 473
22.0	.(6)154 7566 781	—.(9)262 6168 770	.(12)468 4939 329
22.5	.(6)132 5752 232	—.(9)215 2622 450	.(12)367 4462 219
23.0	.(6)113 9556 568	—.(9)177 2103 622	.(12)289 7172 134
23.5	.(7)982 6695 408	—.(9)146 4894 049	.(12)229 5872 257
24.0	.(7)849 9994 800	—.(9)121 5749 393	.(12)182 8194 575
24.5	.(7)741 7936 742	—.(9)101 8827 904	.(12)147 1231 630
25.0	.(7)641 5689 752	—.(10)846 8458 443	.(12)117 5267 941

TABLE 45

COEFFICIENTS FOR THE SEXTIC FOR THE RANGE FROM

$x = -p$ to $x = +p$

Dsscription: The coefficients $A, B, C, D, E, F, G, H, I, J,$ are computed to ten significant figures for the range $p = 3.0$ to $p = 25.0$ by increments of .5.

The sextic,

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 \;,$$

is obtained from the equations,

$$a_0 = A\, M_0 + B\, M_2 + C\, M_4 + D\, M_6 \;,$$

$$a_2 = B\, M_0 + E\, M_2 + F\, M_4 + G\, M_6 \;,$$

$$a_4 = C\, M_0 + F\, M_2 + H\, M_4 + I\, M_6 \;,$$

$$a_6 = D\, M_0 + G\, M_2 + I\, M_4 + J\, M_6 \;,$$

combined with the values a_1, a_3, a_5 given by the tables for the quintic. The moments, $M_0, M_1, M_2, M_3, M_4, M_5, M_6$ are computed over range $x = -p$ to $x = +p$.

(The numbers in parentheses denote the number of ciphers between the decimal point and the first significant figure.)

p	A	B	C
3.0	1.000 0000 000	—1.361 1111 111	.388 8888 889
3.5	.745 3327 179	—.617 5664 266	.116 7093 913
4.0	.619 2696 193	—.362 6910 127	.(1)498 5754 986
4.5	.536 5078 449	—.238 2141 749	.(1)251 6301 473
5.0	.475 9358 289	—.167 1854 290	.(1)140 7742 584
5.5	.428 9313 952	—.122 7793 517	.(2)846 3824 237
6.0	.391 0671 372	—.(1)932 5031 693	.(2)537 1649 335
6.5	.359 7514 629	—.(1)727 0467 546	.(2)355 7417 128
7.0	.333 3333 333	—.(1)578 9958 809	.(2)243 8852 284
7.5	.310 6966 019	—.(1)469 2645 603	.(2)172 0852 322
8.0	.291 0527 351	—.(1)386 0218 617	.(2)124 4257 604
8.5	.273 8253 011	—.(1)321 6238 375	.(3)918 7769 007
9.0	.258 5812 357	—.(1)270 9606 497	.(3)690 9857 765
9.5	.244 9877 912	—.(1)230 5173 392	.(3)528 1236 437
10.0	.232 7844 334	—.(1)197 8161 991	.(3)409 4730 527
10.5	.221 7638 626	—.(1)171 0732 926	.(3)321 5757 191
11.0	.211 7588 265	—.(1)148 9801 195	.(3)255 4794 154
11.5	.202 6327 307	—.(1)130 5610 007	.(3)205 1024 695
12.0	.194 2728 127	—.(1)115 0776 715	.(3)166 2345 586
12.5	.186 5850 870	—.(1)101 9640 947	.(3)135 9108 167
13.0	.179 4905 415	—.(2)907 8107 302	.(3)112 0109 002
13.5	.172 9222 326	—.(2)811 8410 815	.(4)929 9691 087
14.0	.166 8230 401	—.(2)729 0028 821	.(4)777 3890 832
14.5	.161 1439 098	—.(2)657 1143 441	.(4)653 9677 383
15.0	.155 8424 640	—.(2)594 4165 421	.(4)513 3898 749
15.5	.150 8818 928	—.(2)539 4804 055	.(4)470 8598 806
16.0	.146 2300 606	—.(2)491 1364 663	.(4)402 7015 521
16.5	.141 8587 811	—.(2)448 4212 624	.(4)346 0719 079
17.0	.137 7432 239	—.(2)410 5360 674	.(4)298 7541 988
17.5	.133 8614 260	—.(2)376 8148 358	.(4)259 0066 153
18.0	.130 1938 866	—.(2)346 6991 057	.(4)225 4506 124
18.5	.126 7232 293	—.(2)319 7181 992	.(4)196 9877 124
19.0	.123 4339 187	—.(2)295 4734 917	.(4)172 7369 850
19.5	.120 3120 213	—.(2)273 6258 312	.(4)151 9876 810
20.0	.117 3450 034	—.(2)253 8854 120	.(4)134 1630 697
20.5	.114 5607 471	—.(2)236 0035 778	.(4)118 7926 279
21.0	.111 8314 598	—.(2)219 7661 473	.(4)105 4905 051
21.5	.109 2654 341	—.(2)204 9879 526	.(5)939 3873 877
22.0	.106 8150 524	—.(2)191 5083 451	.(5)838 7409 116
22.5	.104 4726 355	—.(2)179 1874 832	.(5)750 7766 502
23.0	.102 2311 723	—.(2)167 9032 479	.(5)673 6666 593
23.5	.100 0842 483	—.(2)157 5486 730	.(5)605 8783 224
24.0	.(1)980 2598 324	—.(2)148 0297 913	.(5)546 1216 848
24.5	.(1)960 5097 234	—.(2)139 2638 202	.(5)493 3070 093
25.0	.(1)941 5425 829	—.(2)131 1776 633	.(5)446 5105 454

p	D	E	F
3.0	−.(1)277 7777 778	2.988 9351 852	−.948 1481 481
3.5	−.(2)581 8684 896	.875 4725 025	−.189 2894 604
4.0	−.(2)185 1851 852	.370 6973 366	−.(1)590 7882 241
4.5	−.(3)727 3356 120	.186 3394 083	−.(1)229 8587 984
5.0	−.(3)326 7973 856	.104 0300 478	−.(1)102 7515 921
5.5	−.(3)161 6301 360	.(1)624 7216 370	−.(2)506 6826 853
6.0	−.(4)859 9931 201	.(1)396 1994 936	−.(2)269 0942 361
6.5	−.(4)484 8904 080	.(1)262 2649 076	−.(2)151 5410 433
7.0	−.(4)286 6643 734	.(1)179 7450 169	−.(3)895 1744 340
7.5	−.(4)176 3237 847	.(1)126 8009 428	−.(3)550 2997 477
8.0	−.(4)112 1730 157	.(2)916 6922 412	−.(3)349 9462 528
8.5	−.(5)734 6824 363	.(2)676 8238 027	−.(3)229 1312 716
9.0	−.(5)493 5612 689	.(2)508 9783 157	−.(3)153 8970 152
9.5	−.(5)339 0842 014	.(2)388 9905 578	−.(3)105 7119 802
10.0	−.(5)237 6406 110	.(2)301 5839 811	−.(4)740 7668 221
10.5	−.(5)169 6421 007	.(2)236 8373 742	−.(4)528 4390 327
11.0	−.(5)122 9175 574	.(2)188 1526 456	−.(4)383 0865 153
11.5	−.(6)904 2245 370	.(2)151 0481 065	−.(4)281 7940 194
12.0	−.(6)674 0640 244	.(2)122 4214 851	−.(4)210 0557 062
12.5	−.(6)508 6263 021	.(2)100 0884 111	−.(4)158 4944 724
13.0	−.(6)388 0974 686	.(3)824 6707 418	−.(4)120 9320 228
13.5	−.(6)299 1919 424	.(3)684 8389 962	−.(5)932 2623 816
14.0	−.(6)232 8584 812	.(3)572 4725 971	−.(5)725 5546 509
14.5	−.(6)182 8395 204	.(3)481 5811 670	−.(5)569 6913 075
15.0	−.(6)144 7498 667	.(3)407 5132 846	−.(5)451 0040 878
15.5	−.(6)115 4775 918	.(3)346 7368 814	−.(5)359 7939 891
16.0	−.(7)927 8837 607	.(3)296 5444 286	−.(5)289 0976 344
16.5	−.(7)750 6043 467	.(3)254 8421 133	−.(5)233 8608 712
17.0	−.(7)611 0454 032	.(3)219 9973 645	−.(5)190 3777 170
17.5	−.(7)500 4028 978	.(3)190 7274 117	−.(5)155 9046 664
18.0	−.(7)412 1003 883	.(3)166 0170 278	−.(5)128 3924 319
18.5	−.(7)341 1837 940	.(3)145 0572 508	−.(5)106 2973 088
19.0	−.(7)283 8913 786	.(3)127 1993 361	−.(6)884 4715 492
19.5	−.(7)237 3452 480	.(3)111 9198 707	−.(6)739 4524 672
20.0	−.(7)199 3279 892	.(4)987 9413 925	−.(6)621 0067 139
20.5	−.(7)168 1195 507	.(4)874 7564 250	−.(6)523 7749 416
21.0	−.(7)142 3771 352	.(4)776 8023 679	−.(6)443 5733 082
21.5	−.(7)121 0460 765	.(4)691 6366 980	−.(6)377 1157 568
22.0	−.(7)103 2932 157	.(4)617 6240 310	−.(6)321 8064 139
22.5	−.(8)884 5674 819	.(4)552 8493 126	−.(6)275 5833 129
23.0	−.(8)760 0821 534	.(4)496 0674 883	−.(6)236 7999 628
23.5	−.(8)655 2351 718	.(4)446 1499 515	−.(6)204 1350 268
24.0	−.(8)566 6066 961	.(4)402 1467 932	−.(6)176 5230 172
24.5	−.(8)491 4263 585	.(4)365 3958 471	−.(6)154 0028 718
25.0	−.(8)427 4401 392	.(4)328 7959 685	−.(6)133 1661 235

p	G	H	I
3.0	.(1)703 2407 407	.311 9212 963	—.(1)234 9537 037
3.5	.(2)996 0214 120	.(1)431 6767 940	—.(2)232 8292 407
4.0	.(2)233 6419 753	.(1)100 2196 106	—.(3)408 9506 173
4.5	.(3)711 2449 363	.(2)303 4820 991	—.(4)972 9456 019
5.0	.(3)256 2636 166	.(2)109 0241 118	—.(4)282 5435 730
5.5	.(3)104 2151 284	.(3)442 5904 068	—.(5)947 9582 728
6.0	.(4)464 8740 588	.(3)197 2080 911	—.(5)355 3443 795
6.5	.(4)223 1993 819	.(4)946 1621 940	—.(5)145 5344 260
7.0	.(4)113 8216 820	.(4)482 2607 431	—.(6)641 0133 904
7.5	.(5)610 4831 370	.(4)258 5707 479	—.(6)300 1017 659
8.0	.(5)341 8161 061	.(4)144 7403 415	—.(6)148 0060 623
8.5	.(5)198 6271 380	.(5)840 9223 280	—.(7)763 5619 773
9.0	.(5)119 2274 520	.(5)504 7006 725	—.(7)409 7430 989
9.5	.(6)736 4557 705	.(5)311 7158 600	—.(7)227 6566 932
10.0	.(6)466 6351 290	.(5)197 4943 272	—.(7)130 4646 031
10.5	.(6)302 4952 338	.(5)128 0172 024	—.(8)768 7010 766
11.0	.(6)200 1684 248	.(6)847 0785 482	—.(8)464 4029 703
11.5	.(6)134 9506 134	.(6)571 0651 132	—.(8)287 0085 966
12.0	.(7)925 4160 570	.(6)391 5919 775	—.(8)181 0859 646
12.5	.(7)644 5499 552	.(6)272 7356 553	—.(8)116 4409 022
13.0	.(7)455 3925 745	.(6)192 6912 065	—.(9)761 8900 625
13.5	.(7)326 0189 201	.(6)137 9465 952	—.(9)506 5928 672
14.0	.(7)236 2653 096	.(7)999 6815 163	—.(9)341 8901 625
14.5	.(7)173 1717 709	.(7)732 7121 006	—.(9)233 9443 041
15.0	.(7)128 2726 524	.(7)542 7324 021	—.(9)162 1522 356
15.5	.(8)959 5450 485	.(7)405 9879 530	—.(9)113 7486 502
16.0	.(8)724 4284 089	.(7)306 5066 664	—.(10)806 9508 539
16.5	.(8)551 6645 046	.(7)233 4084 850	—.(10)578 5246 623
17.0	.(8)423 5236 197	.(7)179 1912 403	—.(10)418 8850 767
17.5	.(8)327 6400 420	.(7)138 6225 823	—.(10)306 1363 264
18.0	.(8)255 2963 323	.(7)108 0139 227	—.(10)225 7107 282
18.5	.(8)200 2846 930	.(8)847 3860 468	—.(10)167 8017 503
19.0	.(8)158 1422 442	.(8)669 0830 603	—.(10)125 7344 857
19.5	.(8)125 6316 099	.(8)531 5324 962	—.(11)949 1786 023
20.0	.(8)100 3843 545	.(8)424 7132 721	—.(11)721 6269 402
20.5	.(9)806 5374 215	.(8)341 2348 327	—.(11)552 3276 496
21.0	.(9)651 4166 128	.(8)275 6048 185	—.(11)425 4604 167
21.5	.(9)528 7636 968	.(8)223 7117 748	—.(11)329 7379 935
22.0	.(9)431 2539 387	.(8)182 4566 123	—.(11)257 0422 413
22.5	.(9)353 3296 685	.(8)149 4878 961	—.(11)201 4893 910
23.0	.(9)290 7474 080	.(8)123 0102 298	—.(11)158 7837 578
23.5	.(9)240 2476 147	.(8)101 6445 207	—.(11)125 7671 107
24.0	.(9)199 3121 346	.(9)843 2535 668	—.(11)100 1019 200
24.5	.(9)166 9628 406	.(9)706 3889 111	—.(12)805 1862 542
25.0	.(9)138 7382 906	.(9)586 9755 711	—.(12)642 9736 393

p	J
3.0	.(2)178 2407 407
3.5	.(3)127 3148 148
4.0	.(4)169 7530 864
4.5	.(5)318 2870 370
5.0	.(6)748 9106 754
5.5	.(6)208 0307 432
6.0	.(7)656 9391 889
6.5	.(7)229 9287 161
7.0	.(8)875 9189 186
7.5	.(8)358 3304 667
8.0	.(8)155 7958 551
8.5	.(9)714 0643 358
9.0	.(9)342 7508 812
9.5	.(9)171 3754 406
10.0	.(10)888 6133 957
10.5	.(10)476 0428 905
11.0	.(10)262 6443 534
11.5	.(10)148 8318 003
12.0	.(11)864 1846 467
12.5	.(11)513 1096 340
13.0	.(11)310 9755 357
13.5	(11)192 0731 250
14.0	.(11)120 7316 786
14.5	.(12)771 3412 798
15.0	.(12)500 3294 788
15.5	.(12)329 1641 308
16.0	.(12)219 4427 539
16.5	.(12)148 1238 589
17.0	.(12)101 1577 573
17.5	.(13)698 4702 287
18.0	.(13)487 3048 107
18.5	.(13)343 3283 894
19.0	.(13)244 1446 325
19.5	.(13)175 1472 363
20.0	.(13)126 7022 561
20.5	.(14)923 8706 171
21.0	.(14)678 7620 861
21.5	.(14)502 2839 437
22.0	.(14)374 2507 816
22.5	.(14)280 6880 862
23.0	.(14)211 8400 650
23.5	.(14)160 8415 309
24.0	.(14)122 8244 418
24.5	.(15)948 6730 535
25.0	.(15)728 0195 608

TABLE 46

COEFFICIENTS FOR THE SEPTIMIC FOR THE RANGE FROM $x = -p$ to $x = +p$

Description: The coefficients A', B', C', D' E', F', G', H' I' J', are computed to ten significant figures for the range $p = 3.5$ to $p = 25.0$ by increments of .5.

The septimic,

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 \;,$$

is obtained from the equations,

$$a_1 = A' M_1 + B' M_3 + C' M_5 + D' M_7 \;,$$

$$a_3 = B' M_1 + E' M_3 + F' M_5 + G' M_7 \;,$$

$$a_5 = C' M_1 + F' M_3 + H' M_5 + I' M_7 \;,$$

$$a_7 = D' M_1 + G' M_3 + I' M_5 + J' M_7 \;,$$

combined with the values a_0, a_2, a_4, a_6 given by the tables for the sextic. The moments, M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7, are computed over the range $x = -p$ to $x = +p$.

(The numbers in parentheses denote the number of ciphers between the decimal point and the first significant figure.)

p	A'	B'	C'
3.5	28.751 2015 275	−2.018 4913 853	.357 6197 452
4.0	1.362 9280 045	−.639 9553 571	.(1)798 5119 048
4.5	.826 9536 776	−.286 6103 525	.(1)269 7269 968
5.0	.557 2123 756	−.150 4344 206	.(1)111 5137 722
5.5	.399 2748 295	−.(1)869 6309 477	.(2)523 4528 750
6.0	.298 3433 421	−.(1)537 5591 718	.(2)268 7939 211
6.5	.229 9463 348	−.(1)349 3835 704	.(2)147 7366 521
7.0	.181 5696 107	−.(1)236 2100 221	.(3)856 9155 732
7.5	.146 2040 440	−.(1)164 8961 618	.(3)519 3903 212
8.0	.119 6560 886	−.(1)118 2279 748	.(3)326 6117 660
8.5	.(1)992 8681 536	−.(2)867 1239 512	.(3)211 9254 433
9.0	.(1)833 6752 283	−.(2)648 5413 101	.(3)141 2859 194
9.5	.(1)707 2793 855	−.(2)493 4138 461	.(4)964 5009 167
10.0	.(1)605 5349 149	−.(2)381 0899 486	.(4)672 3458 517
10.5	.(1)522 6404 211	−.(2)298 3079 274	.(4)477 5025 292
11.0	.(1)454 3790 237	−.(2)236 3306 957	.(4)344 8411 857
11.5	.(1)397 6190 688	−.(2)189 2706 474	.(4)252 8236 420
12.0	.(1)350 0205 660	−.(2)153 0798 651	.(4)187 9176 343
12.5	.(1)309 7896 316	−.(2)124 9247 269	.(4)141 4318 835
13.0	.(1)275 5429 712	−.(2)102 7891 576	.(4)107 6721 454
13.5	.(1)246 1999 134	−.(3)852 1744 589	.(5)828 3962 809
14.0	.(1)220 9073 683	−.(3)711 4425 518	.(5)643 5779 618
14.5	.(1)198 9854 583	−.(3)597 8026 293	.(5)504 5238 369
15.0	.(1)179 8875 910	−.(3)505 3397 258	.(5)398 8437 502
15.5	.(1)163 1707 725	−.(3)429 5745 974	.(5)317 7726 592
16.0	.(1)148 4732 835	−.(3)367 0820 942	.(5)255 0351 049
16.5	.(1)135 4977 156	−.(3)315 2193 163	.(5)206 0875 950
17.0	.(1)123 9979 553	−.(3)271 9296 956	.(5)167 6061 262
17.5	.(1)113 7691 065	−.(3)235 6002 537	.(5)137 1349 784
18.0	.(1)104 6396 241	−.(3)204 9565 434	.(5)112 8433 182
18.5	.(2)964 6512 317	−.(3)178 9845 869	.(6)933 5432 357
19.0	.(2)891 2347 318	−.(3)156 8723 569	.(6)776 2421 574
19.5	.(2)842 5297 627	−.(3)137 9655 374	.(6)648 5562 423
20.0	.(2)765 3875 595	−.(3)121 7338 175	.(6)544 3500 271
20.5	.(2)711 3114 773	−.(3)107 7450 175	.(6)458 8701 286
21.0	.(2)662 2270 210	−.(4)956 4508 777	.(6)388 4099 727
21.5	.(2)617 5695 829	−.(4)851 4293 652	.(6)330 0128 092
22.0	.(2)576 8498 581	−.(4)759 9627 856	.(6)281 5293 946
22.5	.(2)539 6419 780	−.(4)680 0595 916	.(6)240 9925 109
23.0	.(2)505 5744 586	−.(4)610 0441 976	.(6)206 9976 250
23.5	.(2)474 3260 497	−.(4)548 5160 065	.(6)178 3794 880
24.0	.(2)445 5992 272	−.(4)494 2969 537	.(6)154 1991 740
24.5	.(2)419 1587 794	−.(4)446 3969 384	.(6)133 6979 531
25.0	.(2)394 7661 411	−.(4)403 9598 359	.(6)116 2537 745

p	D′	E′	F′
3.5	−.(1)173 0659 191	1.579 8913 122	−.291 1769 387
4.0	−.(2)282 3837 868	.344 9276 620	−.(1)455 1504 630
4.5	−.(3)732 8051 663	.115 4442 217	−.(1)115 8781 297
5.0	−.(3)241 1381 219	.(1)475 1410 972	−.(2)377 5757 988
5.5	−.(4)925 0558 091	.(1)222 4862 588	−.(2)144 0406 919
6.0	−.(4)396 2769 318	.(1)114 0808 001	−.(3)614 9607 748
6.5	−.(4)184 7282 763	.(2)626 4468 334	−.(3)286 0508 258
7.0	−.(5)921 2018 141	.(2)363 1390 089	−.(3)142 4423 327
7.5	−.(5)485 5539 780	.(2)220 0141 501	−.(4)750 0507 440
8.0	−.(5)268 1183 076	.(2)138 3131 064	−.(4)413 8794 788
8.5	−.(5)154 0554 574	.(3)897 2722 669	−.(4)237 6853 941
9.0	−.(6)916 1750 131	.(3)598 0994 308	−.(4)141 2990 508
9.5	−.(6)561 5350 601	.(3)408 2508 260	−.(5)865 7917 011
10.0	−.(6)353 4749 049	.(3)284 5633 018	−.(5)544 8786 925
10.5	−.(6)227 8602 485	.(3)202 0841 107	−.(5)351 1847 710
11.0	−.(6)150 0558 578	.(3)145 9325 974	−.(5)231 2393 114
11.5	−.(6)100 7434 807	.(3)106 9873 276	−.(5)155 2312 455
12.0	−.(7)688 3248 748	.(4)795 1834 088	−.(5)106 0518 396
12.5	−.(7)477 8804 517	.(4)598 4598 613	−.(6)736 2301 182
13.0	−.(7)336 6794 370	.(4)455 5973 535	−.(6)518 6638 399
13.5	−.(7)240 4245 028	.(4)350 5159 780	−.(6)370 3629 282
14.0	−.(7)173 8435 375	.(4)272 3103 032	−.(6)267 7874 363
14.5	−.(7)127 1625 885	.(4)213 4710 252	−.(6)195 8739 375
15.0	−.(8)940 2155 070	.(4)168 7544 633	−.(6)144 8213 551
15.5	−.(8)702 1757 322	.(4)134 4512 878	−.(6)108 1535 449
16.0	−.(8)529 3336 632	.(4)107 9058 431	−.(7)815 2967 606
16.5	−.(8)402 5511 467	.(5)871 9546 641	−.(7)620 0115 559
17.0	−.(8)308 6648 353	.(5)709 1357 523	−.(7)475 4002 735
17.5	−.(8)238 5149 470	.(5)580 2104 892	−.(7)367 3519 091
18.0	−.(8)185 6576 541	.(5)477 4317 347	−.(7)285 9398 751
18.5	−.(8)145 5130 248	.(5)394 9737 353	−.(7)224 1091 135
19.0	−.(8)114 7942 689	.(5)328 4199 721	−.(7)176 7967 053
19.5	−.(9)911 2100 394	.(5)274 3965 893	−.(7)140 3360 406
20.0	−.(9)727 5436 127	.(5)230 3075 611	−.(7)112 0487 209
20.5	−.(9)584 1368 200	.(5)194 1416 638	−.(8)899 6217 202
21.0	−.(9)471 4844 101	.(5)164 3306 063	−.(8)726 1234 346
21.5	−.(9)382 4793 533	.(5)139 6233 668	−.(8)589 0458 851
22.0	−.(9)311 7704 559	.(5)119 1105 861	−.(8)480 1471 084
22.5	−.(9)255 3016 408	.(5)101 9599 333	−.(8)393 1799 685
23.0	−.(9)209 9789 105	.(6)875 7714 928	−.(8)323 3791 671
23.5	−.(9)173 4277 959	.(6)754 6921 922	−.(8)267 0876 406
24.0	−.(9)143 8154 281	.(6)652 3888 702	−.(8)221 4825 166
24.5	−.(9)119 7203 889	.(6)565 6513 791	−.(8)184 3745 724
25.0	−.(9)100 0289 166	.(6)491 8478 631	−.(8)154 0485 226

p	G'	H'	I'
3.5	.(1)143 3986 442	.(1)545 8043 981	−.(2)270 9986 772
4.0	.(2)165 3852 513	.(2)616 8981 481	−.(3)227 3478 836
4.5	.(3)325 3719 022	.(2)120 1878 234	−.(4)343 6965 064
5.0	.(4)847 0633 624	.(3)311 2518 155	−.(5)713 2482 623
5.5	.(4)264 7923 510	.(4)969 9931 081	−.(5)182 5352 461
6.0	.(5)944 9259 723	.(4)345 4902 916	−.(6)544 3210 423
6.5	.(5)373 3302 227	.(4)136 3314 118	−.(6)182 6693 153
7.0	.(5)160 0116 167	.(5)583 8397 737	−.(7)674 0025 445
7.5	.(6)733 3626 200	.(5)267 4300 767	−.(7)268 9333 213
8.0	.(6)355 6040 392	.(5)129 6221 514	−.(7)114 6212 362
8.5	.(6)180 9471 484	.(6)659 3767 449	−.(8)516 9083 906
9.0	.(7)960 0363 156	.(6)349 7616 947	−.(8)244 8220 580
9.5	.(7)528 3674 230	.(6)192 4628 423	−.(8)121 0509 065
10.0	.(7)300 3768 815	.(6)109 4008 386	−.(9)621 7703 060
10.5	.(7)175 7761 725	.(7)640 1325 618	−.(9)330 4159 767
11.0	.(7)105 5698 126	.(7)384 4277 647	−.(9)181 0370 007
11.5	.(8)649 1137 736	.(7)236 3567 152	−.(9)101 9713 676
12.0	.(8)407 7298 452	.(7)148 4557 926	−.(10)588 9829 883
12.5	.(8)261 1497 436	.(8)950 8148 611	−.(10)348 0938 563
13.0	.(8)170 2826 250	.(8)619 9571 695	−.(10)210 1044 796
13.5	.(8)112 8752 998	.(8)410 9394 710	−.(10)129 2993 719
14.0	.(9)759 6818 749	.(8)276 5670 073	−.(11)810 1043 368
14.5	.(9)518 5449 520	.(8)188 7758 141	−.(11)516 0618 886
15.0	.(9)358 6183 357	.(8)130 5524 791	−.(11)333 8664 755
15.5	.(9)251 0639 560	.(9)913 9675 989	−.(11)219 1292 642
16.0	.(9)177 7849 669	.(9)647 1965 291	−.(11)145 7726 865
16.5	.(9)127 2480 223	.(9)463 2203 741	−.(12)982 0445 539
17.0	.(10)919 9584 852	.(9)334 8890 426	−.(12)669 4697 055
17.5	.(10)671 4105 266	.(9)244 4091 600	−.(12)461 4992 604
18.0	.(10)494 3960 450	.(9)179 9705 267	−.(12)321 4946 024
18.5	.(10)367 1241 847	.(9)133 6401 141	−.(12)226 1959 235
19.0	.(10)274 7920 816	.(9)100 0289 750	−.(12)160 6464 099
19.5	.(10)207 2366 516	.(10)754 3731 214	−.(12)115 1112 743
20.0	.(10)157 4099 489	.(10)572 9940 142	−.(13)831 8162 819
20.5	.(10)120 3776 663	.(10)438 1898 319	−.(13)605 9221 411
21.0	.(11)926 5399 250	.(10)337 2711 214	−.(13)444 7507 764
21.5	.(11)717 5522 633	.(10)261 1964 982	−.(13)328 8288 593
22.0	.(11)558 9721 931	.(10)203 4711 966	−.(13)244 8106 616
22.5	.(11)437 8836 453	.(10)159 3934 600	−.(13)183 4686 278
23.0	.(11)344 8668 374	.(10)125 5342 827	−.(13)138 3685 504
23.5	.(11)273 0031 904	.(11)993 7516 417	−.(13)104 9876 917
24.0	.(11)217 1767 993	.(11)790 5382 130	−.(14)801 2235 814
24.5	.(11)173 5819 878	.(11)631 8492 407	−.(14)614 8704 752
25.0	.(11)139 3623 589	.(11)507 2869 968	−.(14)474 3701 124

p	J	p	J'
3.5	.(3)135 1095 994	14.5	.(13)144 2831 239
4.0	.(5)844 4349 962	15.0	.(14)873 2925 919
4.5	.(6)993 4529 367		
5.0	.(6)165 5754 895	15.5	.(14)537 4108 258
		16.0	.(14)335 8817 661
5.5	.(7)348 5799 778	16.5	.(14)212 9981 931
6.0	.(8)871 4499 445	17.0	.(14)136 9274 099
6.5	.(8)248 9856 984	17.5	.(15)891 6203 434
7.0	.(9)792 2272 223	18.0	.(15)587 6588 627
7.5	.(9)275 5572 947	18.5	.(15)391 7725 751
8.0	.(9)103 3339 855	19.0	.(15)264 0206 484
8.5	.(10)413 3359 421	19.5	.(15)179 7587 394
9.0	.(10)174 8728 986	20.0	.(15)123 5841 333
9.5	.(11)777 2128 825		
10.0	.(11)360 8488 383	20.5	.(16)857 5225 577
		21.0	.(16)600 2657 904
10.5	.(11)174 2028 875	21.5	.(16)423 7170 285
11.0	.(12)871 0144 373	22.0	.(16)301 4909 626
11.5	.(12)449 5558 386	22.5	.(16)216 1633 317
12.0	.(12)238 8265 393	23.0	.(16)156 1179 618
12.5	.(12)130 2690 214	23.5	.(16)113 5403 358
13.0	.(13)727 9739 432	24.0	.(17)831 2774 587
13.5	.(13)415 9851 104	24.5	.(17)612 5263 839
14.0	.(13)242 6579 811	25.0	.(17)454 1098 277

FUNCTIONS OF POLYNOMIAL APPROXIMATION

FUNCTIONS OF POLYNOMIAL APPROXIMATION

1. *Description.* The functions tabulated in this section are designed to aid in fitting polynomials from the first to the third degree to data given as a series of equally spaced items, i. e.

	y_1	y_2	y_3	y_4	y_5	.	y_p
x	x_1	x_2	x_3	x_4	x_5		

where $x_{i+1} - x_i$ is constant.

It is clear that there is no loss of generality in replacing the x-series by the set of integers, 1, 2, 3, 4, 5, $\cdots\cdots$, p .

If we denote by $m_0, m_1, m_2, m_3, \cdots, m_n$ the moments

$$m_r = \sum_{i=1}^{p} y_i\, i^r$$

and by s_n the series, $s_r = 1^r + 2^r + 3^r + \quad + p^r$, then the coefficients of the polynomial

$$y = a \;+ a_1x + a_2x^2 + a_3x^3 + \cdots \quad a_nx^n \;,$$

fitted to the data by the method of least squares, will be determined as solutions of the following set of equations:

$$s_0a_0 + s_1a_1 + s_2a_2 + \cdots + s_na_n \;= m_0 \;,$$

$$s_1a_0 + s_2a_1 + s_3a_2 + \cdots + s_{n+1}a_n = m_1 \;,$$

$$s_na_0 + s_{m+1}a_1 + s_{n+2}a_2 + \cdots + s_{2n}a_n = m_n \;.$$

The coefficients of the moments in the solutions of these equations, regarded as functions of p, are recorded in the tables which follow.

2. Functions of the Approximation. If we solve the system of equations given in the last section, we obtain the following explicit results:

The straight line, $y = a_0 + a_1 x$.

$$a_0 = Am_0 + Bm_1$$

$$a_1 = Bm_0 + Cm_1 \ ,$$

where we write,

$$A = 2(2p+1)/p(p-1),\ B = -6/p(p-1),\ C = 12/p(p^2-1) \ .$$

The parabola, $y = a_0 + a_1 x + a_2 x^2$.

$$a_0 = Am_0 + Bm_1 + Cm_2 \ ,$$

$$a_1 = Bm_0 + Dm_1 + Em_2 \ ,$$

$$a_2 = Cm_0 + Em_1 + Fm_2 \ ,$$

where we have,

$$A = 3(3p^2 + 3p + 2)/p(p-1)(p-2) \ ,$$

$$B = -18(2p+1)/p(p-1)(p-2) \ ,$$

$$C = 30/p(p-1)(p-2) \ ,$$

$$D = 12(2p+1)(8p+11)/p(p^2-1)(p^2-4) \ ,$$

$$E = -180/p(p-1)(p^2-4) \ ,\ F = 180/p(p^2-1)(p^2-4).$$

The cubic, $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$.

$$a_0 = Am_0 + Bm_1 + Cm_2 + Dm_3 \ ,$$

$$a_1 = Bm_0 + Em_1 + Fm_2 + Gm_3 \ ,$$

$$a_2 = Cm_0 + Fm_1 + Hm_2 + Im_3 \ ,$$

$$a_3 = Dm_0 + Gm_1 + Im_2 + Jm_3 \ ,$$

where we abbreviate,

$$A = 8(2p+1)(p^2+p+3)/p(p-1)(p-2)(p-3) \ ,$$

$$B = -20(6p^2+6p+5)/p(p-1)(p-2)(p-3) \ ,$$

$$C = 120(2p+1)/p(p-1)(p-2)(p-3) \ ,$$

$$D = -140/p(p-1)(p-2)(p-3) \ ,$$

$$E = 200(6p^4 + 27p^3 + 42p^2 + 30p + 11)/p(p^2-1) \times (p^2-4)(p^2-9) \ ,$$

$$F = -300(3p+2)(3p+5)/p(p-1)(p^2-4)(p^2-9) \ ,$$

$$G = 280(6p^2 + 15p + 11)/p(p^2-1)(p^2-4)(p^2-9) \ ,$$

$$H = 360(2p+1)(9p+13)/p(p^2-1)(p^2-4)(p^2-9) \ ,$$

$$I = -4200/p(p-1)(p^2-4)(p^2-9) \ ,$$

$$J = 2800/p(p^2-1)(p^2-4)(p^2-9) \ .$$

These functions of polynomial approximation have been tabulated for the following ranges: (1) Straight line: from $p = 2$ to $p = 240$; (2) Parabola: from $p = 3$ to $p = 120$; (3) Cubic: from $p = 4$ to $p = 60$.

3. Application. The following computation illustrates the application of these formulas, the data being given by E. T. Whittaker and C. Martin to show the observed magnitudes of the varistar R. W. Cassiopeiae over a period of 14.81 days::

Days	x	Magnitude (y)	$x \cdot y$	$x^2 \cdot y$	$x^3 \cdot y$
0.00	1	9.20	9.20	9.20	9.20
0.62	2	9.52	19.04	38.08	76.16
1.23	3	9.64	28.92	86.76	260.28
1.85	4	9.72	38.88	155.52	622.08
2.47	5	9.89	49.45	247.25	1236.25
3.08	6	9.98	59.88	359.28	2155.68
3.70	7	10.05	75.35	492.45	3447.15
4.32	8	10.21	81.68	653.44	5227.52
4.94	9	10.36	93.24	839.16	7552.44
5.55	10	10.48	104.80	1048.00	10480.00

**Monthly Notices, R. A. S.*, vol. 71 (1911), p. 511; also E. T. Whittaker and G. Robinson: *The Calculus of Observations*, London, (1924) p. 280.

Days	x	Magnitude (y)	$x \cdot y$	$x^2 \cdot y$	$x^3 \cdot y$
6.17	11	10.62	116.82	1285.02	14135.22
6.79	12	10.67	128.04	1536.48	18437.76
7.41	13	10.75	139.75	1816.75	23617.75
8.02	14	10.88	152.32	2132.48	29854.72
8.64	15	10.95	164.25	2463.75	36956.25
9.26	16	11.10	177.60	2841.60	45465.60
9.88	17	11.12	189.04	3213.68	54632.56
10.49	18	11.03	198.54	3573.72	64326.96
11.11	19	10.80	205.20	3898.80	74077.20
11.73	20	10.64	212.80	4256.00	85120.00
12.35	21	10.53	221.13	4643.73	97518.33
12.96	22	10.53	231.66	5096.52	112123.44
13.58	23	10.42	239.66	5512.18	126780.14
14.19	24	9.66	231.84	5564.16	133539.84
14.81	25	9.20	230.00	5750.00	143750.00
		257.95	3394.09	57514.01	1091402.53

From these totals we have, $M_0 = 257.95$, $M_1 = 3394.09$, $M_2 = 57514.01$, $M_3 = 1091402.53$.

Substituting these moments in the formulas of section 2, the values of the coefficients being taken from the tables corresponding to $p = 25$, we obtain the following least square approximations:

$$y = 9.9106 + .031338x ,$$

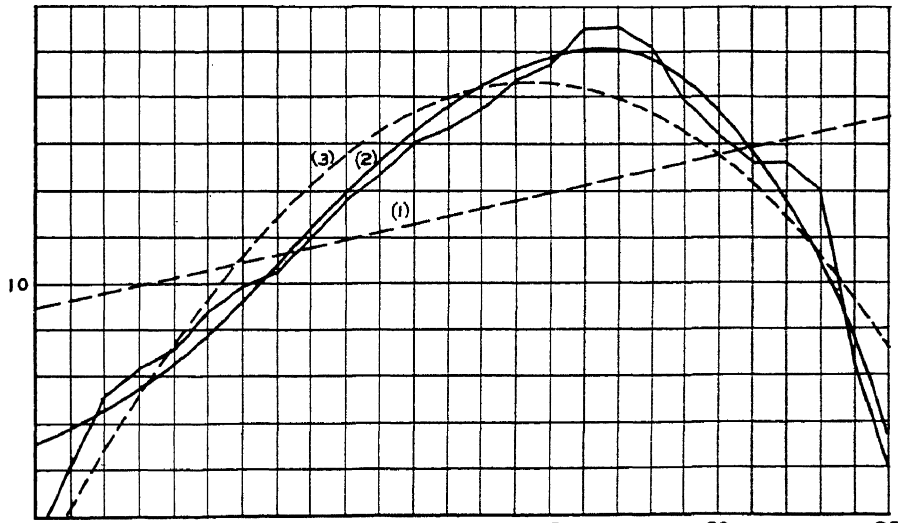
$$y = 8.7102 + .298092x - .010259755x^2 ,$$

$$y = 9.3380 + .033877x + .014654169x^2 - .000638818x^3 .$$

From these equations we compute the following approximating values:

x	Straight line	Parabola	Cubic	x	Straight line	Parabola	Cubic
				13	10.3180	10.8515	10.8515
1	9.9419	8.9980	9.3859	14	10.3493	10.8725	10.9316
2	9.9733	9.2654	9.4593	15	10.3807	10.8731	10.9874
3	10.0046	9.5122	9.5543	16	10.4120	10.8531	11.0150
4	10.0360	9.7384	9.6671	17	10.4434	10.8127	11.0104
5	10.0673	9.9442	9.7939	18	10.4747	10.7517	10.9701
6	10.0986	10.1294	9.9309	19	10.5060	10.6701	10.8902
7	10.1300	10.2942	10.0741	20	10.5374	10.5681	10.7667
8	10.1613	10.4383	10.2199	21	10.5687	10.4454	10.5959
9	10.1926	10.5619	10.3642	22	10.6000	10.3024	10.3737
10	10.2240	10.6651	10.5034	23	10.6314	10.1388	10.0967
11	10.2553	10.7477	10.6336	24	10.6627	9.9546	9.7609
12	10.2867	10.8099	10.7509	25	10.6941	9.7500	9.3623

The closeness of the approximation is exhibited in the accompanying figure.



POLYNOMIAL APPROXIMATION
(The numbers refer to the degrees of the polynomials used in the approximation).

TABLE 47

Coefficients for the Straight Line

Description: The coefficients A, B, and C are computed to ten significant figures for the range $p = 2$ to $p = 240$ by unit intervals. The straight line,

$$y = a_0 + a_1 x \ ,$$

is obtained from the formulas,

$$a_0 = A\, m_0 + B\, m_1 \ ,$$

$$a_1 = B\, m_0 + C\, m_1 \ ,$$

where m_0, m_1 are the moments computed for the range $x = 1$ to $x = p$.

(The numbers in parentheses denote the number of ciphers between the decimal point and the first significant figure.)

p	A	B	C
2	5.000 0000 000	−3.000 0000 000	2.000 0000 000
3	2.333 3333 333	−1.000 0000 000	.500 0000 000
4	1.500 0000 000	−.500 0000 000	.200 0000 000
5	1.100 0000 000	−.300 0000 000	.100 0000 000
6	.866 6666 667	−.200 0000 000	(1) 571 4285 714
7	.714 2857 143	−.142 8571 429	(1) 357 1428 571
8	.607 1428 571	−.107 1428 571	(1) 238 0952 381
9	.527 7777 778	−(1) 833 3333 333	(1) 166 6666 667
10	.466 6666 667	−(1) 666 6666 667	(1) 121 2121 212
11	.418 1818 182	−(1) 545 4545 455	(2) 909 0909 091
12	.378 7878 788	−(1) 454 5454 545	(2) 699 3006 993
13	.346 1538 462	−(1) 384 6153 846	(2) 549 4505 495
14	.318 6813 187	−(1) 329 6703 297	(2) 439 5604 396
15	.295 2380 952	−(1) 285 7142 857	(2) 357 1428 571
16	.275 0000 000	−(1) 250 0000 000	(2) 294 1176 471
17	.257 3529 412	−(1) 220 5882 353	(2) 245 0980 392
18	.241 8300 654	−(1) 196 0784 314	(2) 206 3983 488
19	.228 0701 754	−(1) 175 4385 965	(2) 175 4385 965
20	.215 7894 737	−(1) 157 8947 368	(2) 150 3759 398
21	.204 7619 048	−(1) 142 8571 429	(2) 129 8701 299
22	.194 8051 948	−(1) 129 8701 299	(2) 112 9305 477
23	.185 7707 510	−(1) 118 5770 751	(3) 988 1422 925
24	.177 5362 319	−(1) 108 6956 522	(3) 869 5652 174
25	.170 0000 000	−(1) 100 0000 000	(3) 769 2307 692
26	.163 0769 231	−(2) 923 0769 231	(3) 683 7606 838
27	.156 6951 567	−(2) 854 7008 547	(3) 610 5006 105
28	.150 7936 508	−(2) 793 6507 937	(3) 547 3453 749
29	.145 3201 970	−(2) 738 9162 562	(3) 492 6108 374
30	.140 2298 851	−(2) 689 6551 724	(3) 444 9388 209
31	.135 4838 710	−(2) 645 1612 903	(3) 403 2258 065
32	.131 0483 871	−(2) 604 8387 097	(3) 366 5689 150
33	.126 8939 394	−(2) 568 1818 182	(3) 334 2245 989
34	.122 9946 524	−(2) 534 7593 583	(3) 305 5767 762
35	.119 3277 311	−(2) 504 2016 807	(3) 280 1120 448
36	.115 8730 159	−(2) 476 1904 762	(3) 257 4002 574
37	.112 6126 126	−(2) 450 4504 505	(3) 237 0791 844
38	.109 5305 832	−(2) 426 7425 320	(3) 218 8423 241
39	.106 6126 856	−(2) 404 8582 996	(3) 202 4291 498
40	.103 8461 538	−(2) 384 6153 846	(3) 187 6172 608
41	.101 2195 122	−(2) 365 8536 585	(3) 174 2160 279
42	(1) 987 2241 580	−(2) 348 4320 557	(3) 162 0614 213
43	(1) 963 4551 495	−(2) 332 2259 136	(3) 151 0117 789
44	(1) 940 8033 827	−(2) 317 1247 357	(3) 140 9443 270
45	(1) 919 1919 192	−(2) 303 0303 030	(3) 131 7523 057
46	(1) 898 5507 246	−(2) 289 8550 725	(3) 123 3425 840
47	(1) 878 8159 112	−(2) 277 5208 141	(3) 115 6336 725
48	(1) 859 9290 780	−(2) 265 9574 468	(3) 108 5540 599
49	(1) 841 8367 347	−(2) 255 1020 408	(3) 102 0408 163
50	(1) 824 4897 959	−(2) 244 8979 592	(4) 960 3841 537

p	A	B	C
51	(1)807 8431 373	−(2)235 2941 176	(4)904 9773 756
52	(1)791 8552 036	−(2)226 2443 439	(4)853 7522 411
53	(1)776 4876 633	−(2)217 7068 215	(4)806 3215 610
54	(1)761 7051 013	−(2)209 6436 059	(4)762 3403 843
55	(1)747 4747 475	−(2)202 0202 020	(4)721 5007 215
56	(1)733 7662 338	−(2)194 8051 948	(4)683 5269 993
57	(1)720 5513 784	−(2)187 9699 248	(4)648 1721 545
58	(1)707 8039 927	−(2)181 4882 033	(4)615 2142 484
59	(1)695 4997 078	−(2)175 3360 608	(4)584 4535 359
60	(1)683 6158 192	−(2)169 4915 254	(4)555 7099 194
61	(1)672 1311 475	−(2)163 9344 262	(4)528 8207 298
62	(1)661 0259 122	−(2)158 6462 189	(4)503 6387 903
63	(1)650 2816 180	−(2)153 6098 310	(4)480 0307 220
64	(1)639 8809 524	−(2)148 8095 238	(4)457 8754 579
65	(1)629 8076 923	−(2)144 2307 692	(4)437 0629 371
66	(1)620 0466 200	−(2)139 8601 399	(4)417 4929 548
67	(1)610 5834 464	−(2)135 6852 103	(4)399 0741 480
68	(1)601 4047 410	−(2)131 6944 688	(4)381 7230 981
69	(1)592 4978 687	−(2)127 8772 379	(4)365 3635 367
70	(1)583 8509 317	−(2)124 2236 025	(4)349 9256 408
71	(1)575 4527 163	−(2)120 7243 461	(4)335 3454 058
72	(1)567 2926 448	−(2)117 3708 920	(4)321 5640 877
73	(1)559 3607 306	−(2)114 1552 511	(4)308 5277 058
74	(1)551 6475 379	−(2)111 0699 741	(4)296 1865 976
75	(1)544 1441 441	−(2)108 1081 081	(4)284 4950 213
76	(1)536 8421 053	−(2)105 2631 579	(4)273 4107 997
77	(1)529 7334 245	−(2)102 5290 499	(4)262 8949 997
78	(1)522 8105 228	−(3)999 0009 990	(4)252 9116 453
79	(1)516 0662 123	−(3)973 7098 345	(4)243 4274 586
80	(1)509 4936 709	−(3)949 3670 886	(4)234 4116 268
81	(1)503 0864 198	−(3)925 9259 259	(4)225 8355 917
82	(1)496 8383 017	−(3)903 3423 668	(4)217 6728 595
83	(1)490 7434 617	−(3)881 5750 808	(4)209 8988 288
84	(1)484 7963 282	−(3)860 5851 979	(4)202 4906 348
85	(1)478 9915 966	−(3)840 3361 345	(4)195 4270 080
86	(1)473 3242 134	−(3)820 7934 337	(4)188 6881 457
87	(1)467 7893 611	−(3)801 9246 191	(4)182 2555 952
88	(1)462 3824 451	−(3)783 6990 596	(4)176 1121 482
89	(1)457 0990 807	−(3)766 0878 447	(4)170 2417 433
90	(1)451 9350 811	−(3)749 0636 704	(4)164 6293 781
91	(1)446 8864 469	−(3)732 6007 326	(4)159 2610 288
92	(1)441 9493 550	−(3)716 6746 297	(4)154 1235 763
93	(1)437 1201 496	−(3)701 2622 721	(4)149 2047 387
94	(1)432 3953 329	−(3)686 3417 982	(4)144 4930 102
95	(1)427 7715 566	−(3)671 8924 972	(4)139 9776 036
96	(1)423 2456 140	−(3)657 8947 368	(4)135 6483 993
97	(1)418 8144 330	−(3)644 3298 969	(4)131 4958 973
98	(1)414 4750 681	−(3)631 1803 072	(4)127 5111 732
99	(1)410 2246 959	−(3)618 4291 899	(4)123 6858 380
100	(1)406 0606 061	−(3)606 0606 061	(4)120 0120 012

p	A	B	C
101	(1)401 9801 980	−(3)594 0594 059	(4)116 4822 365
102	(1)397 9809 746	−(3)582 4111 823	(4)113 0895 500
103	(1)394 0605 368	−(3)571 1022 273	(4)109 8273 514
104	(1)390 2165 795	−(3)560 1194 922	(4)106 6894 271
105	(1)386 4468 864	−(3)549 4505 495	(4)103 6699 150
106	(1)382 7493 261	−(3)539 0835 580	(4)100 7632 819
107	(1)379 1218 480	−(3)529 0072 298	(5)979 6430 181
108	(1)375 5624 784	−(3)519 2107 996	(5)952 6803 662
109	(1)372 0693 170	−(3)509 6839 959	(5)926 6981 744
110	(1)368 6405 338	−(3)500 4170 142	(5)901 6522 778
111	(1)365 2743 653	−(3)491 4004 914	(5)877 5008 775
112	(1)361 9691 120	−(3)482 6254 826	(5)854 2043 940
113	(1)358 7231 353	−(3)474 0834 387	(5)831 7253 310
114	(1)355 5348 548	−(3)465 7661 854	(5)810 0281 485
115	(1)352 4027 460	−(3)457 6659 039	(5)789 0791 446
116	(1)349 3253 373	−(3)449 7751 124	(5)768 8463 461
117	(1)346 3012 084	−(3)442 0866 490	(5)749 2994 051
118	(1)343 3289 874	−(3)434 5936 549	(5)730 4095 041
119	(1)340 4073 494	−(3)427 2895 599	(5)712 1492 665
120	(1)337 5350 140	−(3)420 1680 672	(5)694 4926 731
121	(1)334 7107 438	−(3)413 2231 405	(5)677 4149 844
122	(1)331 9333 424	−(3)406 4489 907	(5)660 8926 677
123	(1)329 2016 527	−(3)399 8400 640	(5)644 9033 290
124	(1)326 5145 555	−(3)393 3910 307	(5)629 4256 491
125	(1)323 8709 677	−(3)387 0967 742	(5)614 4393 241
126	(1)321 2698 413	−(3)380 9523 810	(5)599 9250 094
127	(1)318 7101 612	−(3)374 9531 309	(5)585 8642 670
128	(1)316 1909 449	−(3)369 0944 882	(5)572 2395 166
129	(1)313 7112 403	−(3)363 3720 930	(5)559 0339 893
130	(1)311 2701 252	−(3)357 7817 531	(5)546 2316 842
131	(1)308 8667 058	−(3)352 3194 363	(5)533 8173 277
132	(1)306 5001 157	−(3)346 9812 630	(5)521 7763 354
133	(1)304 1695 147	−(3)341 7634 997	(5)510 0947 756
134	(1)301 8740 882	−(3)336 6625 519	(5)498 7593 362
135	(1)299 6130 459	−(3)331 6749 585	(5)487 7572 920
136	(1)297 3856 209	−(3)326 7973 856	(5)477 0764 754
137	(1)295 1910 691	−(3)322 0266 209	(5)466 7052 476
138	(1)293 0286 681	−(3)317 3595 684	(5)456 6324 725
139	(1)290 8977 166	−(3)312 7932 437	(5)446 8474 910
140	(1)288 7975 334	−(3)308 3247 688	(5)437 3400 975
141	(1)286 7274 569	−(3)303 9513 678	(5)428 1005 180
142	(1)284 6868 445	−(3)299 6703 626	(5)419 1193 883
143	(1)282 6750 714	−(3)295 4791 687	(5)410 3877 343
144	(1)280 6915 307	−(3)291 3752 914	(5)401 8969 536
145	(1)278 7356 322	−(3)287 3563 218	(5)393 6387 970
146	(1)276 8068 021	−(3)283 4199 339	(5)385 6053 522
147	(1)274 9044 823	−(3)279 5638 803	(5)377 7890 275
148	(1)273 0281 302	−(3)275 7859 901	(5)370 1825 370
149	(1)271 1772 175	−(3)272 0841 647	(5)362 7788 863
150	(1)269 3512 304	−(3)268 4563 758	(5)355 5713 587

p	A	B	C
151	(1)267 5496 689	−(3)264 9006 623	(5)348 5535 030
152	(1)265 7720 460	−(3)261 4151 272	(5)341 7191 206
153	(1)264 0178 879	−(3)257 9979 360	(5)335 0622 546
154	(1)262 2867 329	−(3)254 6473 135	(5)328 5771 787
155	(1)260 5781 315	−(3)251 3615 417	(5)322 2583 868
156	(1)258 8916 460	−(3)248 1389 578	(5)316 1005 832
157	(1)257 2268 496	−(3)244 9779 520	(5)310 0986 734
158	(1)255 5833 266	−(3)241 8769 653	(5)304 2477 550
159	(1)253 9606 719	−(3)238 8344 877	(5)298 5431 096
160	(1)252 3584 906	−(3)235 8490 566	(5)292 9801 945
161	(1)250 7763 975	−(3)232 9192 547	(5)287 5546 354
162	(1)249 2140 173	−(3)230 0437 083	(5)282 2622 188
163	(1)247 6709 839	−(3)227 2210 861	(5)277 0988 855
164	(1)246 1469 400	−(3)224 4500 973	(5)272 0607 240
165	(1)244 6415 373	−(3)221 7294 900	(5)267 1439 639
166	(1)243 1544 359	−(3)219 0580 504	(5)262 3449 709
167	(1)241 6853 041	−(3)216 4346 007	(5)257 6602 389
168	(1)240 2338 181	−(3)213 8579 983	(5)253 0863 885
169	(1)238 7996 619	−(3)211 3271 344	(5)248 6201 581
170	(1)237 3825 270	−(3)208 8409 328	(5)244 2584 010
171	(1)235 9821 121	−(3)206 3983 488	(5)239 9980 800
172	(1)234 5981 232	−(3)203 9983 680	(5)235 8362 636
173	(1)233 2302 729	−(3)201 6400 054	(5)231 7701 211
174	(1)231 8782 805	−(3)199 3223 042	(5)227 7969 190
175	(1)230 5418 719	−(3)197 0443 350	(5)223 9140 170
176	(1)229 2207 792	−(3)194 8051 948	(5)220 1188 642
177	(1)227 9147 406	−(3)192 6040 062	(5)216 4089 957
178	(1)226 6235 003	−(3)190 4399 162	(5)212 7820 293
179	(1)225 3468 081	−(3)188 3120 959	(5)209 2356 621
180	(1)224 0844 196	−(3)186 2197 393	(5)205 7676 677
181	(1)222 8360 958	−(3)184 1620 626	(5)202 3758 930
182	(1)221 6016 028	−(3)182 1383 037	(5)199 0582 554
183	(1)220 3807 122	−(3)180 1477 211	(5)195 8127 404
184	(1)219 1732 003	−(3)178 1895 937	(5)192 6373 986
185	(1)217 9788 484	−(3)176 2632 197	(5)189 5303 438
186	(1)216 7974 426	−(3)174 3679 163	(5)186 4897 501
187	(1)215 6287 735	−(3)172 5030 188	(5)183 5138 498
188	(1)214 4726 362	−(3)170 6678 803	(5)180 6009 315
189	(1)213 3288 304	−(3)168 8618 710	(5)177 7493 379
190	(1)212 1971 596	−(3)167 0843 776	(5)174 9574 635
191	(1)211 0774 318	−(3)165 3348 030	(5)172 2237 531
192	(1)209 9694 590	−(3)163 6125 654	(5)169 5466 999
193	(1)208 8730 570	−(3)161 9170 984	(5)166 9248 438
194	(1)207 7880 455	−(3)160 2478 501	(5)164 3567 692
195	(1)206 7142 480	−(3)158 6042 823	(5)161 8411 044
196	(1)205 6514 914	−(3)156 9858 713	(5)159 3765 191
197	(1)204 5996 063	−(3)155 3921 061	(5)156 9617 233
198	(1)203 5584 269	−(3)153 8224 888	(5)154 5954 662
199	(1)202 5277 905	−(3)152 2765 342	(5)152 2765 342
200	(1)201 5075 377	−(3)150 7537 688	(5)150 0037 501

B

201	(1)200 4975 124	-(3)149 2537 313	(5)147 7759 716
202	(1)199 4975 617	-(3)147 7759 716	(5)145 5920 903
203	(1)198 5075 355	-(3)146 3200 507	(5)143 4510 301
204	(1)197 5272 868	-(3)144 8855 404	(5)141 3517 468
205	(1)196 5566 714	-(3)143 4720 230	(5)139 2932 262
206	(1)195 5955 482	-(3)142 0790 907	(5)137 2744 838
207	(1)194 6437 784	-(3)140 7063 459	(5)135 2945 633
208	(1)193 7012 263	-(3)139 3534 002	(5)133 3525 361
209	(1)192 7677 586	-(3)138 0198 749	(5)131 4474 999
210	(1)191 8432 445	-(3)136 7053 999	(5)129 5785 781
211	(1)190 9275 559	-(3)135 4096 141	(5)127 7449 189
212	(1)190 0205 669	-(3)134 1321 649	(5)125 9456 947
213	(1)189 1221 543	-(3)132 8727 079	(5)124 1801 009
214	(1)188 2321 969	-(3)131 6309 069	(5)122 4473 553
215	(1)187 3505 760	-(3)130 4064 334	(5)120 7466 976
216	(1)186 4771 748	-(3)129 1989 664	(5)119 0773 884
217	(1)185 6118 792	-(3)128 0081 925	(5)117 4387 087
218	(1)184 7545 766	-(3)126 8338 054	(5)115 8299 593
219	(1)183 9051 569	-(3)125 6755 058	(5)114 2504 599
220	(1)183 0635 118	-(3)124 5330 012	(5)112 6995 486
221	(1)182 2295 352	-(3)123 4060 058	(5)111 1765 818
222	(1)181 4031 226	-(3)122 2942 399	(5)109 6809 327
223	(1)180 5841 716	-(3)121 1974 306	(5)108 2119 916
224	(1)179 7725 817	-(3)120 1153 107	(5)106 7691 651
225	(1)178 9682 540	-(3)119 0476 190	(5)105 3518 753
226	(1)178 1710 914	-(3)117 9941 003	(5)103 9595 597
227	(1)177 3809 988	-(3)116 9545 047	(5)102 5916 708
228	(1)176 5978 824	-(3)115 9285 880	(5)101 2476 751
229	(1)175 8216 502	-(3)114 9161 112	(6)999 2705 325
230	(1)175 0522 119	-(3)113 9168 407	(6)986 2929 931
231	(1)174 2894 786	-(3)112 9305 477	(6)973 5392 044
232	(1)173 5333 632	-(3)111 9570 085	(6)961 0043 649
233	(1)172 7837 798	-(3)110 9960 041	(6)948 6837 961
234	(1)172 0406 441	-(3)110 0473 203	(6)936 5729 391
235	(1)171 3038 734	-(3)109 1107 474	(6)924 6673 509
236	(1)170 5733 862	-(3)108 1860 801	(6)912 9627 009
237	(1)169 8491 025	-(3)107 2731 174	(6)901 4547 677
238	(1)169 1309 435	-(3)106 3716 626	(6)890 1394 359
239	(1)168 4188 320	-(3)105 4815 232	(6)879 0126 929
240	(1)167 7126 918	-(3)104 6025 105	(6)868 0706 262

TABLE 48

Coefficients for the Parabola

Description: The coefficients A, B, C, D, E, F are computed to ten significant figures for the range $p = 3$ to $p = 120$ by unit intervals. The parabola,

$$y = a_0 + a_1 + a_2 x^2 \ ,$$

is obtained from the formulas,

$$a_0 = A\, m_0 + B\, m_1 + C\, m_2 \ ,$$

$$a_1 = B\, m_0 + D\, m_1 + E\, m_2 \ ,$$

$$a_2 = C\, m_0 + E\, m_1 + F\, m_2 \ ,$$

where m_0, m_1, m_2 are the moments computed for the range $x = 1$ to $x = p$.

(The numbers in parentheses denote the number of ciphers between the decimal point and the first significant figure.)

p	A	B	C
3	19.000 0000 0	−21.000 0000 0	5.000 0000 00
4	7.750 0000 00	−6.750 0000 00	1.250 0000 00
5	4.600 0000 00	−3.300 0000 00	.500 0000 000
6	3.200 0000 00	−1.950 0000 00	.250 0000 000
7	2.428 5714 29	−1.285 7142 86	.142 8571 429
8	1.946 4285 71	−.910 7142 857	(1)892 8571 429
9	1.619 0476 19	−.678 5714 286	(1)595 2380 952
10	1.383 3333 33	−.525 0000 000	(1)416 6666 667
11	1.206 0606 06	−.418 1818 182	(1)303 0303 030
12	1.068 1818 18	−.340 9090 909	(1)227 2727 273
13	.958 0419 580	−.283 2167 832	(1)174 8251 748
14	.868 1318 681	−.239 0109 890	(1)137 3626 374
15	.793 4065 934	−.204 3956 044	(1)109 8901 099
16	.730 3571 429	−.176 7857 143	(2)892 8571 429
17	.676 4705 882	−.154 4117 647	(2)735 2941 176
18	.629 9019 608	−.136 0294 118	(2)612 7450 980
19	.589 2672 859	−.120 7430 341	(2)515 9958 720
20	.553 5087 719	−.107 8947 368	(2)438 5964 912
21	.521 8045 113	−(1)969 9248 120	(2)375 9398 496
22	.493 5064 935	−(1)876 6233 766	(2)324 6753 247
23	.468 0971 203	−(1)796 1603 614	(2)282 3263 693
24	.445 1581 028	−(1)726 2845 850	(2)247 0355 731
25	.424 3478 261	−(1)665 2173 913	(2)217 3913 043
26	.405 3846 154	−(1)611 5384 615	(2)192 3076 923
27	.388 0341 880	−(1)564 1025 641	(2)170 9401 709
28	.372 1001 221	−(1)521 9780 220	(2)152 6251 526
29	.357 4165 298	−(1)484 4006 568	(2)136 8363 437
30	.343 8423 645	−(1)450 7389 163	(2)123 1527 094
31	.331 2569 522	−(1)420 4671 857	(2)111 2347 052
32	.319 5564 516	−(1)393 1451 613	(2)100 8064 516
33	.308 6510 264	−(1)368 4017 595	(3)916 4222 874
34	.298 4625 668	−(1)345 9224 599	(3)835 5614 973
35	.288 9228 419	−(1)325 4392 666	(3)763 9419 404
36	.279 9719 888	−(1)306 7226 891	(3)700 2801 120
37	.271 5572 716	−(1)289 5752 896	(3)643 5006 435
38	.263 6320 531	−(1)273 8264 580	(3)592 6979 611
39	.256 1549 404	−(1)259 3281 541	(3)547 1058 103
40	.249 0890 688	−(1)245 9514 170	(3)506 0728 745
41	.242 4015 009	−(1)233 5834 897	(3)469 0431 520
42	.236 0627 178	−(1)222 1254 355	(3)435 5400 697
43	.230 0461 874	−(1)211 4901 548	(3)405 1535 532
44	.224 3279 976	−(1)201 6007 248	(3)377 5294 473
45	.218 8865 398	−(1)192 3890 063	(3)352 3608 175
46	.213 7022 398	−(1)183 7944 664	(3)329 3807 642
47	.208 7573 235	−(1)175 7631 822	(3)308 3564 601
48	.204 0356 152	−(1)168 2469 935	(3)289 0841 813
49	.199 5223 621	−(1)161 2027 790	(3)271 3851 498
50	.195 2040 816	−(1)154 5918 367	(3)255 1020 408

p	A	B	C
51	.191 0684 274	−(1)148 3793 517	(3)240 0960 384
52	.187 1040 724	−(1)142 5339 367	(3)226 2443 439
53	.183 3006 062	−(1)137 0272 347	(3)213 4380 603
54	.179 6484 438	−(1)131 8335 752	(3)201 5803 903
55	.176 1387 460	−(1)126 9296 741	(3)190 5850 962
56	.172 7633 478	−(1)122 2943 723	(3)180 3751 804
57	.169 5146 958	−(1)117 9084 074	(3)170 8817 498
58	.166 3857 921	−(1)113 7542 131	(3)162 0430 386
59	.163 3701 437	−(1)109 8157 433	(3)153 8035 621
60	.160 4617 183	−(1)106 0783 168	(3)146 1133 840
61	.157 6549 041	−(1)102 5284 801	(3)138 9274 799
62	.154 9444 738	−(2)991 5388 683	(3)132 2051 824
63	.152 3255 521	−(2)959 4318 954	(3)125 9096 976
64	.149 7935 868	−(2)928 8594 470	(3)120 0076 805
65	.147 3443 223	−(2)899 7252 747	(3)114 4688 645
66	.144 9737 762	−(2)871 9405 594	(3)109 2657 343
67	.142 6782 173	−(2)845 4232 335	(3)104 3732 387
68	.140 4541 464	−(2)820 0973 741	(4)997 6853 699
69	.138 2982 784	−(2)795 8926 595	(4)954 3077 452
70	.136 2075 265	−(2)772 7438 801	(4)913 4088 418
71	.134 1789 870	−(2)750 5904 995	(4)874 8141 020
72	.132 2099 262	−(2)729 3762 575	(4)838 3635 144
73	.130 2977 683	−(2)709 0488 134	(4)803 9102 193
74	.128 4400 839	−(2)689 5594 224	(4)771 3192 645
75	.126 6345 798	−(2)670 8626 435	(4)740 4664 939
76	.124 8790 896	−(2)652 9160 740	(4)711 2375 533
77	.123 1715 653	−(2)635 6801 094	(4)683 5269 993
78	.121 5100 689	−(2)619 1177 243	(4)657 2374 993
79	.119 8927 655	−(2)603 1942 741	(4)632 2791 133
80	.118 3179 163	−(2)587 8773 126	(4)608 5686 465
81	.116 7838 725	−(2)573 1364 276	(4)586 0290 670
82	.115 2890 696	−(2)558 9430 894	(4)564 5889 792
83	.113 8320 219	−(2)545 2705 128	(4)544 1821 486
84	.112 4113 177	−(2)532 0935 309	(4)524 7470 719
85	.111 0256 151	−(2)519 3884 783	(4)506 2265 870
86	.109 6736 369	−(2)507 1330 858	(4)488 5675 200
87	.108 3541 676	−(2)495 3063 824	(4)471 7203 642
88	.107 0660 494	−(2)483 8886 054	(4)455 6389 881
89	.105 8081 786	−(2)472 8611 180	(4)440 2803 705
90	.104 5795 029	−(2)462 2063 330	(4)425 6043 582
91	.103 3790 180	−(2)451 9076 429	(4)411 5734 453
92	.102 2057 652	−(2)441 9493 550	(4)398 1525 721
93	.101 0588 290	−(2)432 3166 315	(4)385 3089 407
94	(1)999 3733 401	−(2)422 9954 343	(4)373 0118 469
95	(1)988 4044 359	−(2)413 9724 741	(4)361 2324 254
96	(1)977 6735 722	−(2)405 2351 624	(4)349 9440 090
97	(1)967 1730 874	−(2)396 7715 681	(4)339 1209 984
98	(1)956 8956 449	−(2)388 5703 766	(4)328 7397 433
99	(1)946 8342 163	−(2)380 6208 519	(4)318 7779 329
100	(1)936 9820 656	−(2)372 9128 015	(3)309 2145 949

p	A	B	C
101	(1)927 3327 333	—(2)365 4365 437	(4)300 0300 030
102	(1)917 8800 233	—(2)358 1828 771	(4)291 2055 911
103	(1)908 6179 892	—(2)351 1430 526	(4)282 7238 749
104	(1)899 5409 217	—(2)344 3087 467	(4)274 5683 785
105	(1)890 6433 372	—(2)337 6720 368	(4)266 7235 677
106	(1)881 9199 668	—(2)331 2253 784	(4)259 1747 875
107	(1)873 3657 455	—(2)324 9615 840	(4)251 9082 047
108	(1)864 9758 028	—(2)318 8738 024	(4)244 9107 545
109	(1)856 7454 533	—(2)312 9555 003	(4)238 1700 916
110	(1)848 6701 881	—(2)307 2004 448	(4)231 6745 436
111	(1)840 7456 664	—(2)301 6026 369	(4)225 4130 695
112	(1)832 9677 080	—(2)296 1565 462	(4)219 3752 194
113	(1)825 3322 855	—(2)290 8565 962	(4)213 5510 985
114	(1)817 8355 180	—(2)285 6976 512	(4)207 9313 328
115	(1)810 4736 640	—(2)280 6747 534	(4)202 5070 371
116	(1)803 2431 153	—(2)275 7831 611	(4)197 2697 862
117	(1)796 1403 913	—(2)271 0183 370	(4)192 2115 865
118	(1)789 1621 334	—(2)266 3759 385	(4)187 3248 513
119	(1)782 3050 994	—(2)261 8518 072	(4)182 6023 760
120	(1)775 5661 587	—(2)257 4419 598	(4)178 0373 166

p	D	E	F
3	24.500 0000 0	—6.000 0000 00	1.500 0000 00
4	6.450 0000 00	—1.250 0000 00	.250 0000 000
5	2.671 4285 71	—.428 5714 286	(1)714 2857 143
6	1.369 6428 57	—.187 5000 000	(1)267 8571 429
7	.797 6190 476	—(1)952 3809 524	(1)119 0476 190
8	.505 9523 810	—(1)535 7142 857	(2)595 2380 952
9	.341 3419 913	—(1)324 6753 247	(2)324 6753 247
10	.241 2878 788	—(1)208 3333 333	(2)189 3939 394
11	.176 9230 769	—(1)139 8601 399	(2)116 5501 166
12	.133 6163 836	—(2)974 0259 740	(3)749 2507 493
13	.103 3966 034	—(2)699 3006 993	(3)499 5004 995
14	(1)816 6208 791	—(2)515 1098 901	(3)343 4065 934
15	(1)656 2702 004	—(2)387 8474 467	(3)242 4046 542
16	(1)535 3641 457	—(2)297 6190 476	(3)175 0700 280
17	(1)442 4664 603	—(2)232 1981 424	(3)128 9989 680
18	(1)369 9045 408	—(2)183 8235 294	(4)967 4922 601
19	(1)312 3986 437	—(2)147 4273 920	(4)737 1369 600
20	(1)266 2337 662	—(2)119 6172 249	(4)569 6058 328
21	(1)228 7437 962	—(3)980 7126 512	(4)445 7784 778
22	(1)197 9813 665	—(3)811 6883 117	(4)352 9079 616
23	(1)172 5014 116	—(3)677 5832 863	(4)282 3263 693
24	(1)151 2161 751	—(3)570 0820 918	(4)228 0328 367
25	(1)133 2961 724	—(3)483 0917 874	(4)185 8045 336
26	(1)118 1013 431	—(3)412 0879 121	(4)152 6251 526
27	(1)105 1324 155	—(3)353 6693 192	(4)126 3104 711
28	(2)939 9604 227	—(3)305 2503 053	(4)105 2587 260
29	(2)843 7946 925	—(3)264 8445 363	(5)882 8151 209
30	(2)760 3190 052	—(3)230 9113 301	(5)744 8752 582

p	D	E	F
31	(2)687 5063 202	−(3)202 2449 186	(5)632 0153 706
32	(2)623 7062 274	−(3)177 8937 381	(5)539 0719 338
33	(2)567 5657 360	−(3)157 1009 636	(5)462 0616 575
34	(2)517 9685 511	−(3)139 2602 496	(5)397 8864 273
35	(2)473 9881 210	−(3)123 8824 768	(5)344 1179 912
36	(2)434 8510 386	−(3)110 5705 440	(5)298 8393 081
37	(2)399 9082 946	−(4)990 0009 900	(5)260 5265 763
38	(2)368 6125 397	−(4)889 0469 417	(5)227 9607 543
39	(2)340 4999 746	−(4)800 6426 492	(5)200 1606 623
40	(2)315 1758 383	−(4)722 9612 493	(5)176 3320 120
41	(2)292 3027 058	−(4)654 4788 167	(5)155 8282 897
42	(2)271 5910 012	−(4)593 9182 768	(5)138 1205 295
43	(2)252 7912 624	−(4)540 2047 376	(5)122 7738 040
44	(2)235 6878 040	−(4)492 4297 139	(5)109 4288 253
45	(2)220 0934 979	−(4)449 8223 202	(6)977 8746 091
46	(2)205 8454 573	−(4)411 7259 552	(6)876 0126 706
47	(2)192 8014 499	−(4)377 5793 389	(6)786 6236 226
48	(2)180 8369 046	−(4)346 9010 176	(6)707 9612 604
49	(2)169 8424 050	−(4)319 2766 468	(6)638 5532 937
50	(2)159 7215 809	−(4)294 3485 086	(6)577 1539 385
51	(2)150 3893 284	−(4)271 8068 359	(6)522 7054 537
52	(2)141 7703 027	−(4)251 3826 043	(6)474 3068 006
53	(2)133 7976 366	−(4)232 8415 203	(6)431 1880 006
54	(2)126 4118 481	−(4)215 9789 896	(6)392 6890 719
55	(2)119 5599 061	−(4)200 6158 908	(6)358 2426 621
56	(2)113 1944 281	−(4)186 5950 142	(6)327 3596 740
57	(2)107 2729 909	−(4)173 7780 507	(6)299 6173 287
58	(2)101 7575 353	−(4)162 0430 386	(6)274 6492 180
59	(3)966 1385 069	−(4)151 2821 922	(6)252 1369 870
60	(3)918 1112 910	−(4)141 4000 490	(6)231 8033 590
61	(3)873 2157 636	−(4)132 3118 856	(6)213 4062 671
62	(3)831 2007 378	−(4)123 9423 585	(6)196 7339 024
63	(3)791 8388 239	−(4)116 2243 362	(6)181 6005 253
64	(3)754 9238 396	−(4)109 0978 914	(6)167 8429 098
65	(3)720 2685 374	−(4)102 5094 309	(6)155 3173 195
66	(3)687 7026 069	−(5)964 1094 200	(6)143 8969 284
67	(3)657 0709 132	−(5)907 5933 800	(6)133 4696 147
68	(3)628 2319 429	−(5)855 1588 885	(6)123 9360 708
69	(3)601 0564 283	−(5)806 4572 494	(6)115 2081 785
70	(3)575 4261 288	−(5)761 1740 348	(6)107 2076 105
71	(3)551 2327 489	−(5)719 0252 893	(7)998 6462 352
72	(3)528 3769 754	−(5)679 7542 009	(7)931 1701 382
73	(3)506 7676 204	−(5)643 1281 754	(7)869 0921 290
74	(3)486 3208 558	−(5)608 9362 614	(7)811 9150 152
75	(3)466 9595 297	−(5)576 9868 784	(7)759 1932 610
76	(3)448 6125 539	−(5)547 1058 103	(7)710 5270 263
77	(3)431 2143 553	−(5)519 1344 299	(7)665 5569 614
78	(3)414 7043 829	−(5)492 9281 245	(7)623 9596 513
79	(3)399 0266 648	−(5)468 3548 987	(7)585 4436 234
80	(3)384 1294 093	−(5)445 2941 316	(7)549 7458 415

p	D	E	F
81	(3)369 9646 447	-(5)423 6354 702	(7)516 6286 221
82	(3)356 4878 950	-(5)403 2778 423	(7)485 8769 184
83	(3)343 6578 863	-(5)384 1285 755	(7)457 2959 232
84	(3)331 4362 805	-(5)366 1026 083	(7)430 7089 510
85	(3)319 7874 352	-(5)349 1217 842	(7)405 9555 630
86	(3)308 6781 844	-(5)333 1142 182	(7)382 8899 060
87	(3)298 0776 407	-(5)318 0137 287	(7)361 3792 371
88	(3)287 9570 144	-(5)303 7593 254	(7)341 3026 128
89	(3)278 2894 492	-(5)290 2947 498	(7)322 5497 220
90	(3)269 0498 721	-(5)277 5680 597	(7)305 0198 458
91	(3)260 2148 575	-(5)265 5312 550	(7)288 6209 294
92	(3)251 7625 015	-(5)254 1399 396	(7)273 2687 523
93	(3)243 6723 081	-(5)243 3530 152	(7)258 8861 864
94	(3)235 9250 851	-(5)233 1324 043	(7)245 4025 308
95	(3)228 5028 476	-(5)223 4427 992	(7)232 7529 158
96	(3)221 3887 310	-(5)214 2514 341	(7)220 8777 671
97	(3)214 5669 100	-(5)205 5278 778	(7)209 7223 243
98	(3)208 0225 248	-(5)197 2438 460	(7)199 2362 081
99	(3)201 7416 133	-(5)189 3730 295	(7)189 3730 295
100	(3)195 7110 477	-(5)181 8909 382	(7)180 0900 378
101	(3)189 9184 779	-(5)174 7747 590	(7)171 3478 030
102	(3)184 3522 774	-(5)168 0032 257	(7)163 1099 278
103	(3)179 0014 951	-(5)161 5564 999	(7)155 3427 884
104	(3)173 8558 092	-(5)155 4160 633	(7)148 0152 984
105	(3)168 9054 859	-(5)149 5646 174	(7)141 0986 957
106	(3)164 1413 408	-(5)143 9859 930	(7)134 5663 486
107	(3)159 5547 023	-(5)138 6650 668	(7)128 3935 804
108	(3)155 1373 795	-(5)133 5876 843	(7)122 5575 085
109	(3)150 8816 308	-(5)128 7405 900	(7)117 0369 000
110	(3)146 7801 353	-(5)124 1113 626	(7)111 8120 384
111	(3)142 8259 669	-(5)119 6883 555	(7)106 8646 031
112	(3)139 0125 691	-(5)115 4606 418	(7)102 1775 591
113	(3)135 3337 328	-(5)111 4179 644	(8)977 3505 652
114	(3)131 7835 742	-(5)107 5506 894	(8)935 2233 857
115	(3)128 3565 161	-(5)103 8497 626	(8)895 2565 744
116	(3)125 0472 684	-(5)100 3066 709	(8)857 3219 737
117	(3)121 8508 119	-(6)969 1340 497	(8)821 3000 421
118	(3)118 7623 815	-(6)936 6242 563	(8)787 0792 070
119	(3)115 7774 520	-(6)905 4663 274	(8)754 5552 728
120	(3)112 8917 233	-(6)875 5933 604	(8)723 6308 764

TABLE 49

Coefficients for the Cubic

Description: The coefficients $A, B, C, D, E, F, G, H, I, J$ are computed to ten significant figures for the range $p = 4$ to $p = 60$ by unit intervals. The cubic,

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ ,$$

is obtained from the formulas,

$$a_0 = A\, m_0 + B\, m_1 + C\, m_2 + D\, m_3 \ ,$$

$$a_1 = B\, m_0 + E\, m_1 + F\, m_2 + G\, m_3 \ ,$$

$$a_2 = C\, m_0 + F\, m_1 + H\, m_2 + I\, m_3 \ ,$$

$$a_3 = D\, m_0 + G\, m_1 + I\, m_2 + J\, m_3 \ ,$$

where m_0, m_1, m_2, m_3 are the moments computed for the range $x = 1$ to $x = p$.

(The numbers in parentheses denote the number of ciphers between the decimal point and the first significant figure.)

p	A	B	C
4	69.000 0000 00	−104.166 6666 67	45.000 0000 00
5	24.200 0000 00	−30.833 3333 33	11.000 0000 00
6	13.000 0000 00	−14.277 7777 78	4.333 3333 33
7	8.428 5714 29	−8.119 0476 19	2.142 8571 43
8	6.071 4285 71	−5.202 3809 52	1.214 2857 14
9	4.674 6031 75	−3.604 4973 54	.753 9682 540
10	3.766 6666 67	−2.638 8888 89	.500 0000 000
11	3.136 3636 36	−2.012 6262 63	.348 4848 485
12	2.676 7676 77	−1.584 1750 84	.252 5252 525
13	2.328 6713 29	−1.278 5547 79	.188 8111 888
14	2.056 9430 57	−1.053 1135 53	.144 8551 449
15	1.839 5604 40	−.882 1733 822	.113 5531 136
16	1.662 0879 12	−.749 5421 245	(1)906 5934 066
17	1.514 7058 82	−.644 6078 431	(1)735 2941 176
18	1.390 5228 76	−.560 1851 852	(1)604 5751 634
19	1.284 5717 23	−.491 2710 698	(1)503 0959 752
20	1.193 1888 54	−.434 2965 256	(1)423 1166 151
21	1.113 6173 77	−.386 6610 972	(1)359 2314 119
22	1.043 7457 28	−.346 4342 675	(1)307 5871 497
23	.981 9311 124	−.312 1588 556	(1)265 3867 871
24	.926 8774 704	−.282 7184 892	(1)230 5665 349
25	.877 5494 071	−.257 2463 768	(1)201 5810 277
26	.833 1103 679	−.235 0613 155	(1)177 2575 251
27	.792 8774 929	−.215 6220 323	(1)156 6951 567
28	.756 2881 563	−.198 4940 985	(1)139 1941 392
29	.722 8748 263	−.183 3256 144	(1)124 2052 966
30	.692 2459 405	−.169 8291 066	(1)111 2935 596
31	.664 0711 902	−.157 7678 902	(1)100 1112 347
32	.638 0700 779	−.146 9456 804	(2)903 7819 800
33	.614 0029 326	−.137 1985 989	(2)818 6705 767
34	.591 6637 916	−.128 3889 656	(2)743 9192 686
35	.570 8747 135	−.120 4004 329	(2)677 9984 721
36	.551 4811 985	−.113 1341 425	(2)619 6417 961
37	.533 3484 745	−.106 5056 653	(2)567 7946 854
38	.516 3584 637	−.100 4425 478	(2)521 5742 058
39	.500 4072 899	−(1)948 8233 172	(2)480 2373 223
40	.485 4032 170	−(1)897 7094 503	(2)443 1557 063
41	.471 2649 353	−(1)850 6138 705	(2)409 7955 959
42	.457 9201 287	−(1)807 1264 779	(2)379 7015 992
43	.445 3042 703	−(1)766 8881 506	(2)352 4835 913
44	.433 3596 069	−(1)729 5833 303	(2)327 8060 567
45	.422 0342 965	−(1)694 9338 345	(2)305 3793 751
46	.411 2816 742	−(1)662 6936 708	(2)284 9526 611
47	.401 0596 249	−(1)632 6446 706	(2)266 3078 519
48	.391 3300 442	−(1)604 5927 982	(2)249 2548 052
49	.382 0583 738	−(1)578 3650 185	(2)233 6272 159
50	.373 2132 002	−(1)553 8066 290	(2)219 2792 010

p	A	B	C
51	.364 7659 064	−(1)530 7789 783	(2)206 0824 330
52	.356 6903 685	−(1)509 1575 092	(2)193 9237 233
53	.348 9626 910	−(1)488 8300 748	(2)182 7029 796
54	.341 5609 753	−(1)469 6954 845	(2)172 3314 709
55	.334 4651 156	−(1)451 6622 441	(2)162 7303 514
56	.327 6566 201	−(1)434 6474 629	(2)153 8293 991
57	.321 1184 527	−(1)418 5759 010	(2)145 5659 350
58	.314 8348 929	−(1)403 3791 375	(2)137 8838 947
59	.308 7914 116	−(1)388 9948 425	(2)130 7330 278
60	.302 9745 609	−(1)375 3661 379	(2)124 0682 068

p	D	E	F
4	−5.833 3333 33	161.388 8889	−70.833 3333 3
5	−1.166 6666 67	41.349 2063 5	−15.178 5714 3
6	− .388 8888 889	16.877 2045 9	−5.324 0740 74
7	− .166 6666 667	8.580 0264 55	−2.373 0158 73
8	−(1)833 3333 333	4.971 0197 21	−1.224 0259 74
9	−(1)462 9629 630	3.143 1377 26	−.697 4506 975
10	−(1)277 7777 778	2.116 1939 91	−.427 3504 274
11	−(1)176 7676 768	1.493 9458 69	−.276 8065 268
12	−(1)117 8451 178	1.094 5289 28	−.187 3496 873
13	−(2)815 8508 159	.826 2015 762	−.131 4102 564
14	−(2)582 7505 828	.639 1627 163	−(1)949 4182 288
15	−(2)427 3504 274	.504 7428 516	−(1)703 2727 621
16	−(2)320 5128 205	.405 6230 004	−(1)532 1798 085
17	−(2)245 0980 392	.330 9110 194	−(1)410 2167 183
18	−(2)190 6318 083	.273 5184 230	−(1)321 3507 625
19	−(2)150 4987 960	.228 6944 150	−(1)255 3453 598
20	−(2)120 3990 368	.193 1724 542	−(1)205 4805 696
21	−(3)974 6588 694	.164 6555 270	−(1)167 2357 222
22	−(3)797 4481 659	.141 4959 500	−(1)137 5028 480
23	−(3)658 7615 283	.122 4906 736	−(1)114 1032 880
24	−(3)548 9679 403	.106 7472 070	−(2)954 8288 534
25	−(3)461 1330 698	(1)935 9387 258	−(2)805 1529 791
26	−(3)390 1895 206	(1)825 1854 435	−(2)683 7240 720
27	−(3)332 3836 657	(1)731 2608 280	−(2)584 3730 556
28	−(3)284 9002 849	(1)651 0827 916	−(2)502 4485 670
29	−(3)245 6036 939	(1)582 2231 745	−(2)434 4044 597
30	−(3)212 8565 347	(1)522 7513 012	−(2)377 5093 574
31	−(3)185 3911 754	(1)471 1189 217	−(2)329 6393 894
32	−(3)162 2172 785	(1)426 0746 079	−(2)289 1284 434
33	−(3)142 5545 780	(1)386 5993 829	−(2)254 6587 224
34	−(3)125 7834 512	(1)351 8578 323	−(2)225 1798 979
35	−(3)111 4081 996	(1)321 1606 298	−(2)199 8487 873
36	−(4)990 2951 079	(1)293 9355 623	−(2)177 9839 241
37	−(4)883 2361 774	(1)269 7049 446	−(2)159 0310 414
38	−(4)790 2639 482	(1)248 0678 813	−(2)142 5366 316
39	−(4)709 2112 355	(1)228 6862 337	−(2)128 1275 351
40	−(4)638 2901 120	(1)211 2734 458	−(2)115 4950 665

p	D	E	F
41	−(4)576 0179 059	(1)195 5855 872	−(2)104 3825 864
42	−(4)521 1590 577	(1)181 4141 318	−(3)945 7570 433
43	−(4)472 6791 454	(1)168 5801 023	−(3)858 9451 054
44	−(4)429 7083 140	(1)156 9292 958	−(3)781 8737 837
45	−(4)391 5120 194	(1)146 3283 718	−(3)713 2599 200
46	−(4)357 4674 960	(1)136 6616 307	−(3)652 0134 174
47	−(4)327 0447 304	(1)127 8283 489	−(3)597 2046 543
48	−(4)299 7910 028	(1)119 7405 642	−(3)548 0381 072
49	−(4)275 3182 679	(1)112 3212 277	−(3)503 8306 506
50	−(4)253 2928 065	(1)105 5026 561	−(3)463 9937 685
51	−(4)233 4267 040	(2)992 2522 916	−(3)428 0188 665
52	−(4)215 4708 037	(2)934 3629 062	−(3)395 4651 013
53	−(4)199 2088 563	(2)880 8921 721	−(3)365 9492 561
54	−(4)184 4526 447	(2)831 4262 696	−(3)339 1372 874
55	−(4)171 0379 069	(2)785 5970 435	−(3)314 7372 425
56	−(4)158 8209 135	(2)743 0762 284	−(3)292 4933 039
57	−(4)147 6755 863	(2)703 5704 944	−(3)272 1807 658
58	−(4)137 4910 631	(2)666 8171 844	−(3)253 6017 829
59	−(4)128 1696 351	(2)632 5806 352	−(3)236 5817 616
60	−(4)119 6249 927	(2)600 6489 945	−(3)220 9662 884

p	G	H	I
4	9.277 7777 778	31.500 0000 00	−4.166 6666 667
5	1.638 8888 889	5.696 4285 71	−.625 0000 000
6	.489 1975 309	1.728 1746 03	−.162 0370 370
7	.189 8148 148	.678 5714 286	−(1)555 5555 556
8	(1)867 0033 670	.312 7705 628	−(1)227 2727 273
9	(1)443 3221 100	.161 0750 361	−(1)105 2188 552
10	(1)246 3739 964	(1)900 3496 503	−(2)534 1880 342
11	(1)146 0113 960	(1)536 1305 361	−(2)291 3752 914
12	(2)910 8175 775	(1)335 7753 358	−(2)168 3501 684
13	(2)592 4630 925	(1)219 1558 442	−(2)101 9813 520
14	(2)399 0698 844	(1)148 0504 789	−(3)642 7396 133
15	(2)276 8700 073	(1)102 9770 883	−(3)418 9710 072
16	(2)197 0266 289	(2)734 4435 750	−(3)281 1515 969
17	(2)143 3321 867	(2)535 3457 172	−(3)193 4984 520
18	(2)106 3047 051	(2)397 7468 180	−(3)136 1655 773
19	(3)802 0087 354	(2)300 5508 423	−(4)977 2649 092
20	(3)614 3455 464	(2)230 5519 670	−(4)713 8282 815
21	(3)477 0563 504	(2)179 2607 342	−(4)529 7059 073
22	(3)375 0317 824	(2)141 0888 882	−(4)398 7240 829
23	(3)298 1318 199	(2)112 2790 253	−(4)304 0437 823
24	(3)239 4188 385	(3)902 5595 982	−(4)234 6016 839
25	(3)194 0625 129	(3)732 2387 757	−(4)182 9893 134
26	(3)158 6457 484	(3)599 1052 093	−(4)144 1586 899
27	(3)130 7157 437	(3)494 0142 871	−(4)114 6150 571
28	(3)108 4881 693	(3)410 3053 046	−(5)919 0331 771
29	(4)906 4889 024	(3)343 0670 491	−(5)742 7531 065
30	(4)762 1897 299	(3)288 6370 726	−(5)604 7060 644

p	G	H	I
31	(4) 644 6143 698	(3) 244 2553 521	−(5) 495 6983 299
32	(4) 548 1597 365	(3) 207 8215 248	−(5) 408 9511 222
33	(4) 468 5267 177	(3) 177 7226 042	−(5) 339 4156 620
34	(4) 402 3883 293	(3) 152 7092 964	−(5) 283 2960 613
35	(4) 347 1491 475	(3) 131 8062 464	−(5) 237 7130 860
36	(4) 300 7691 346	(3) 114 2462 443	−(5) 200 4645 968
37	(4) 261 6333 886	(4) 994 2153 905	−(5) 169 8531 110
38	(4) 228 4549 781	(4) 868 4748 736	−(5) 144 5604 783
39	(4) 200 2018 476	(4) 761 3518 525	−(5) 123 5559 644
40	(4) 176 0413 887	(4) 669 7069 867	−(5) 106 0282 578
41	(4) 155 2980 924	(4) 590 9918 365	−(6) 913 3476 310
42	(4) 137 4209 637	(4) 523 1265 867	−(6) 789 6349 360
43	(4) 121 9582 775	(4) 464 4052 586	−(6) 685 0422 397
44	(4) 108 5378 981	(4) 413 4217 613	−(6) 596 2650 055
45	(5) 968 5184 014	(4) 369 0118 490	−(6) 520 6276 854
46	(5) 866 4408 821	(4) 330 2073 010	−(6) 455 9534 388
47	(5) 777 0093 339	(4) 296 1995 495	−(6) 400 4629 352
48	(5) 698 4326 605	(4) 266 3106 562	−(6) 352 6952 975
49	(5) 629 2039 812	(4) 239 9700 363	−(6) 311 4460 044
50	(5) 568 0508 727	(4) 216 6957 015	−(6) 275 7178 590
51	(5) 513 8951 063	(4) 196 0790 767	−(6) 244 6820 797
52	(5) 465 8199 257	(4) 177 7726 528	−(6) 217 6472 765
53	(5) 423 0433 383	(4) 161 4871 545	−(6) 194 0346 002
54	(5) 384 8962 284	(4) 146 9470 334	−(6) 173 3577 488
55	(5) 350 8043 493	(4) 133 9561 487	−(6) 155 2068 121
56	(5) 320 2734 532	(4) 122 3196 643	−(6) 139 2351 667
57	(5) 292 8769 663	(4) 111 8756 310	−(6) 125 1488 019
58	(5) 268 2457 390	(4) 102 4838 617	−(6) 112 6975 927
59	(5) 246 0594 916	(5) 940 2269 581	−(6) 101 6681 399
60	(5) 226 0396 541	(5) 863 8628 886	−(7) 918 7787 461

p	J	p	J	p	J
		21	(5) 160 5169 416	41	(7) 144 9758 144
		22	(5) 115 5721 980	42	(7) 122 4240 211
		23	(6) 844 5660 620	43	(7) 103 7942 787
4	.555 5555 556	24	(6) 625 6044 903	44	(8) 883 3555 638
5	(1) 694 4444 444	25	(6) 469 2033 678	45	(8) 754 5328 774
6	(1) 154 3209 877	26	(6) 355 9473 824	46	(8) 646 7424 663
7	(2) 462 9629 630	27	(6) 272 8929 932	47	(8) 556 1985 210
8	(2) 168 3501 684	28	(6) 211 2719 947	48	(8) 479 8575 476
9	(3) 701 4590 348	29	(6) 165 0562 459	49	(8) 415 2613 392
10	(3) 323 7503 238	30	(6) 130 0443 149	50	(8) 360 4155 020
11	(3) 161 8751 619	31	(6) 103 2704 854	51	(8) 313 6949 739
12	(4) 863 3341 967	32	(7) 826 1638 832	52	(8) 273 7701 591
13	(4) 485 6254 856	33	(7) 665 5209 059	53	(8) 239 5488 892
14	(4) 285 6620 504	34	(7) 539 6115 453	54	(8) 210 1306 046
15	(4) 174 5712 530	35	(7) 440 2094 185	55	(8) 184 7700 144
16	(4) 110 2555 282	36	(7) 361 1974 716	56	(8) 162 8481 482
17	(5) 716 6609 334	37	(7) 297 9879 141	57	(8) 143 8491 976
18	(5) 477 7739 556	38	(7) 247 1119 288	58	(8) 127 3419 126
19	(5) 325 7549 697	39	(7) 205 9266 073	59	(8) 112 9645 999
20	(5) 226 6121 528	40	(7) 172 4036 712	60	(8) 100 4129 777

ERRATA VOLUME I

Page vii. Line 1, for 19 read 10.

Page 12. Read last two lines: "to date as far back as 1900 B. C."

Page 53. Read "in which we write,

$$\Psi(s) = \psi[g^{-1}(s+g)]/g^1[g^{-1}(s+g)] \ ,$$

where $g^{-1}(t)$ is the function inverse to $g(t)$."

Page 86. Change caption of table to read $\log_{10} \Gamma(x)$.

Page 93. In fourth line below the table replace "ten values" by "nine values."

Page 163. Horsburgh. For London read Edinburgh.

Page 164. Ives. For London read New York and London.

Page 192. Table 2. For points read places.

Page 229. For $x = 1.564$, correct last three figures of $\log \Gamma(x)$ to read 446 and for $\Gamma(x)$ correct last figure to 5.

Page 237. For $x = 1.986$ correct last figure of $\Gamma(x)$ to read 0.

Page 245. Read $\text{Log}\, \Gamma(7.56) = 3.3229\ 7194\ 5671$.

Page 270. For $r = 1.0$ in first table read 1.0000 0000 0000.

Page 280. Replace $R(x) < 1$ by $|x| < 1$.

Page 286. Replace formula for $\Psi^{(m)}(nx)$ by,

$$\Psi^{(m)}(nx) = (1/n^{m+1})[\Psi^{(m)}(x+1-1/n) + \Psi^{(m)}(x+1-2/n) + \cdots + \Psi^{(m)}(x+1/n) + \Psi^{(m)}(x)] \ .$$

Page 289. Table 7. Read .0001 for .001.

Page 322. From $x = 1.431$ to $x = 1.439$ inclusive add one unit to the sixth decimal in each value of $\Psi(x)$. Thus change .03006 39111 to .03006 49111. The corresponding values of the logarithms should be changed to the following: 8.47805 99240; 8.46344 25443; 8.44833 04245; 8.43268 83717; 8.41647 73000; 8.39965 36327; 8.38216 85866; 8.36396 73167; 8.34498 78566.

Page 329. Read $\text{Log}|\Psi(1.794)|$ as 9.44802 59585.

Page 332. From $x = 1.927$ to $x = 1.929$ inclusive, reduce end figures both in $\Psi(x)$ and its logarithm by one unit.

INDEX OF NAMES

INDEX OF SUBJECTS